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Strangeness in Neutron Stars

FRIDOLIN WEBER,* ALEXANDER HO,† RODRIGO P. NEGREIROS,‡
PHILIP ROSENFIELD§

Department of Physics, San Diego State University, 5500 Campanile Drive
San Diego, California 92182-1233, USA

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It is generally agreed on that the tremendous densities reached in the centers of neutron stars provide a high-pressure environment in which several intriguing particles processes may compete with each other. These range from the generation of hyperons to quark deconfinement to the formation of kaon condensates and H-matter. There are theoretical suggestions of even more exotic processes inside neutron stars, such as the formation of absolutely stable strange quark matter. In the latter event, neutron stars would be largely composed of strange quark matter possibly enveloped in a thin nuclear crust. This paper gives a brief overview of these striking physical possibilities with an emphasis on the role played by strangeness in neutron star matter, which constitutes compressed baryonic matter at ultra-high baryon number density but low temperature which is no accessible to relativistic heavy ion collision experiments.

Keywords: neutron stars; quark stars; strangeness.

1. Introduction

Neutron stars, spotted as pulsars by radio telescopes and x-ray satellites, are among the most fascinating objects in the Universe. They are more massive than our sun but are typically only about 10 kilometers across so that the matter in their centers is compressed to densities that are up to an order of magnitude higher than the density inside atomic nuclei[1,2]. At such extreme densities numerous subatomic particle processes are expected to compete with each other and novel phases of matter–like the quark-gluon plasma being sought at the most powerful terrestrial particle colliders–could exist. Figure 1 summarizes the situation graphically[2,3]. The strangeness-carrying s quark is likely to play a key for the composition of neutron star matter, since several potential building blocks of such matter contain s quarks

*fweber@sciences.sdsu.edu
†aho@rohan.sdsu.edu
‡negreiro@sciences.sdsu.edu
§philrose@sciences.sdsu.edu

as one of their constituents. Examples of which are the Λ, Σ and Ξ hyperons, the K^- meson, and the H-dibaryon. First and foremost, strangeness may also exist in the form of unconfined s quarks, which could populate, in chemical equilibrium with u and d quarks and/or hadrons, extended regions inside neutron stars. This paper aims at giving a brief overview of the possible manifestations of strangeness inside neutron stars.

2. Proposed Particle Compositions

2.1. *Hyperons*

Only in the most primitive conception, a neutron star is constituted from neutrons. At a more accurate representation, neutron stars will contain neutrons, n, and a small number of protons, p, whose charge is balanced by leptons, e^-, μ^-. At the densities that exist in the interiors of neutron stars, the neutron chemical potential, μ^n, easily exceeds the mass of the Λ so that neutrons would be replaced with Λ hyperons. From the threshold relation $\mu^n = \mu^\Lambda$ it follows that this would happen for neutron Fermi momenta greater than $k_{F_n} \sim 3\,\mathrm{fm}^{-1}$. Such Fermi momenta correspond to densities of just $\sim 2\rho_0$, with $\rho_0 = 0.16\,\mathrm{fm}^{-3}$ the baryon number density of infinite nuclear matter. Hence, in addition to nucleons and electrons, neutron

