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November 2006

Publication Number: CSRCR2006-19

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International Journal of Modern Physics E

Ultra-Dense Neutron Star Matter, Strange Quark Stars, and the Nuclear Equation of State

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With central densities way above the density of atomic nuclei, neutron stars contain matter in one of the densest forms found in the universe. Depending of the density reached in the cores of neutron stars, they may contain stable phases of exotic matter found nowhere else in space. This article gives a brief overview of the phases of ultra-dense matter predicted to exist deep inside neutron stars and discusses the equation of state (EoS) associated with such matter.

1. Introduction

Neutron stars are dense, neutron-packed remnants of stars that blew apart in supernova explosions. Many neutron stars form radio pulsars, emitting radio waves that appear from the Earth to pulse on and off like a lighthouse beacon as the star rotates at very high speeds. Neutron stars in x-ray binaries accrete material from a companion star and flare to life with a burst of x-rays. Measurements of radio pulsars and neutron stars in x-ray binaries comprise most of the neutron star observations. Improved data on isolated neutron stars (e.g. RX J1856.5-3754, PSR 0205+6449) are now becoming available, and future investigations at gravitational wave observatories focus on neutron stars as major potential sources of gravitational waves (see Ref. 1 for a recent overview). Depending on star mass and rotational frequency, the matter in the core regions of neutron stars may be compressed to densities that are up to an order of magnitude greater than the density of ordinary atomic nuclei. This extreme compression provides a high-pressure environment in which numerous subatomic particle processes are believed to compete with each other[2,3]. The most spectacular ones stretch from the generation of hyperons

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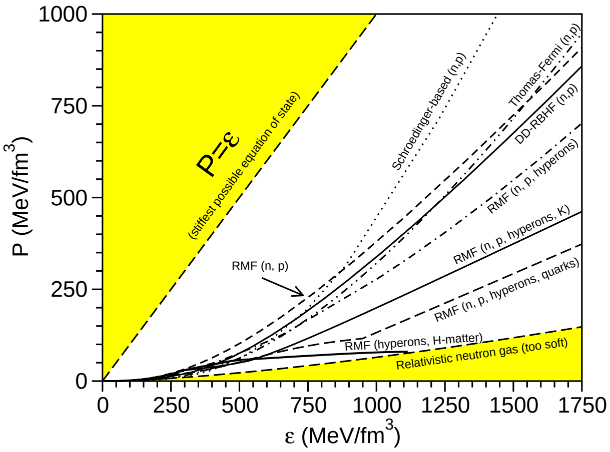

Fig. 1. Models for the EoS (pressure versus energy density) of neutron star matter[6]. The notation is as follows: RMF=relativistic mean-field model; DD-RBHF=density dependent relativistic Brueckner-Hartree-Fock model; n=neutrons; p=protons; K=$K^-[u,\bar{s}]$ meson condensate; quarks=u,d,s; H-matter=H-dibaryon condensate.

and baryon resonances $(\Sigma, \Lambda, \Xi, \Delta)$ to quark (u, d, s) deconfinement to the formation of boson condensates (π^-, K^-, H-matter)[2,3,4,5,6,7]. It has also been suggested (strange matter hypothesis) that strange quark matter may be more stable than ordinary atomic nuclei. In the latter event, neutron stars could in fact be made of absolutely stable strange quark matter rather than ordinary hadronic matter[8,9,10]. Another striking implication of the strange matter hypothesis is the possible existence of a new class of white-dwarfs-like strange stars (strange dwarfs)[11]. The quark matter in neutron stars, strange stars, or strange dwarfs ought to be in a color superconducting state[12,13,14,15]. This fascinating possibility has renewed tremendous interest in the physics of neutron stars and the physics and astrophysics of (strange) quark matter[6,12,13]. This paper reviews the possible phases of ultra-dense nuclear matter expected to exist deep inside neutron stars and its associated equation of state[2,3,4,5,6,7].

