hp-Adaptive FEM for Elliptic Obstacle Problems CG vs. IPDG

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Abstract

The solution of an elliptic obstacle problem is typically only of reduced regularity with a singularity across the a priori unknown free boundary. It is therefore apparent that *hp*-adaptive FEM is an appropriate tool for solving that problem approximately.
Here, the non-penetration condition is enforced weakly by a Lagrange multiplier λ leading to a mixed method. On the discrete level the Lagrange multiplier λ_{hp} is a linear combination of basis functions which are biorthogonal to the basis functions of the primal variable. This allows to use the same mesh and polynomial degree for both the primal and dual variable. For both the continuous Galerkin (cG) and the interior penalty discontinuous Galerkin (IPDG) method a residual based a posteriori error estimator is derived. These consist of a discretization error of a linear variational equality problem, a consistency term in λ_{hp}, a complementarity term and an obstacle penetration term. For the IPDG an additional consistency term in u_{hp} arises.
In the second part of the talk we describe how the connectivity matrices for biorthogonal basis functions can be computed efficiently, even in the presents of multiple order hanging nodes and anisotropic meshes.
In the numerical experiments we compare cG with IPDG and show the applicability of the derived error estimations and the superiority of *hp*-adaptivity compared to low order uniform and adaptive approaches.
The results on cG, IPDG are based on a joint work with Andreas Schröder, Ernst P. Stephan, respectively.

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