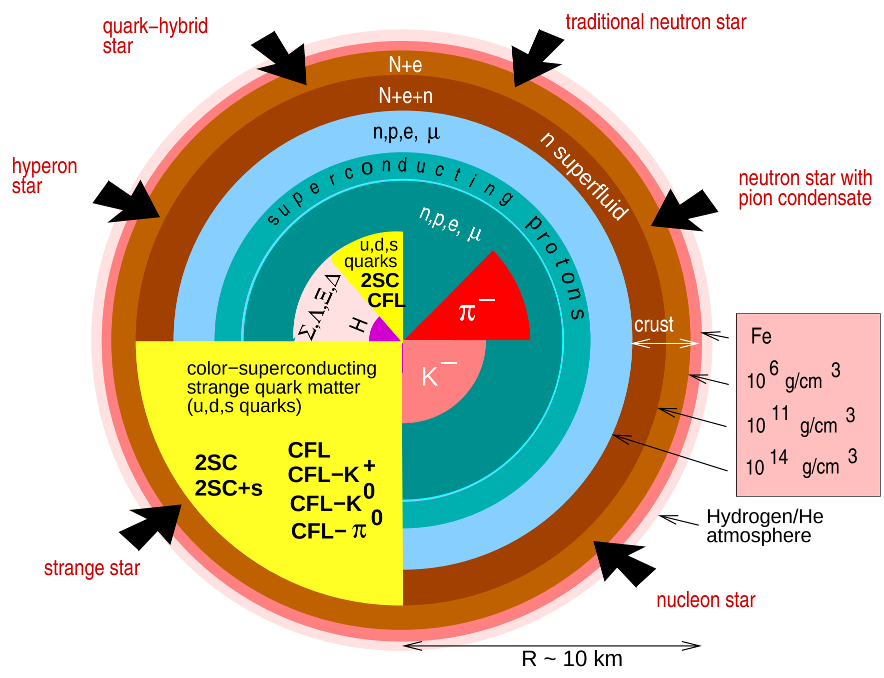

Fig. 1. Competing structures and novel phases of subatomic matter predicted by theory to make their appearances in the cores ($R \lesssim 8$ km) of neutron stars[3].

stars may be expected to contain considerable populations of strangeness-carrying Λ hyperons, possibly accompanied with somewhat smaller populations of Σ and Ξ hyperons[4]. The total hyperon population may be as large as 20% [4].

2.2. *Meson condensates and nucleon matter*

The condensation of negatively charged mesons in neutron star matter is favored because such mesons would replace electrons with very high Fermi momenta. Early estimates predicted the onset of a negatively charged pion condensate at around $2\rho_0$ (see, for instance, Ref. 5). However, these estimates are very sensitive to the strength of the effective nucleon particle-hole repulsion in the isospin $T = 1$, spin $S = 1$ channel, described by the Landau Fermi-liquid parameter g', which tends to suppress the condensation mechanism[6]. Measurements in nuclei tend to indicate that the repulsion is too strong to permit condensation in nuclear matter[7,8]. Nevertheless, some authors argue to the contrary in the case of neutron star matter[9,10]. In the mid 1980s it was discovered that the in-medium properties of K^- mesons may be such that they could condense in neutron star matter as well[11,12,13]. Pion as well as kaon condensates would have two important effects on neutron stars. Firstly,

Table 1. Properties of non-rotating neutron stars composed of nucleons and hyperons (HV), nucleons, hyperons, and normal quarks ($\mathrm{G}_{300}^{\mathrm{B}180}$), and nucleons, hyperons, and color-superconducting quarks (CFL).

Stellar property	HV	$\mathrm{G}_{300}^{\mathrm{B}180}$	CFL
ϵ_{c} (MeV/fm^3)	361.0	814.3	2300.0
M ($M_\odot$)	1.39	1.40	1.36
R (km)	14.1	12.2	9.0
Z	0.1889	0.2322	0.3356
$g_{\mathrm{s},14}$ (cm/s^2)	1.1086	1.5447	3.0146
BE ($M_\odot$)	0.0937	0.1470	0.1534

condensates soften the equation of state above the critical density for onset of condensation, which reduces the maximal possible neutron mass. At the same time, however, the central stellar density increases, because of the softening. Secondly, meson condensates would lead to neutrino luminosities which are considerably enhanced over those of normal neutron star matter. This would speed up neutron star cooling considerably[14]. The condensation of K^- mesons in neutron stars is initiated by the reaction

$$e^- \rightarrow K^- + \nu \,, \tag{1}$$

which is shorthand for $p + e^- \rightarrow n + \nu$ and $n \rightarrow p + K^-$. If this reaction becomes possible in a neutron star, it is energetically advantageous for the star to replace the fermionic electrons with the bosonic K^- mesons. Whether or not this happens depends on the behavior of the K^- mass in neutron star matter. Experiments which shed light on the properties of the K^- in nuclear matter have been performed with the Kaon Spectrometer (KaoS) and the FOPI detector at the heavy-ion synchrotron SIS at GSI[15,16,17,18,19]. An analysis of the early K^- kinetic energy spectra extracted from Ni+Ni collisions [15] showed that the attraction from nuclear matter would bring the K^- mass down to $m^*_{K^-} \simeq 200$ MeV at $\rho \sim 3\,\rho_0$. For neutron-rich matter, the relation

$$m^*_{K^-}(\rho) \simeq m_{K^-}\left(1 - \frac{1}{5}\,\frac{\rho}{\rho_0}\right) \tag{2}$$

