Reaction-Diffusion Models: Dynamics and Control

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Reaction-diffusion equations are ubiquitous and its applications include combustion and population dynamics modelling.

There is an extensive mathematical literature addressing the analysis of steady state solutions, traveling waves, and their stability, among other properties.

Control problems arise in many applications involving these models. And, often times, they involve control and/or state constraints, as intrinsic requirements of the processes under consideration.

In this lecture we shall present the recent work of our team on the Fisher-KPP and Allen-Canh or bistable model. We show that these systems can be controlled fulfilling the natural constraints if time is large enough. This is in contrast with the unconstrained case where parabolic systems can be controlled in an arbitrarily small time, thanks to the infinite velocity of propagation.

The method of proof combines various methods and, in particular, employs phase-plane analysis techniques allowing to build paths of steady-state solutions. The control strategy consists then in building trajectories of the time-evolving system in the vicinity of those paths.

We shall conclude our lecture with a number of challenging open problems.