PERIODIC PROBLEM FOR THE DAMPED DISCRETE WAVE EQUATION

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In this presentation is studied the time asymptotic behavior of solutions to the periodic Cauchy problem for the following linear damped discrete wave equation with step h:

$$\begin{gathered}\partial_t^2 u_j + \partial_t u_j - \frac{1}{h^2}\left(u_{j+1} - 2u_j + u_{j-1}\right)(t) = f_j(t),\ j \in \mathbf{Z},\ t > 0;\\ u_j(0) = \phi_j,\ \partial_t u_j(0) = \psi_j,\ j \in \mathbf{Z}.\end{gathered} \tag{0.1}$$

The solutions of equation (0.1) which satisfy the periodic boundary condition $u_j(t) = u_{j+k}(t)$ for all $j \in \mathbf{Z}$ and $t > 0$, with $k \in \mathbf{Z}$ such that $kh = 2\pi$, are considered. The sequences $\{f_j\}$, $\{\phi_j\}$ and $\{\psi_j\}$ are k - periodic with respect to j: $f_{j+k}(t) = f_j(t)$, $\phi_j = \phi_{j+k}$, $\psi_j = \psi_{j+k}$, $k \in \mathbf{Z}$, for all $j \in \mathbf{Z}$, $t \geq 0$. Eq. (0.1) is interpolated to the whole real axis, using the Whittaker–Kotelnikov interpolation, which is defined by $v_h(x) = [\mathrm{Kot}_h v_j](x) = \sum_{j=-\infty}^{\infty} v_j \operatorname{sinc}\pi(x/h - j)$ for a given sequence $\{v_j\} \in \boldsymbol{\ell_2}$. Here $\operatorname{sinc} x \equiv (\sin x)/x$, $x \neq 0$ and $\operatorname{sinc} 0 = 1$. Defining $D_h = (E_{h/2} - E_{-h/2})/h$, where $E_y : L_2(\mathbb{R}) \longrightarrow L_2(\mathbb{R})$ is the operator of translation by $y \in \mathbb{R}$ such that $[E_y u](t,x) = u(t, x + y)$, $u \in L_2(\mathbb{R}_x)$, the solutions $u(t,x)$ of the corresponding interpolated equation

$$\begin{gathered}\mathcal{L}u = f(t,x),\ x \in \Omega,\ t > 0,\\ u(0,x) = \phi(x),\ \partial_t u(0,x) = \psi(x),\ x \in \Omega,\end{gathered} \tag{0.2}$$

are considered now. Here $\mathcal{L} = \partial_t^2 + \partial_t - {D_h}^2$, $u(t, jh) = u_j(t)$, $f(t, jh) = f_j(t)$, $\Omega = [0, 2\pi]$, $\phi(jh) = \phi_j$ and $\psi(jh) = \psi_j$. The periodic boundary condition $u(t,x) = u(t, 2\pi + x)$ for all $x \in \mathbb{R}$ and $t > 0$ is satisfied. The function f and the initial data ϕ and ψ are 2π - periodic with respect to the spacial variable x. The Sobolev space for the case of the periodic functions is defined as follows

$$\mathbf{H}^s = \left\{ \varphi \in \mathbf{D}' : \|\varphi\|^2_{\mathbf{H}^s} = \sum_{n=-\infty}^{\infty} \langle n \rangle^{2s} |\widehat{\varphi}_n|^2 < \infty \right\}$$

for any $s \in \mathbb{R}$, where $\langle n \rangle = \sqrt{1 + n^2}$ and $\widehat{\varphi}_n = (1/2\pi) \int_\Omega e^{-inx} \varphi(x) dx$ are the Fourier coefficients of the 2π - periodic function $\varphi(x)$. Some simple estimates for the solutions of the periodic problem (0.2) in the spaces $\mathbf{H}^s$ with $s > 1/2$ are collected following the methods of [1].

REFERENCES

[1] N. Hayashi, E.I. Kaikina, P.I. Naumkin and I.A. Shishmarev, *Asymptotics for dissipative nonlinear equations*, Springer, Berlin Heidelberg New York (2006).