was established[20,21,22], with $m_K = 495$ MeV the K^- vacuum mass. Values around $m^*_{K^-} \simeq 200$ MeV lie in the vicinity of the electron chemical potential, μ^e, in neutron star matter[2,4] so that the threshold condition for the onset of K^- condensation, $\mu^e = m^*_K$, which follows from Eq. (1), could be fulfilled in the centers of neutron stars. Equation (1) is followed by

$$n \rightarrow p + K^- \,, \tag{3}$$

with the neutrinos again leaving the star. By this conversion the nucleons in the cores of newly formed neutron stars can become half neutrons and half protons, which lowers the energy per baryon of the matter[23]. The relatively isospin symmetric composition achieved in this way resembles the one of atomic nuclei, which are made up of roughly equal numbers of neutrons and protons. Neutron stars are therefore referred to in this description as nucleon stars. The maximal possible mass of nucleon stars, where Eq. (3) has gone to completion, has been calculated to be around $1.5\,M_\odot$ [14]. Consequently, the collapsing core of a supernova (e.g. 1987A), if heavier than this value, should go into a black hole rather than forming a neutron star[20,21,24]. Another striking implication, pointed out by Brown and Bethe, would be the existence of a large number of low-mass black holes in our galaxy[24].

2.3. *H-dibaryons*

A novel particle that could make its appearance in the center of a neutron star is Jaffe's H-dibaryon, a doubly strange six-quark composite with spin and isospin zero, and baryon number two[25]. In neutron star matter, which may contain a significant fraction of Λ hyperons, the Λ's could combine to form H-dibaryons which could give way to the formation of H-matter at densities somewhere between $3\,\epsilon_0$ and $6\,\epsilon_0$, depending on the in-medium properties of the H-dibaryon. H-matter could thus exist in the cores of moderately dense neutron stars[26,27,28]. If formed, however, H-matter may not remain dormant in the centers but, because of its instability against compression, could trigger the conversion of neutron stars into hypothetical strange stars[28,29,30].

2.4. *Quark deconfinement*

It has been suggested already many decades ago[31,32,33,34,35,36,37] that neutrons, protons plus the heavier constitutes (Σ, Λ, Ξ, Δ) may melt under the enormous pressure that exists in the cores of neutron stars, creating a new state of matter know as quark matter. At present one does not know from experiment at what density the expected phase transition to quark matter occurs, and one has no conclusive guide yet from lattice QCD simulations either. From simple geometrical considerations it follows that nuclei begin to touch each other at densities around $(4\pi r_N^3/3)^{-1} \simeq 0.24 \text{ fm}^{-3} = 1.5\,\rho_0$ (less than twice the density of nuclear matter!) for a characteristic nucleon radius of $r_N \sim 1$ fm. This figure increases to $\sim 11\,\rho_0$ for a nucleon radius of $r_N = 0.5$ fm. One may thus expect that the nuclear boundaries of hadrons in the cores of neutron stars begin to dissolve at densities somewhere between $\sim 2 - 10\,\rho_0$ and that the quarks, originally confined into these hadrons, begin to populate free states outside of them. Depending on rotational frequency and stellar mass, densities as large as two to three times ρ_0 are easily surpassed in the cores of neutron stars of canonical mass, as can be seen from Fig. 2 and Tables 1 and 2[38], so that the neutrons and protons in the centers of neutron stars may indeed have been broken up into their constituent quarks by gravity[39]. More than that, since the mass of the strange quark is only $m_s \sim 150$ MeV, high-energetic up and down quarks will readily transform to strange quarks at about the same density at which up and down quark deconfinement sets[3,40,41], giving way to the existence of three-flavor quark matter in the centers of neutron stars[1,2,39,42]. We also note that in contrast to relativistic heavy ion collisions, which can only provide a fleeting glimpse of quark matter, neutron stars would contain quark matter as a permanent component of matter in their centers. Whether or not quark deconfinement exists in static (non-rotating) neutron stars makes only very little difference to their properties, such as the range of possible masses and radii, which renders the detection of quark matter in such objects extremely complicated. This turns out to be strikingly different for rotating neutron stars which develop quark matter cores in the course of spin-down. The reason being that as such stars spin down, because of the emission of magnetic dipole radiation and a wind of electron-positron pairs, they become more and more compressed. For some rotating neutron stars the mass and initial rotational frequency may be just such that the central density rises from below to above the critical density for the dissolution of baryons into their quark constituents. This could effect the star's moment of inertia dramatically[42]. Depending on the rate at which quark matter is produced, the moment of inertia can decrease very anomalously, and could even introduce an era of stellar spin-up ("backbending") lasting for $\sim 10^8$ years[42]. Since the dipole age of millisecond pulsars is about 10^9 years, one may estimate that roughly about 10% of the ~ 30 solitary millisecond pulsars presently known could be in the quark transition epoch and thus could be signaling the ongoing process of quark deconfinement. Changes in the moment of inertia reflect themselves in the braking index, n, of a rotating neutron star, as can be seen from[3,42,43]