2. Neutron Star Masses

In 1939, Tolman, Oppenheimer and Volkoff performed the first neutron star calculations, assuming that such objects are entirely made of a gas of relativistic neutrons[16,17]. The EoS of such a gas is extremely soft (i.e. very little additional pressure is gained with increasing density), as can be seen from Fig. 1, and leads to a maximum neutron star mass of just 0.7 $M_\odot$. A relativistic neutron gas thus fails to accommodate neutron stars such as the Hulse-Taylor pulsar ($M = 1.44\,M_\odot$)[18], and also conflicts with the average neutron star mass of $1.350 \pm 0.004\,M_\odot$ derived by Thorsett and Chakrabarty[19] from observations of radio pulsar systems. More than that, recent observations indicate that neutron star masses may be as high as $\sim 2\ M_\odot$. Examples of such very heavy neutron stars are $M_{\rm J0751+1807} = 2.1 \pm 0.2\ M_\odot$[20], $M_{\rm 4U\,1636+536} = 2.0 \pm 0.1\ M_\odot$[21], $M_{\rm Vela\,X-1} = 1.86 \pm 0.16\, M_\odot$[22], $M_{\rm Cyg\,X-2} = 1.78 \pm 0.23\, M_\odot$[23,24]. Large masses have also been reported for the high-mass x-ray binary 4U 1700–37 and the compact object in the low-mass x-ray binary 2S0921–630, $M_{\rm 4U\,1700-37} = 2.44 \pm 0.27\ M_\odot$[25] and $M_{\rm 2S0921-630} = 2.0 - 4.3 M_\odot$[26]. respectively. The latter two objects may be either massive neutron stars or low-mass black holes with masses slightly higher than the maximum possible neutron star mass of $\sim 3M_\odot$. This value follows from a general, theoretical estimate of the maximal possible mass of a stable neutron star as performed by Rhoades and Ruffini[27] on the basis that (1) Einstein's theory of general relativity is the correct theory of gravity, (2) the EoS satisfies both the microscopic stability condition $\partial P/\partial\epsilon \geq 0$ and the causality condition $\partial P/\partial\epsilon \leq c^2$, and (3) that the EoS below some matching density is known. From these assumptions, it follows that the maximum mass of the equilibrium configuration of a neutron star cannot be larger than $3.2\, M_\odot$. This value increases to about $5\, M_\odot$ if one abandons the causality constraint $\partial P/\partial\epsilon \leq c^2$,[28,29] since it allows the EoS to behave stiffer at asymptotically high nuclear densities. If either one of the two objects 4U 1700–37 or 2S0921–630 were a black hole, it would confirms the prediction of the existence of low-mass black holes[30]. Conversely, if these objects were massive neutron stars, their high masses would severely constrain the EoS of nuclear matter.

3. Composition of Ultra-Dense Neutron Star Matter

Models for the EoS of neutron star matter are being computed in different theoretical frameworks. The most popular ones are the semi-classical Thomas-Fermi theory[31,32], Schroedinger-based treatments (e.g. variational approach, Monte Carlo techniques, hole line expansion (Brueckner theory), coupled cluster method, Green function method)[4,33,34,35], or relativistic field-theoretical treatments (relativistic mean field (RMF), Hartree-Fock (RHF), standard Brueckner-Hartree-Fock (RBHF), density dependent RBHF (DD-RBHF)[36,37,38,39,40,41], the Nambu-Jona-Lasinio (NJL) model[42,43], and the chiral SU(3) quark mean field model[44]). An overview of EoSs computed for several of these methods is given in Fig. 1. Mass-radius relationships of models of neutron stars based on these EoSs are shown in Fig. 2. Any acceptable nuclear many-body calculation must reproduce the bulk properties of nuclear matter at saturation density and of finite nuclei. The nuclear matter properties are the binding energy, $E/A = -16.0$ MeV, effective nucleon mass, $m^*_{\rm N} = 0.79\, m_{\rm N}$, incompressibility, $K \simeq 240$ MeV, and the symmetry energy, $a_{\rm s} = 32.5$ MeV, at a saturation density of $n_0 = 0.16$ fm^{-3}.

