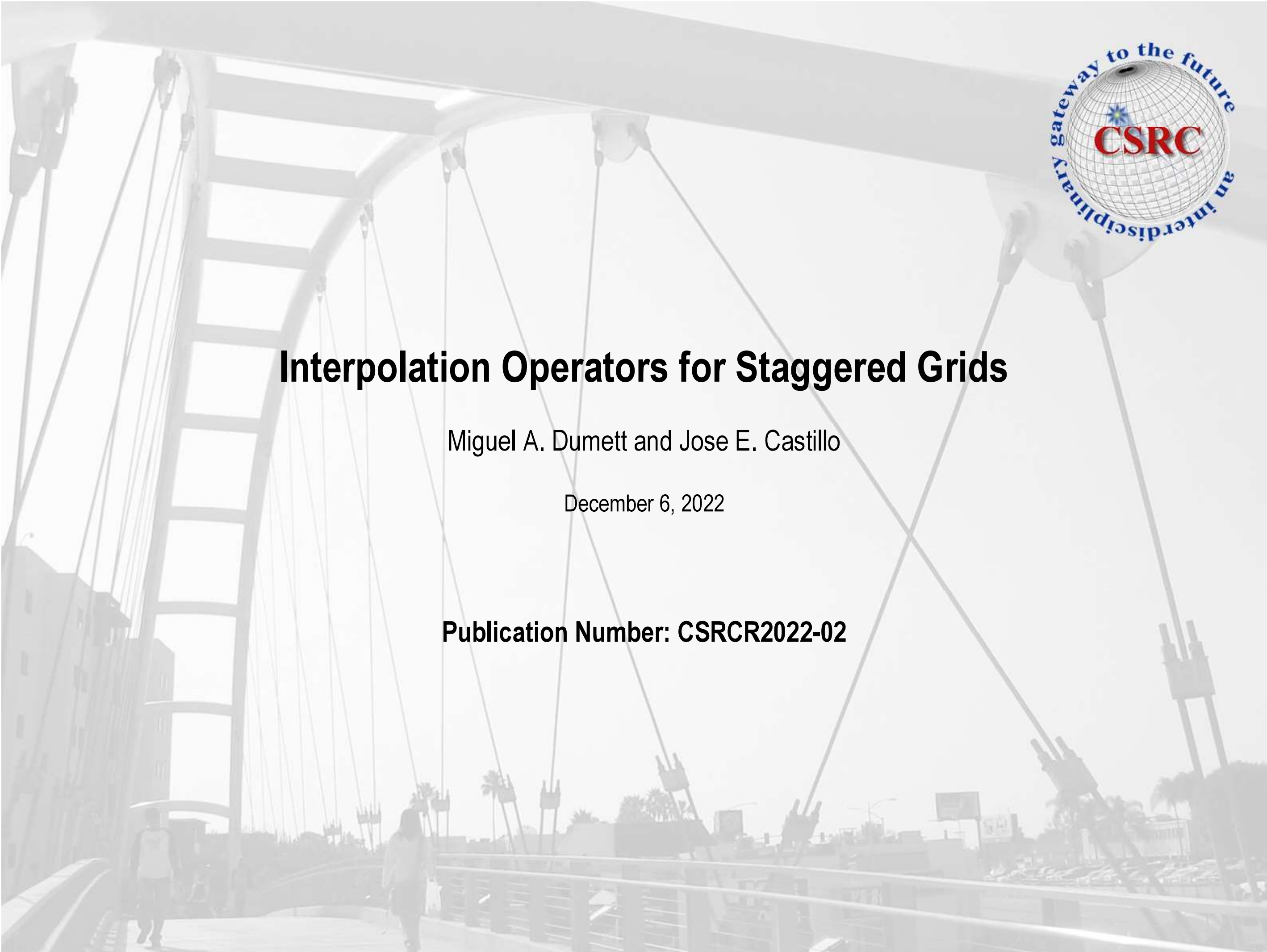

Interpolation Operators for Staggered Grids

Miguel A. Dumett and Jose E. Castillo

December 6, 2022

Publication Number: CSRCR2022-02

Computational Science &
Engineering Faculty and Students
Research Articles

Database Powered by the
Computational Science Research Center
Computing Group & Visualization Lab

COMPUTATIONAL SCIENCE & ENGINEERING

SAN DIEGO STATE UNIVERSITY

Computational Science Research Center
College of Sciences
5500 Campanile Drive
San Diego, CA 92182-1245
(619) 594-3430

and

$$I_x^{f\to c,(2)} = I_G^2 = \frac{1}{2}\begin{bmatrix} 2 & & & & \\ 1 & 1 & & & \\ & \ddots & \ddots & & \\ & & & 1 & 1 \\ & & & & 2 \end{bmatrix}.$$