$$n(\Omega) \equiv \frac{\Omega\,\ddot{\Omega}}{\dot{\Omega}^2} = 3 - \frac{I + 3\,I'\,\Omega + I''\,\Omega^2}{I + I'\,\Omega} \to 3 - \frac{3\,I'\,\Omega + I''\,\Omega^2}{2\,I + I'\,\Omega} \qquad (4)$$

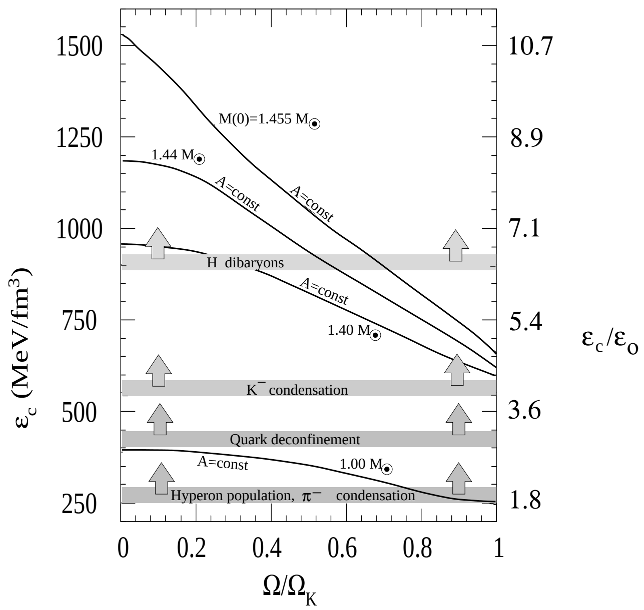

Fig. 2. Central density versus rotational frequency for several sample neutron stars[2]. The stars' baryon number, A, is constant in each case. Theory predicts that the interior stellar density could become so great that the threshold densities of various novel phases of superdense matter are reached. $\epsilon_0 = 140$ MeV/fm^3 denotes the density of nuclear matter, Ω_K is the Kepler frequency, and $M(0)$ is the :

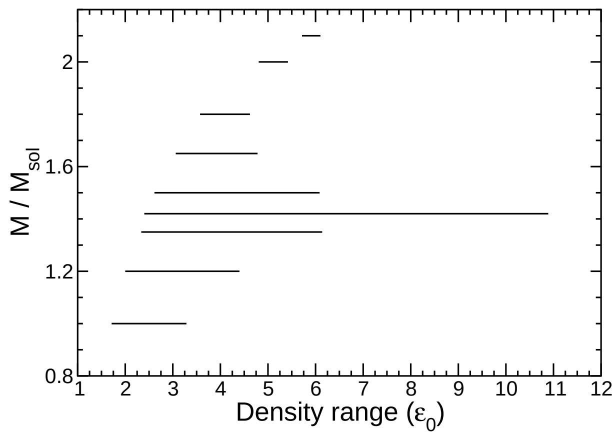

Fig. 3. Spread of central density (in units of the density of nuclear matter, $\epsilon_0 = 140$ MeV/fm^3) of non-rotating neutron stars for a broad collection of modern equations of state[2,3]. The very wide density range of the $1.42\,M_\odot$ solar mass model has its origin in quark deconfinement.

where dots and primes denote derivatives with respect to time and Ω, respectively. The last relation in (4) constitutes the non-relativistic limit of the braking index[44].

Table 2. Same as Table 1 but for neutron stars rotating at the Kepler frequency, ν_{K}.