3.1. *Hyperons*

Only in the simplest conception, a neutron star is constituted from only neutrons. As already discussed in Sect. 2, this model fails by far to accommodate observed

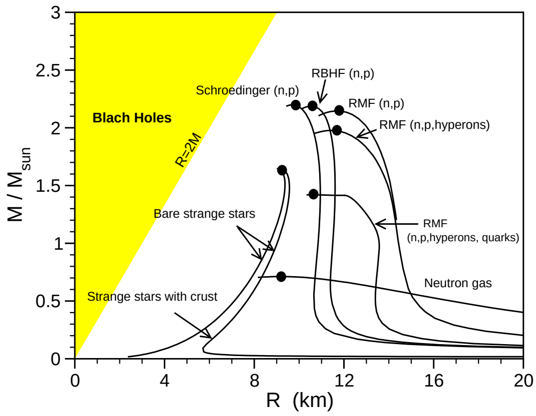

Fig. 2. Mass-radius relationship of neutron stars and strange stars. The strange stars may be enveloped in a crust of ordinary nuclear material whose density is below neutron drip density[8,45,46]. The labels are explained in Fig. 1. (Figure from Ref. 6.)

neutron star masses. At a more accurate representation, neutron stars will contain neutrons, n, and a small number of protons, p, whose charge is balanced by electrons, e^-, according to the reaction $n \to p + e^- + \bar{\nu}_e$ and its inverse. At the densities that exist in the interiors of neutron stars, the neutron chemical potential, μ^n, easily exceeds the mass of the Λ so that neutrons would be replaced with Λ hyperons. From the threshold relation $\mu^n = \mu^\Lambda$ it follows that this would happen for neutron Fermi momenta greater than $k_{F_n} \sim 3\,\mathrm{fm}^{-1}$.[6] Such Fermi momenta correspond to densities of just $\sim 2n_0$. Hence, in addition to nucleons and electrons, neutron stars may be expected to contain considerable populations of strangeness-carrying Λ hyperons, possibly accompanied by smaller populations of the charged states of the Σ and Ξ hyperons[47]. The total hyperon population may be as large as 20% [47].

3.2. *Meson condensation*

The condensation of negatively charged mesons in neutron star matter is favored because such mesons would replace electrons with very high Fermi momenta. Early estimates predicted the onset of a negatively charged pion condensate at around $2n_0$ (see, for instance, Ref. 48). However, these estimates are very sensitive to the strength of the effective nucleon particle-hole repulsion in the isospin $T = 1$, spin $S = 1$ channel, described by the Landau Fermi-liquid parameter g', which tends to suppress the condensation mechanism. Measurements in nuclei tend to indicate that the repulsion is too strong to permit condensation in nuclear matter[49,50]. In the mid 1980s, it was discovered that the in-medium properties of $K^-[u\bar{s}]$ mesons may be such that this meson rather than the π^- meson may condense in neutron star matter[51,52,53].

The condensation is initiated by the schematic reaction $e^- \rightarrow K^- + \nu_e$. If this reaction becomes possible in neutron star matter, it is energetically advantageous to replace the fermionic electrons with the bosonic K^- mesons. Whether or not this happens depends on the behavior of the K^- mass, $m^*_{K^-}$, in neutron star matter. Experiments which shed light on the properties of the K^- in nuclear matter have been performed with the Kaon Spectrometer (KaoS) and the FOPI detector at the heavy-ion synchrotron SIS at GSI[54,55,56,57,58]. An analysis of the early K^- kinetic energy spectra extracted from Ni+Ni collisions[54] showed that the attraction from nuclear matter would bring the K^- mass down to $m^*_{K^-} \simeq 200$ MeV at densities $\sim 3\,n_0$. For neutron-rich matter, the relation $m^*_{K^-}/m_{K^-} \simeq 1 - 0.2n/n_0$ was established[59,60,61], with $m_K = 495$ MeV the K^- vacuum mass. Values of around $m^*_{K^-} \simeq 200$ MeV may be reached by the electron chemical potential, μ^e, in neutron star matter[3,47] so that the threshold condition for the onset of K^- condensation, $\mu^e = m^*_K$ might be fulfilled for sufficiently dense neutron stars, provided other negatively charged particles (Σ^-, Δ^-, d and s quarks) are not populated first and prevent the electron chemical potential from increasing monotonically with density.

