Bilevel learning for PDE inverse problems

Speaker:
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Abstract:
In recent years, the integration of optimization techniques with machine learning paradigms has led to significant advances in solving inverse problems, particularly through the optimal selection of parameters or observation strategies. A rigorous and systematic approach to tackle such problems is provided by bilevel optimization, where the lower-level problem corresponds to a model-based inverse problem—often governed by a partial differential equation (PDE)—and the upper-level objective encodes a data-driven loss functional, typically defined over a training set. Unlike classical variational models, the presence of PDE constraints introduces substantial analytical and computational challenges. In this talk, I will present a bilevel optimization framework for PDE-constrained inverse problems, and discuss recent results on the existence of optimal parameters. Furthermore, I will explore the possibility of reformulating the bilevel problem as a single-level optimization problem under suitable assumptions. Finally, I will derive and discuss first-order optimality conditions for the resulting problem, highlighting key mathematical difficulties and future directions.