Stellar property	HV $\nu_{\mathrm{K}} = 850$ Hz	$\mathrm{G}^{\mathrm{B180}}_{300}$ $\nu_{\mathrm{K}} = 940$ Hz	CFL $\nu_{\mathrm{K}} = 1400$ Hz
ϵ_{c} (MeV/fm^3)	280.0	400.0	1100.0
I (km^3)	223.6	217.1	131.8
M ($M_\odot$)	1.39	1.40	1.41
R (km)	17.1	16.0	12.6
Z_{p}	0.2374	0.2646	0.3618
Z_{F}	−0.1788	−0.1817	−0.2184
Z_{B}	0.6046	0.6502	0.9190
$g_{\mathrm{s},14}$ (cm/s^2)	0.7278	0.8487	1.4493
T/W	0.0894	0.0941	0.0787
BE ($M_\odot$)	0.0524	0.1097	0.1203
V_{eq}/c	0.336	0.353	0.424

It is obvious that these expressions reduce to the canonical limit, $n = 3$, if the moment of inertia is completely independent of frequency. Evidently, this is not the case for rapidly rotating neutron stars, and it fails for stars that experience pronounced internal changes (as possibly driven by phase transitions) which alter the moment of inertia significantly. In Ref. 44 it was shown that the changes in the moment of inertia caused by the gradual transformation of hadronic matter into quark matter may lead to $n(\Omega) \rightarrow \pm\infty$ at the transition frequency where pure quark matter is produced. Such dramatic anomalies in $n(\Omega)$ are not known for conventional neutron stars (see, however, Ref. 45), because their moments of inertia appear to vary smoothly with Ω.[2] The future astrophysical observation of a strong anomaly in the braking behavior of a pulsar may thus indicate that quark deconfinement is occurring at the pulsar's center.

Accreting x-ray neutron stars provide a very interesting contrast to the spin-down of isolated neutron stars discussed just above. These x-ray neutron stars are being spun up by the accretion of matter from a lower-mass ($M \lesssim 0.4 M_\odot$), less-dense companion. If the critical deconfinement density falls within that of canonical pulsars, quark matter will already exist in them but will be spun out of x-ray stars as their frequency increases during accretion[46,47].

2.5. *Color superconductivity*

There has been much recent progress in our understanding of quark matter, culminating in the discovery that if quark matter exists it will be a color superconductor[48,49]. The phase diagram of such matter is very complex. At asymptotic densities the ground state of QCD with a vanishing strange quark mass is the color-flavor locked (CFL) phase. This phase is electrically neutral in bulk for a significant range of chemical potentials and strange quark masses[50]. If the strange quark mass is heavy enough to be ignored, then up and down quarks may pair in the two-flavor superconducting (2SC) phase. Other possible condensation patters are the CFL–K^0 phase[51] and the color-spin locked (2SC+s) phase[52]. Depending on the condensation pattern, the magnitude of the supefluid gap energy ranges from several keV's to about one hundred MeV. Color superconductivity has been shown to have consequences for a number of astrophysical phenomena ranging from neutron star cooling, to the arrival times of supernova neutrinos, to the evolution of neutron star magnetic fields, rotational (r-mode) instabilities, and glitches in rotation frequencies of pulsar[48,49,53,54,55,56]. Aside from neutron star properties, an additional test of color superconductivity may be provided by upcoming cosmic ray space experiments such as AMS [57] and ECCO[58]. As shown in Ref. 59, finite lumps of color-flavor locked strange quark matter, which should be present in cosmic rays if strange matter is the ground state of the strong interaction (see Sect. 3), turn out to be significantly more stable than strangelets without color-flavor locking for wide ranges of parameters. In addition, strangelets made of CFL strange matter obey a charge-mass relation of $Z/A \propto A^{-1/3}$, which differs significantly from the charge-mass relation of strangelets made of ordinary strange quark matter. In the latter case, Z/A would be constant for small baryon numbers A and $Z/A \propto A^{-2/3}$ for large A[59,60,61]. This difference may allow an experimental test of CFL locking in strange quark matter[59].