We also note that K^- condensation allows the conversion reaction $n \rightarrow p + K^-$. By this conversion the nucleons in the cores of neutron stars can become half neutrons and half protons, which lowers the energy per baryon of the matter[62]. The relative isospin symmetric composition achieved in this way resembles the one of atomic nuclei, which are made up of roughly equal numbers of neutrons and protons. Neutron stars are therefore referred to, in this picture, as nucleon stars. The maximum mass of such stars has been calculated to be around $1.5\,M_\odot$[63]. Consequently, the collapsing core of a supernova, e.g. 1987A, if heavier than this value, should go into a black hole rather than forming a neutron star, as pointed out by Brown et al.[30,59,60] This would imply the existence of a large number of low-mass black holes in our galaxy[30]. Thielemann and Hashimoto[64] deduced from the total amount of ejected ^{56}Ni in supernova 1987A a neutron star mass range of $1.43 - 1.52\ M_\odot$. If the maximum neutron star mass should indeed be in this range ($\sim 1.5\ M_\odot$), it would pose a problem to the possible discovery of very heavy neutron stars of masses around $2\ M_\odot$ (Sect. 2). In closing, we mention that meson condensates lead to neutrino luminosities which are considerably enhanced over those of normal neutron star matter. This would speed up neutron star cooling considerably[63,65].

3.3. *H-matter and exotic baryons*

A novel particle that could be of relevance for the composition of neutron star matter is the H-dibaryon (H=($[ud][ds][su]$)), a doubly strange six-quark composite with spin and isospin zero, and baryon number two[66]. Since its first prediction in 1977, the H-dibaryon has been the subject of many theoretical and experimental studies as a possible candidate for a strongly bound exotic state. In neutron star matter, which may contain a significant fraction of Λ hyperons, the Λ's could combine to form H-dibaryons, which could give way to the formation of H-dibaryon matter at densities somewhere above $\sim 4\,n_0$[67,68,69]. If formed in neutron stars, however, H-matter appears to unstable against compression which could trigger the conversion of neutron stars into hypothetical strange stars[68,70,71].

Another particle, referred to as exotic baryon, of potential relevance for neutron stars, could be the pentaquark, $\Theta^+([ud]^2\bar{s})$, with a predicted mass of 1540 MeV. The pentaquark, which carries baryon number one, is a hypothetical subatomic particle consisting of a group of four quarks and one anti-quark (compared to three quarks in normal baryons and two in mesons), bound by the strong color-spin correlation force (attraction between quarks in the color $\bar{\mathbf{3}}_c$ channel) that drives color superconductivity[72,73]. The pentaquark decays according to $\Theta^+(1540) \rightarrow K^+[\bar{s}u] + n[udd]$ and thus has the same quantum numbers as the K^+n. The associated reaction in chemically equilibrated matter would imply $\mu^{\Theta^+} = \mu^{K^+} + \mu^n$.

