Simulating The Nonlinear Klein-Gordon-Maxwell System In The Relativistic Limit With Mimetic Differences

The nonlinear Klein-Gordon-Maxwell (KGM) system of equations is particularly useful in modeling the interactions between a new class of an X-ray free electron laser and solid density plasmas. The model also has use in astrophysics with describing the EM wave-plasma interaction with X-rays and γ-rays produced by white dwarf cores and neutron stars. The goal here is to simulate the plasma wake-field generation which is a phenomenon studied in designing next generation plasma/particle accelerators with applications in high energy physics experiments, and the medical and industrial fields. These simulations utilize mimetic differences to solve the explicit 4th order Runge-Kutta time integration scheme in two dimensions, showcasing the nonlinear coupling between the three equations used in the model.

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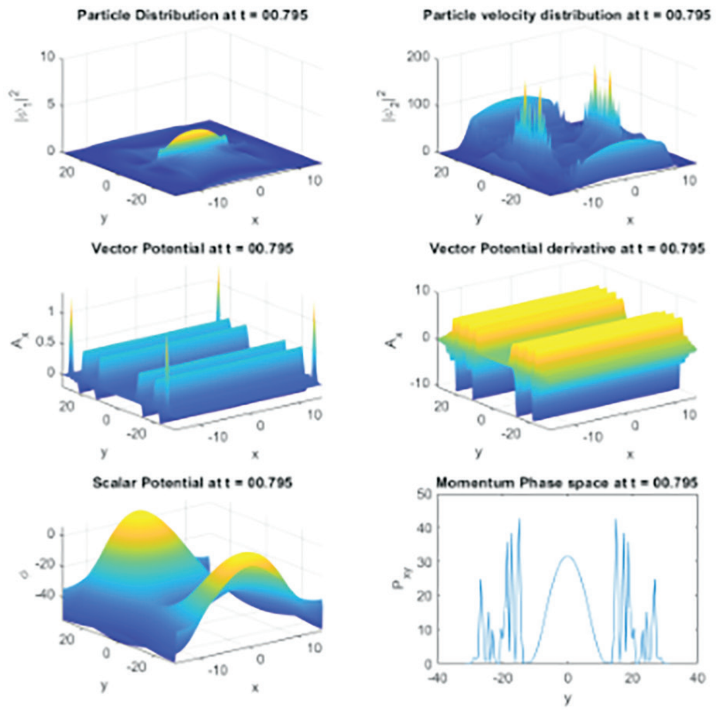

Figure 5: Simulation results showing a spike in particle velocity following the wake of a CPEM wave vector potential. The phase space momentum plot shows a spiking that is reminiscent of the 1D results of [2] where the momentum has a smooth middle part with a sharp and jagged outer part due to the nonlinearity of the system.

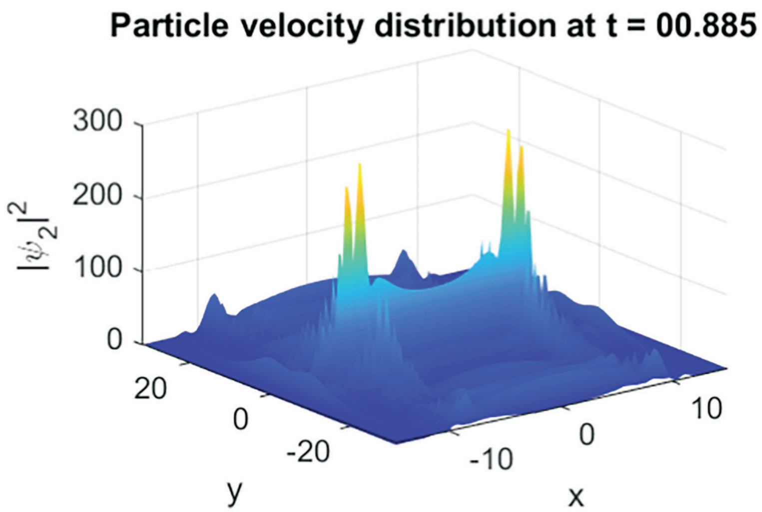

Figure 6: Particle velocity distribution showing nearly the maximum velocity the particles reach before slowing down again, reminiscent of wake-field acceleration

Klein-Gordon Maxwell

$$\partial_{tt}\psi = \nabla^2\psi - (c^4 - \phi^2 + |\mathbf{A}|^2)\psi - 2i\phi\partial_t\psi$$

$$\partial_{tt}\mathbf{A} = c^2\nabla^2\mathbf{A} - \frac{|\psi|^2}{c}\mathbf{A}$$

$$\nabla^2\phi = -\frac{1}{c^2}\left[i(\psi^*\partial_t\psi - \psi\partial_t\psi^*) + \phi|\psi|^2\right]$$