3. Absolutely Stable Strange Quark Matter

It is most intriguing that for strange quark matter made of more than a few hundred up, down, and strange quarks, the energy of strange quark matter may be well below the energy of nuclear matter, $E/A = 930$ MeV[62,63,64]. A simple estimate indicates that for strange quark matter $E/A = 4B\pi^2/\mu^3$, so that bag constants of $B = 57$ MeV/fm^3 (i.e. $B^{1/4} = 145$ MeV) and $B = 85$ MeV/fm^3 ($B^{1/4} = 160$ MeV) would place the energy per baryon of such matter at $E/A = 829$ MeV and 915 MeV, respectively, which correspond obviously to strange quark matter which is absolutely bound with respect to nuclear matter[65,66,67]. If this were indeed the case, neutron star matter would be metastable with respect to strange quark matter, and all neutron stars could in fact be strange quark stars[65,66,67]. As briefly described in Sect. 2.5, strange quark matter is expected to be a color superconductor which, at extremely high densities, should be in the CFL phase. This phase is rigorously electrically neutral with no electrons required[50]. For sufficiently large strange quark masses, however, the low density regime of strange quark matter is rather expected to form a 2-flavor superconductor (2SC) in which electrons are present[48,49]. The presence of electrons causes the formation of an electric dipole layer on the surface of strange matter, with huge electric fields on the order of 10^{19} V/cm, which enables strange quark matter stars to be enveloped in nuclear crusts made of ordinary atomic matter[41,68,69,70]. The maximal possible density at the base of the crust (inner crust density) is determined by neutron drip, which occurs at about 4×10^{11} g/cm^3 or somewhat below[70].

Since the nuclear crust surrounding a strange star would be bound to the star by gravity rather than confinement, the mass-radius relationship of a strange matter star with a nuclear would be qualitatively similar to the one of purely gravitationally bound neutron stars or white dwarfs. The fact that strange stars with crusts tend to possess somewhat smaller radii than neutron stars leads to smaller mass shedding (Kepler) periods $P_{\rm K}$ for strange stars. This is obvious from the classical mass shedding expression $P_{\rm K} = 2\pi\sqrt{R^3/M}$ which carries over to the full general relativistic case[2]. It was found that due to the smaller radii of strange stars the complete sequence of such objects (and not just those close to the mass peak, as is the case for neutron stars) can sustain extremely rapid rotation well below 1 ms [71]. In particular, strange stars with a canonical pulsar mass of around $1.45\, M_\odot$ have Kepler periods in the range of 0.55 ms$\lesssim$P$_{\rm K}\lesssim$0.8 ms, depending on the thickness of the nuclear curst and the bag constant[71,72]. This range is to be compared with $P_{\rm K} \sim 1$ ms obtained for standard neutron stars of the same mass computed for standard equations of state.

The fact that strange stars can carry nuclear crusts is key to reconcile strange stars with superbursts and soft x-ray transient phenomenology[73].

4. Effects of a Net Electric Charge Distributions on the Equation of State of Relativistic Stars

As pointed out just above, the electric field that may exist on the surface of a strange quark star would be as high as 10^{19} V/cm[74].[a] The energy density associated with such ultra-high fields begins to have an influence one the curvature of space as determined by Einstein's field equation[75,76,77],

$$G_\nu{}^\mu = R_\nu{}^\mu - \frac{1}{2} g_\nu{}^\mu R = \frac{8\pi G}{c^4} T_\nu{}^\mu \ , \tag{5}$$

where $G_\nu{}^\mu$ is the Einstein tensor, $R_\nu{}^\mu$ the Ricci tensor, $g_\nu{}^\mu$ the metric tensor, R the Ricci scalar, and $G_\nu{}^\mu$ the energy-momentum tensor. In the following, we will discuss briefly how the equation of state of a compact star is modified by the presence of an ultra-strong interior electric field. The consequences for strange stars, where the electric field is located on the surface, are explored in a paper that is currently under preparation[78]. Our preliminary results indicate that, depending of the amount of electric charge, the strange star structure and specifically the mass-radius relationship might be drastically modified. For the study presented here we adopt a polytropic equation of state to model the compact star[75]. First we will briefly present the structure equations for a electrically charged star and then show how the polytropic equation of state is modified by the presence of charge.

[a]Such ultra-high fields would make the vacuum unstable in free space. The Pauli principle, however, prevents this from happening on the surface of a strange star.

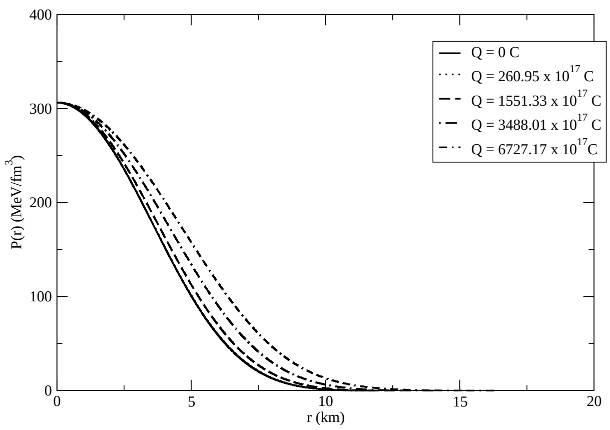

Fig. 4. Influence of electric charge on the pressure profiles of compact stars.

