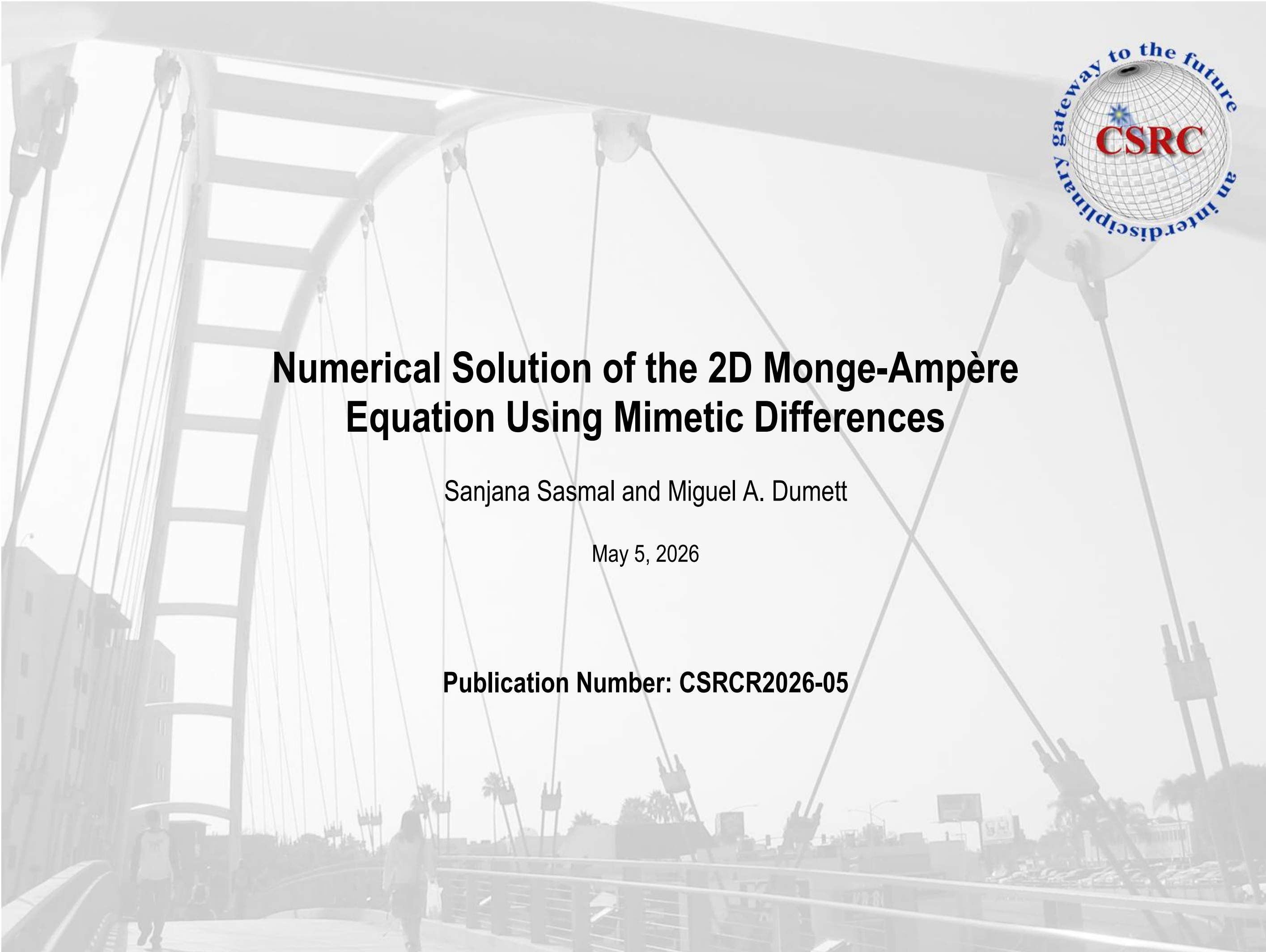

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May 5, 2026

Publication Number: CSRCR2026-05

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Numerical Solution of the 2D Monge–Ampère Equation Using Mimetic Differences

Sanjana Sasmal * Miguel A. Dumett‡

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Abstract

In this work, we numerically solve the two-dimensional Monge–Ampère equation using Method 2 from Benamou, Froese, and Oberman (2010). This approach reduces the nonlinear equation into a sequence of Poisson problems, which are solved using mimetic difference operators from the MOLE library. The numerical solution is validated against the corresponding analytical solution, and convergence behavior is analyzed.