3.4. *Quark deconfinement*

It has been suggested already many decades ago[74,75,76,77,78,79,80] that the nucleons may melt under the enormous pressure that exists in the cores of neutron stars, creating a new state of matter know as quark matter. From simple geometrical considerations it follows that for a characteristic nucleon radius of $r_N \sim 1$ fm, nucleons may begin to touch each other in nuclear matter at densities around $(4\pi r_N^3/3)^{-1} \simeq 0.24\ \mathrm{fm}^{-3} = 1.5\,n_0$, which is less than twice the density of nuclear matter. This figure increases to $\sim 11\,n_0$ for a nucleon radius of $r_N = 0.5$ fm. One may thus speculate that the hadrons of neutron star matter begin to dissolve at densities somewhere between around $2 - 10\,n_0$, giving way to unconfined quarks. Depending on rotational frequency and neutron star mass, densities greater than two to three times n_0 are easily reached in the cores of neutron stars so that the neutrons and protons in the cores of neutron stars may indeed be broken up into their quarks constituents[2,3,6,81]. More than that, since the mass of the strange quark is only $m_s \sim 150$ MeV, high-energetic up and down quarks will readily transform to strange quarks at about the same density at which up and down quark deconfinement sets in. Thus, if quark matter exists in the cores of neutron stars, it should be made of the three lightest quark flavors. A possible astrophysical signal of quark deconfinement in the cores of neutron stars was suggested in Ref. 82. The remaining three quark flavors (charm, top, bottom) are way to massive to be created in neutron stars. For instance, the creation of charm quark requires a density greater than $10^{17}\,\mathrm{g/cm^3}$, which is $\sim 10^2$ times greater than the density reached in neutron stars. A stability analysis of stars with a charm quark population reveals that such objects are unstable against radial oscillations and, thus, can not exist stably in the universe[3,6]. The same is true for ultra-compact stars with unconfined populations of top and bottom quarks, since the pulsation eigen-equations are of Sturm-Liouville type.

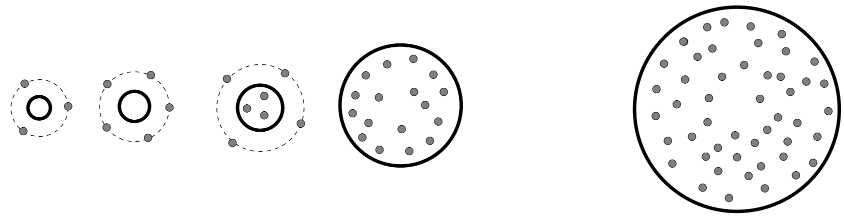

Fig. 3. Radii of quark bags [83]. For masses less than 10^9 GeV the electrons (gray dots) are outside the quark bags (indicated by thick solid circles) and the core+electron system has a size of $\sim 10^5$ fm. For masses between 10^9 and 10^{15} GeV the electrons are partially inside the core. For masses greater than 10^{15} GeV all electrons are inside the core, except for a small electron fraction which forms an electric dipole layer on the surface of the quark bag (strange star).

4. Strange Quark Matter

It is most intriguing that for strange quark matter made of more than a few hundred up, down, and strange quarks, the energy of strange quark matter may be well below the energy of nuclear matter, $E/A = 930$ MeV, which gives rise to new and novel classes of strange matter objects (see Fig. 3), ranging from strangelets at the low baryon-number end to strange stars at the high baryon number end. A simple estimate indicates that for strange quark matter $E/A = 4B\pi^2/\mu^3$, so that bag constants of $B = 57$ MeV/fm^3 (i.e. $B^{1/4} = 145$ MeV) and $B = 85$ MeV/fm^3 ($B^{1/4} = 160$ MeV) would place the energy per baryon of such matter at $E/A =$ 829 MeV and 915 MeV, respectively, which correspond obviously to strange quark matter which is absolutely bound with respect to nuclear matter[84,10].

4.1. *Nuclear crust on strange stars*

Strange quark matter is expected to be a color superconductor which, at extremely high densities, should be in the CFL phase[12,13]. This phase is rigorously electrically neutral with no electrons required[85]. For sufficiently large strange quark masses, however, the low density regime of strange quark matter is rather expected to form other condensation patterns (e.g. 2SC, CFL-K^0, CFL-K^+, CFL-$\pi^{0,-}$) in which electrons are present[12,13]. The presence of electrons causes the formation of an electric dipole layer on the surface of strange matter, with huge electric fields on the order of 10^{19} V/cm, which enables strange quark matter stars to be enveloped in nuclear crusts made of ordinary atomic matter[8,9,46,87].[a] The maximal possible

[a]Depending on the surface tension of blobs of strange matter and screening effects, a heterogeneous crust comprised of blobs of strange quark matter embedded in an uniform electron background may exist in the surface region of strange stars[88]. This heterogeneous strange star surface would have a negligible electric field which would make the existence of an ordinary nuclear crust, which requires a very strong electric field, impossible.