4.1. *Energy-momentum tensor*

We want to maintain spherical symmetry of the star. Thus the natural choice for the metric is

$$ds^2 = e^{\nu(r)}c^2dt^2 - e^{\lambda(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \tag{6}$$

The energy-momentum tensor of the star consists of two terms, the standard term which describes the star's matter as a perfect fluid and the electromagnetic term,

$$T_\nu\mu = (p + \rho c^2)u_\nu u^\mu + p\delta_\nu{}^\mu + \frac{1}{4\pi}\left[F^{\mu l}F_{\nu l} + \frac{1}{4\pi}\delta^\mu_\nu F_{kl}F^{kl}\right]\,, \tag{7}$$

where the components $F^{\nu\mu}$ satisfy the covariant Maxwell equations,

$$[(-g)^{1/2}F^{\nu\mu}]_{,\mu} = 4\pi J^\nu(-g)^{1/2}, \tag{8}$$

where J^ν is the four-current. Imposing that in last equation the only non-vanishing term is the radial component, one obtains for the electric field

$$F^{01}(r) = E(r) = e^{-(\nu+\lambda)/2}\,r^{-2}\int_0^r 4\pi j^0 e^{(\nu+\lambda)/2}dr'\,. \tag{9}$$

From last equation we can define the charge of the system as

$$Q(r) = \int_0^r 4\pi j^0 r'^2 e^{(\nu+\lambda)/2}dr'. \tag{10}$$

The electric field is then given by

$$E(r) = e^{-(\nu+\mu)/2}\,r^{-2}\,Q(r)\,. \tag{11}$$

With the aid of these relations the energy-momentum tensor takes the following form

$$T^{\mu}_{\nu} = \begin{pmatrix} -\left(\epsilon + \frac{Q^2(r)}{8\pi r^4}\right) & 0 & 0 & 0 \\ 0 & p - \frac{Q^2(r)}{8\pi r^4} & 0 & 0 \\ 0 & 0 & p + \frac{Q^2(r)}{8\pi r^4} & 0 \\ 0 & 0 & 0 & p + \frac{Q^2(r)}{8\pi r^4} \end{pmatrix} . \tag{12}$$

Substituting this relation into Einstein's field equation (5) one arrives at

$$e^{-\lambda}\left(-\frac{1}{r^2} + \frac{1}{r}\frac{d\lambda}{dr}\right) + \frac{1}{r^2} = \frac{8\pi G}{c^4}\left(p - \frac{Q^2(r)}{8\pi r^4}\right) , \tag{13}$$

$$e^{-\lambda}\left(\frac{1}{r}\frac{d\nu}{dr} + \frac{1}{r^2}\right) - \frac{1}{r^2} = -\frac{8\pi G}{c^4}\left(\epsilon + \frac{Q^2(r)}{8\pi r^4}\right) . \tag{14}$$

The solution for the metric function $\lambda(r)$ is given by

$$e^{-\lambda} = 1 - \frac{Gm(r)}{rc^2} + \frac{GQ^2}{r^2c^4} . \tag{15}$$

The first two terms on the right-hand-side of this relation correspond to electrically neutral stars, while the third term originates from the net electric charge distribution inside the star. Using Eq. (15) together with Eqs. (13) and (14) we arrive at an equation for the mass of the star within a spherical shell of radius, $m(r)$,

$$\frac{dm(r)}{dr} = \frac{4\pi r^2}{c^2}\epsilon + \frac{Q(r)}{c^2 r}\frac{dQ(r)}{dr} . \tag{16}$$

The first term on the right-hand-side is the standard result for the gravitational mass of electrically uncharged stars, while the second term accounts for the mass change that originates from the electric field.

