

Computational Techniques in Complex Nonlinear Dynamical Systems: Adventures in Applied Mathematics

Speaker:

Dr. Efstathios Charalampidis, Mathematics and Statistics, Computational Science Research Center, San Diego State University, USA

Abstract:

Complex systems are ubiquitous in nature and human-designed environments. The overarching goal of our research is to leverage advanced computational methods with fundamental theoretical analysis to model the nonlinear behavior of systems that are not otherwise amenable to integrable systems techniques. Examples include: Studies of superfluidity and superconductivity in ultra-cold atomic physics (e.g., Bose-Einstein condensation), extreme and rare events (e.g., tsunamis and rogue waves), and collapse phenomena in optics (e.g., light propagation through a medium without diffraction). We have developed computational methods for bifurcation analysis that explain the structure of the parameter space of these systems and continuation methods (pseudo-arclength and Deflated Continuation Method (DCM)) for efficient tracking of solution branches and connecting them to physical observations. The objective is to enable technological innovations, such as the discovery of new materials and development of devices for precision measurements (e.g., interferometers), or to predict extreme phenomena based on the features of the eigenvalue spectra of the system.

In this talk, we will present a wide palette of results that were obtained by using the developed computational methods. Specifically, inconspicuous solutions of the Nonlinear Schrödinger (NLS) equation were discovered by developing DCM specifically for NLS to uncover previously unknown behavior and weakly nonlinear unstable solutions that are potential targets for experimental verification. Furthermore, a novel Kuznetsov-Ma breather (time-periodic) solution to the discrete and non-integrable NLS equation relevant to predicting periodic extreme and rare events in optical systems was discovered by employing pseudo-arclength continuation. The combination of perturbation methods with pseudo-arclength continuation enabled the elucidation of collapsing waveforms associated with the 1D focusing NLS and Korteweg-de Vries equations. Simultaneously, exploiting the symmetries of the underlying PDEs (associated with conservation laws) enables robust convergence in fixed-point iterations. Future research will focus on the development of computational tools for numerical simulation of complex nonlinear systems with

the ultimate goal being the dissemination of a library written in the open-source software FreeFEM that can be used to study bifurcations and perform stability analysis of such systems.