1 Introduction

The Monge–Ampère equation is a fully nonlinear partial differential equation of the form

$$\det(D^2u) = f(x, y), \tag{1}$$

where D^2u is the Hessian matrix. Because of the determinant structure, this equation is more difficult to solve than standard linear PDEs. I arises naturally in different areas of geometry.

Instead of solving the nonlinear equation directly, we use an iterative approach that transforms the problem into a sequence of Poisson equations. This makes the computation more manageable.

2 Problem Setup

We consider the domain

$$\Omega = [-1, 1] \times [-1, 1]. \tag{2}$$

The Monge–Ampère equation is

$$u_{xx}u_{yy} - u_{xy}^2 = f(x, y). \tag{3}$$

We assume the exact solution

$$u(x, y) = \exp\left(\frac{x^2 + y^2}{2}\right), \tag{4}$$

which gives the corresponding right-hand side

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$$f(x,y) = (1 + x^2 + y^2)\exp(x^2 + y^2). \tag{5}$$

Dirichlet boundary conditions are imposed using the exact solution.

3 Numerical Method

We use Method 2 from Benamou, Froese, and Oberman (2010), which reduces the nonlinear equation to a sequence of Poisson problems. At each iteration, we solve

$$\Delta u^{k+1} = \sqrt{(u^k_{xx})^2 + (u^k_{yy})^2 + 2(u^k_{xy})^2 + 2f}. \tag{6}$$

The algorithm is:

- Initialize u^0
- Compute derivatives
- Build the right-hand side
- Solve the Poisson equation
- Apply relaxation
- Repeat until convergence

The stopping condition is

$$\|u^{k+1} - u^k\|_\infty < \varepsilon. \tag{7}$$

4 Mimetic Discretization

We use MOLE [3] operators for the spatial discretization:

- Gradient operator G
- Divergence operator D
- Laplacian operator $L = DG$

Second derivatives are approximated as

$$u_{xx} \approx D_1(G_1 u), \tag{8}$$
$$u_{yy} \approx D_2(G_2 u), \tag{9}$$
$$u_{xy} \approx D_1(G_2 u). \tag{10}$$

5 Implementation Details

The domain is discretized using a uniform grid. The main numerical parameters are:

- Grid size: 40×40
- Tolerance: 10^{-8}
- Maximum iterations: 500
- Relaxation parameter: 0.7

Dirichlet boundary conditions are enforced using values from the exact solution.

6 Results

The numerical solution closely matches the analytical solution.

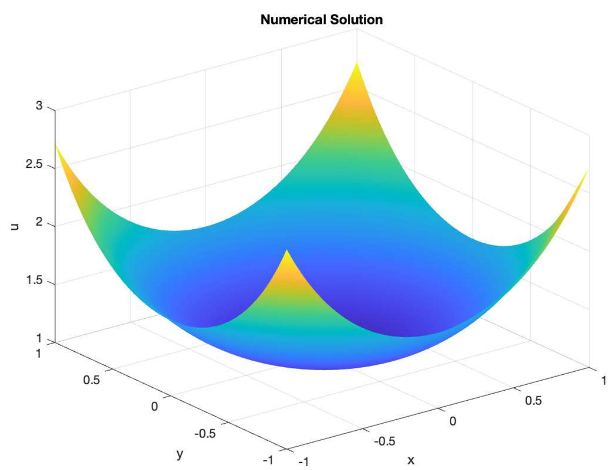

Figure 1: Numerical solution obtained using Method 2.