4.2. *Tolman-Oppenheimer-Volkoff equation*

The equation of general relativistic hydrostatic equilibrium, known as the Tolman-Oppenheimer-Vokoff (TOV) equation, are obtained after imposing ${T_\nu}^{\mu}{}_{;\mu} = 0$. This leads to the

$$\frac{dp}{dr} = -\frac{2G\left(m(r) + \frac{4\pi r^3}{c^2}\left(p - \frac{Q^2(r)}{4\pi r^4 c^2}\right)\right)}{c^2 r^2\left(1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{r^2c^4}\right)}(p+\epsilon) + \frac{Q(r)}{4\pi r^4}\frac{dQ(r)}{dr} . \tag{17}$$

This completes the derivation of the structure equations of a electrically charged compact star. Before we go ahead and solve the TOV equation, we need to make an assumption for the charge distribution inside the star. Only then the problem is fully defined. Here we will follow the approach of Ref. 75, 79 and assume that the charge distribution is proportional to the energy density,

$$j^0(r) = f \times \epsilon , \tag{18}$$

where f is a constant which essentially controls the amount of net electric charge carried by the star. Adopting as initial and boundary conditions of the problem the following values,

$$\epsilon(0) = 1550 \quad \mathrm{MeV/fm}^3 \qquad Q(0) = 0, \tag{19}$$

$$m(0) = 0, \qquad \lambda(0) = 0, \tag{20}$$

$$\nu(0) = 0, \qquad p(R) = 0\,, \tag{21}$$

the properties of five different charged sample stars are compiled in Table 3. As one might expect, both mass and radius of a compact star increase with net electric stellar charge. These features have their origin in the repulsive pressure force introduced into the system by the electric field. Figure 4 shows that in stellar configurations with higher electric charge, the pressure drops at a slower rate, leading to bigger and therefore more massive stars. In Fig. 5 we show the effective (baryonic and electric) equation of state of an electrically charged compact star. The solid line

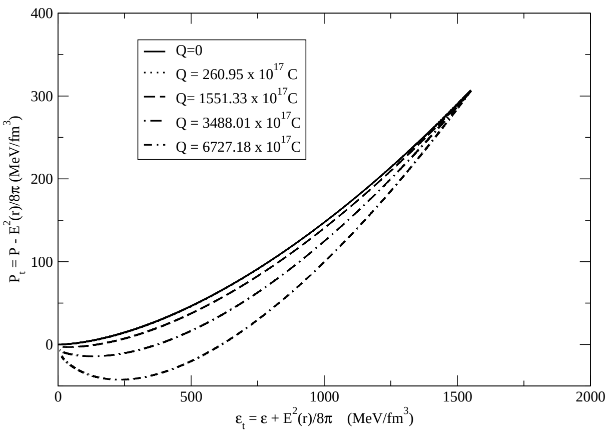

Fig. 5. Influence of electric charge distribution on the equation of state of a compact star.

is for electrically neutral (uncharged) stars computed for the polytropic equation of state. The other curves are for stars with successively increasing charges. Charges

Table 3. Results

f	M ($M_\odot$)	Radius (km)	Charge ($\times 10^{17}$ C)
0.0	1.428	11.85	0
0.0001	1.439	11.879	260.95
0.0005	1.742	12.559	1551.33
0.0008	2.548	14.026	3488.01
0.001	4.156	16.364	6727.17

on this order of magnitude may exist in the surface region of strange stars which, as already mentioned above, is currently under investigation[78].

5. Summary

It is often stressed that there has never been a more exciting time in the overlapping areas of nuclear physics, particle physics and relativistic astrophysics than today. This comes at a time where new orbiting observatories such as the Hubble Space Telescope (HST), Rossi X-ray Timing Explorer, Chandra X-ray satellite, and the X-ray Multi Mirror Mission (XMM) have extended our vision tremendously, allowing us to observe compact star phenomena with an unprecedented clarity and angular resolution that previously were only imagined. On the Earth, radio telescopes (Arecibo, Green Bank, Parkes, VLA) and instruments using adaptive optics and other revolutionary techniques have exceeded previous expectations of what can be accomplished from the ground. Finally, the gravitational wave detectors LIGO, LISA, and VIRGO are opening up a window for the detection of gravitational waves emitted from compact stellar objects such as neutron stars and black holes. This unprecedented situation is providing us with key information on compact stars. As discussed in this paper, a key role in compact star physics is played by strangeness. It alters the masses, radii, cooling behavior, and surface composition of neutron stars, and may even give rise to new classes (strange stars, strange dwarfs) of compact stars. Other important observables may be the spin evolution of isolated neutron stars and neutron stars in low-mass x-ray binaries. All told, these observables are key in exploring the phase diagram of dense nuclear matter at high baryon number density but low temperature, which is not accessible to relativistic heavy ion collision experiments.

Acknowledgments

6. References

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