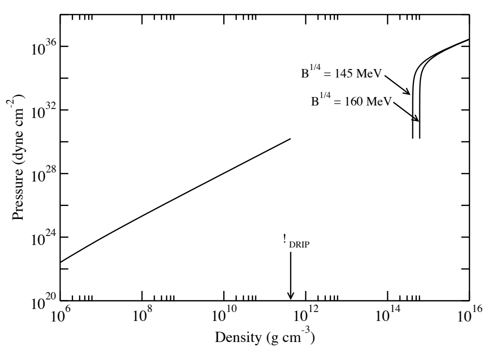

Fig. 4. Illustration of the EoS of strange stars with nuclear crusts (from Ref. 86).

density at the base of the crust (inner crust density) is determined by neutron drip, which occurs at about 4×10^{11} g/cm^3 or somewhat below[46]. The EoS of such a system is shown in Fig. 4. Sequences of compact strange stars with and without (bare) nuclear crusts are shown in Fig. 2. Since the nuclear crust is gravitationally bound to the quark matter core, the mass-radius relationship of strange stars with crusts resembles the one of neutron stars and even that of white dwarfs[11]. Bare strange stars obey $M \propto R^3$ because the mass density of quark matter is almost constant inside strange stars.

4.2. *Strange dwarfs*

For many years only rather vague tests of the theoretical mass-radius relationship of white dwarfs were possible. Recently the quality and quantity of observational data on the mass-radius relation of white dwarfs has been reanalyzed and profoundly improved by the availability of Hipparcos parallax measurements of several white dwarfs[89]. In that work Hipparcos parallaxes were used to deduce luminosity radii for 10 white dwarfs in visual binaries of common proper-motion systems as well as 11 field white dwarfs. Complementary HST observations have been made to better determine the spectroscopy for Procyon B[90] and pulsation of G226-29[91]. Procyon B at first appeared as a rather compact star which, however, was later confirmed to lie on the normal mass-radius relation of white dwarfs. Stars like Sirius B and 40 Erin B, fall nicely on the expected mass-radius relation too. Several other stars of this sample (e.g. GD 140, G156–64, EG 21, EG 50, G181–B5B, GD 279, WD2007–303, G238–44) however appear to be unusually compact and thus could be strange dwarf candidates [92]. The situation is graphically summarized in Fig. 5.

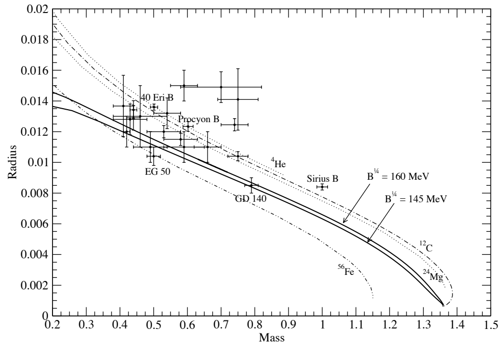

Fig. 5. Comparison of the theoretical mass-radius relationships of strange dwarfs (solid curves) and normal white dwarfs[86]. Radius and mass are in units of $R_\odot$ and $M_\odot$, respectively.

4.3. *Surface properties of strange matter*

The electrons surrounding strange quark matter are held to quark matter electrostatically. Since neither component, electrons and quark matter, is held in place gravitationally, the Eddington limit to the luminosity that a static surface may emit does not apply, and thus the object may have photon luminosities much greater than 10^{38} erg/s. It was shown by Usov[93] that this value may be exceeded by many orders of magnitude by the luminosity of e^+e^- pairs produced by the Coulomb barrier at the surface of a hot strange star. For a surface temperature of $\sim 10^{11}$ K, the luminosity in the outflowing pair plasma was calculated to be as high as $\sim 3\times10^{51}$ erg/s. Such an effect may be a good observational signature of bare strange stars[93,94,95,96]. If the strange star is enveloped by a nuclear crust however, which is gravitationally bound to the strange star, the surface made up of ordinary atomic matter would be subject to the Eddington limit. Hence the photon emissivity of such a strange star would be the same as for an ordinary neutron star. If quark matter at the stellar surface is in the CFL phase the process of e^+e^- pair creation at the stellar quark matter surface may be turned off, since cold CFL quark matter is electrically neutral so that no electrons are required and none are admitted inside CFL quark matter[85]. This may be different for the early stages of a hot CFL quark star[97].