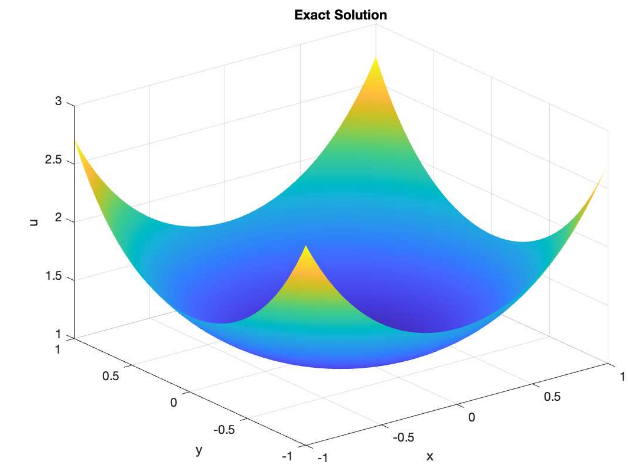

Figure 2: Exact analytical solution.

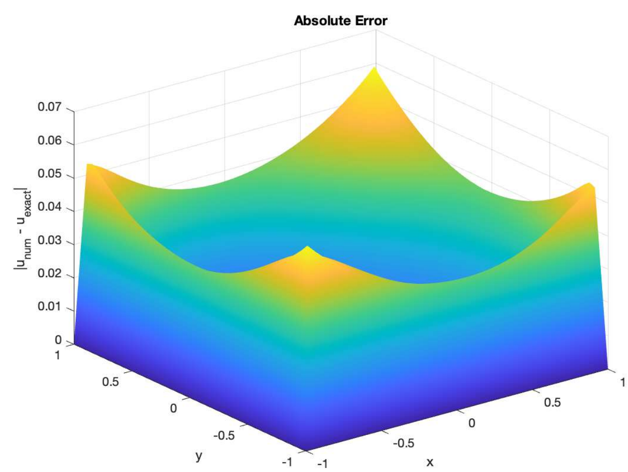

Figure 3: Absolute error between the numerical and exact solutions.

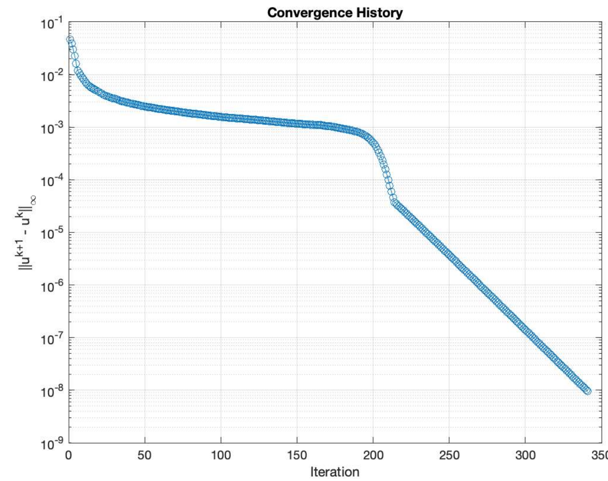

Figure 4: Convergence history showing $\|u^{k+1} - u^k\|_\infty$.

7 Discussion

The key idea of this work is transforming a nonlinear PDE into a sequence of linear Poisson problems. This makes the computation more manageable and aligns well with MOLE [4, 2], since MOLE provides mimetic operators for gradient, divergence, and Laplacian calculations.

The main challenges included:

- Choosing a relaxation parameter for stability
- Setting up the MOLE operators correctly
- Handling the nonlinear iteration
- Enforcing Dirichlet boundary conditions consistently

8 Conclusion

This work demonstrates a numerical solution of the 2D Monge–Ampère equation using Method 2 and mimetic differences. The numerical solution was compared with an exact analytical solution, and the results show that the method can approximate the expected solution while producing convergence behavior over iterations.

Future work could include testing different grid sizes, trying more complex right-hand side functions, and improving the convergence speed.

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