For accuracy $k = 4$, the generator for the interior scheme of the cell faces to cell centers interpolator $I_x^{f\to c,(4)}$ is

$$g_4 = \left[-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}\right]'.$$

The generator associated to the interpolation near the boundary is

$$g_{41} = \left[-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\right]'.$$

Thus, $I_x^{f\to c,(4)}$ is given by

$$I_x^{f\to c,(4)} = I_G^4 = \frac{1}{128}\begin{bmatrix} 128 & & & & & \\ 35 & 140 & -70 & 28 & -5 & \\ -8 & 72 & 72 & -8 & & \\ & \ddots & \ddots & \ddots & \ddots & \\ & & -8 & 72 & 72 & -8 \\ & -5 & 28 & -70 & 140 & 35 \\ & & & & & 128 \end{bmatrix}.$$

For accuracy $k = 6$, the generator for the interior scheme of the cell faces to cell centers interpolator $I_x^{f\to c,(6)}$ is

$$g_6 = \left[-\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}\right]'.$$

So, $I_x^{f\to c,(6)}$ is given by

$$I_x^{f\to c,(6)} = I_G^6 = \frac{1}{1024}\begin{bmatrix} 1024 & & & & & & & \\ 231 & 1386 & -1155 & 924 & -495 & 154 & -21 & \\ -21 & 378 & 945 & -420 & 189 & -54 & 7 & \\ 12 & -100 & 600 & 600 & -100 & 12 & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & 12 & -100 & 600 & 600 & -100 & 12 \\ & 7 & -54 & 189 & -420 & 945 & 378 & -21 \\ & -21 & 154 & -495 & 923 & -1155 & 1278 & 231 \\ & & & & & & & 1024 \end{bmatrix}.$$

For accuracy $k = 8$, the generator for the interior scheme of the cell faces to cell centers interpolator $I_x^{f\to c,(8)}$ is

$$g_8 = \left[-\frac{7}{2}, -\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}\right]'.$$

The generator associated to the interpolation near the boundary are

$$\begin{aligned} g_{81} &= \left[-\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2}, \frac{15}{2}\right]', \\ g_{82} &= \left[-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}, \frac{13}{2}\right]', \\ g_{83} &= \left[-\frac{5}{2}, -\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}\right]'. \end{aligned}$$

Hence, $I_x^{f\to c,(8)}$ is given by

$$I_x^{f\to c,(8)} = \begin{bmatrix} 1 & & & & & & & & & \\ \frac{6435}{32768} & \frac{6435}{4096} & -\frac{15015}{8192} & \frac{9009}{4096} & -\frac{32175}{16384} & \frac{5005}{4096} & -\frac{4095}{8192} & \frac{495}{4096} & -\frac{429}{32768} & \\ -\frac{429}{32768} & \frac{1287}{4096} & \frac{9009}{8192} & -\frac{3003}{4096} & \frac{9009}{16384} & -\frac{1287}{4096} & \frac{1001}{8192} & -\frac{117}{4096} & \frac{99}{32768} & \\ \frac{99}{32768} & -\frac{165}{4096} & \frac{3465}{8192} & \frac{3465}{4096} & -\frac{5775}{16384} & \frac{693}{4096} & -\frac{495}{8192} & \frac{55}{4096} & -\frac{45}{32768} & \\ -\frac{5}{2048} & \frac{49}{2048} & -\frac{245}{2048} & \frac{1225}{2048} & \frac{1225}{2048} & -\frac{245}{2048} & \frac{49}{2048} & -\frac{5}{2048} & & \\ & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \\ & & -\frac{5}{2048} & \frac{49}{2048} & -\frac{245}{2048} & \frac{1225}{2048} & \frac{1225}{2048} & -\frac{245}{2048} & \frac{49}{2048} & -\frac{5}{2048} \\ & -\frac{45}{32768} & \frac{55}{4096} & -\frac{495}{8192} & \frac{693}{4096} & -\frac{5775}{16384} & \frac{3465}{4096} & \frac{3465}{8192} & -\frac{165}{4096} & \frac{99}{32768} \\ & \frac{99}{32768} & -\frac{117}{4096} & \frac{1001}{8192} & -\frac{1287}{4096} & \frac{9009}{16384} & -\frac{3003}{4096} & \frac{9009}{8192} & \frac{1287}{4096} & -\frac{429}{32768} \\ & -\frac{429}{32768} & \frac{495}{4096} & -\frac{4095}{8192} & \frac{5005}{4096} & -\frac{32175}{16384} & \frac{9009}{4096} & -\frac{15015}{8192} & \frac{6435}{4096} & \frac{6435}{32768} \\ & & & & & & & & & 1 \end{bmatrix}.$$

References

[1] Rawashdeh E.A., A simple method for finding the inverse of a Vandermonde matrix, Matematički Vesnik, September 2019.

[2] Srinivasan, A., Corbino, J., and Castillo, J.E., Implementation Of Mimetic Relaxation Runge Kutta Methods, CSRC Report, 3-Jan-2022.