5. Proto-Neutron Star Matter

Here we take a brief look at the composition of proto-neutron star matter. The composition is determined by the requirements of charge neutrality and equilibrium under the weak processes, $B_1 \to B_2 + l + \bar{\nu}_l$ and $B_2 + l \to B_1 + \nu_l$, where B_1 and B_2 are baryons, and l is a lepton, either an electron or a muon. For standard neutron star matter, where the neutrinos have left the system, these two require-

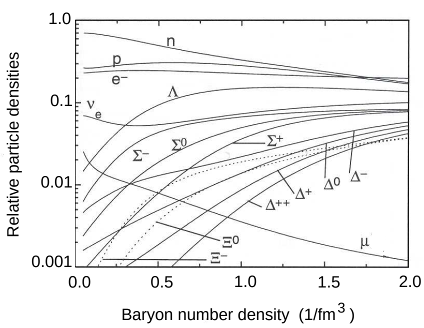

Fig. 6. Composition of hot (T = 40 MeV) proto-neutron star matter for $Y_L = 0.3$

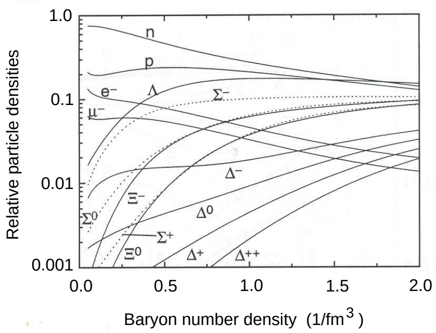

Fig. 7. Same as Fig. 6, but for standard neutron star matter.

ments imply that $Q = \sum_i q_i n_{B_i} + \sum_{l=e,\mu} q_l n_l = 0$ (electric charge neutrality) and $\mu_{B_i} = b_i \mu_n - q_i \mu_l$ (chemical equilibrium), where $q_{i/l}$ denotes the electric charge density of a given particle, and n_{B_i} (n_l) is the baryon (lepton) number density. The subscript i runs over all the baryons considered. The symbol μ_{B_i} refers to the chemical potential of baryon i, b_i is the particle's baryon number, and q_i is its charge. The chemical potential of the neutron is denoted by μ_n. When the neutrinos are trapped, as it is the case for proto-neutron star matter, the chemical equilibrium condition is altered to $\mu_{B_i} = b_i \mu_n - q_i(\mu_l - \mu_{\nu_l})$ and $\mu_e - \mu_{\nu_e} = \mu_\mu - \mu_{\nu_\mu}$, where μ_{ν_l} is the chemical potential of the neutrino ν_l. In proto-neutron star matter, the electron lepton number $Y_L = (n_e + n_{\nu_e})/n_B$ is initially fixed at a value of around $Y_{L_e} = Y_e + Y_{\nu_e} \simeq 0.3 - 0.4$ as suggested by gravitational collapse calculations of massive stars. Also, because no muons are present when neutrinos are trapped, the constraint $Y_{L_\mu} = Y_\mu + Y_{\nu_m u} = 0$ can be imposed. Figures 6 and 7 show sample compositions of proto-neutron star matter and standard neutron star matter (no neutrinos) computed for the RMF approximation. The presence of the Δ particle in (proto) neutron star matter at finite temperature is striking. This particle is generally absent in cold neutron star matter treated in RMF[2,3,98].

6. Electrically Charged Relativistic Star

As described in Sect. 4.1, the electric field existing on the surface of a strange quark star could be as high as 10^{19} V/cm[99]. The energy density associated with such a ultra-high field begins to have an impact one the geometry of space-time, as determined by Einstein's field equation[100,101,102]. In what follows, we investigate the effects of such ultra-high electric fields on the properties of relativistic polytropic stars. The consequences for strange stars are explored in Ref. 103. Since we want to maintain spherical symmetry of the star, the natural choice for the metric is

$$ds^2 = e^{\nu(r)} c^2 dt^2 - e^{\lambda(r)} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \,. \tag{1}$$

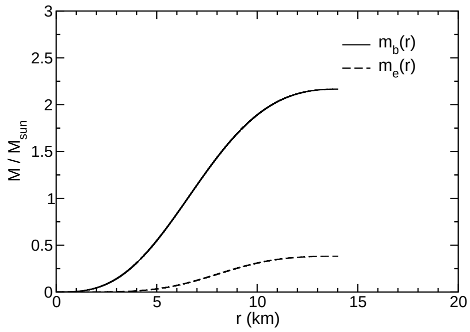

Fig. 8. Star's mass existing as baryonic mass, m_b, and electric mass-energy, m_e, for $f = 0.0008$.

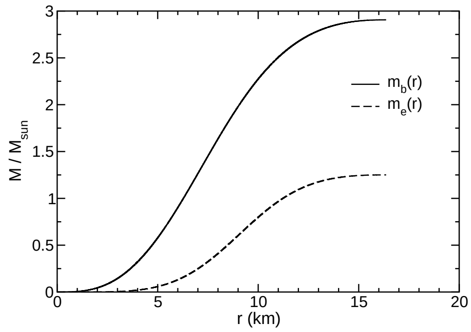

Fig. 9. Same as Fig. 8, but for $f = 0.001$

The energy-momentum tensor of the star consists of two terms, the standard term which describes the star's matter as a perfect fluid and the electromagnetic term,

$$T_\nu{}^\mu = (p + \rho c^2) u_\nu \, u^\mu + p \, \delta_\nu{}^\mu + \frac{1}{4\pi} \left(F^{\mu l} F_{\nu l} + \frac{1}{4\pi} \, \delta_\nu{}^\mu \, F_{kl} \, F^{kl} \right) , \tag{2}$$

where the components $F^{\nu\mu}$ satisfy the covariant Maxwell equations $[(-g)^{1/2} F^{\nu\mu}]_{,\mu} = 4\pi J^\nu (-g)^{1/2}$, with J^ν the four current. For the radial component one obtains

$$F^{01}(r) = E(r) = e^{-(\nu+\lambda)/2} \, r^{-2} \int_0^r 4\pi j^0 e^{(\nu+\lambda)/2} dr' \, . \tag{3}$$

From last equation we can define the charge of the system as

$$Q(r) = \int_0^r 4\pi j^0 r'^2 e^{(\nu+\lambda)/2} dr' . \tag{4}$$

The electric field is then given by

$$E(r) = e^{-(\nu+\mu)/2} \, r^{-2} \, Q(r) \, . \tag{5}$$

With the aid of these relations the energy-momentum tensor takes the form

$$T_\nu{}^\mu = \begin{pmatrix} -\left(\epsilon + \frac{Q^2(r)}{8\pi r^4}\right) & 0 & 0 & 0 \\ 0 & p - \frac{Q^2(r)}{8\pi r^4} & 0 & 0 \\ 0 & 0 & p + \frac{Q^2(r)}{8\pi r^4} & 0 \\ 0 & 0 & 0 & p + \frac{Q^2(r)}{8\pi r^4} \end{pmatrix} . \tag{6}$$

Substituting this relation into Einstein's field equation, $G_\nu{}^\mu = (8\pi G/c^4) T_\nu{}^\mu$, leads to

$$e^{-\lambda} \left(-\frac{1}{r^2} + \frac{1}{r} \frac{d\lambda}{dr} \right) + \frac{1}{r^2} = \frac{8\pi G}{c^4} \left(p - \frac{Q^2(r)}{8\pi r^4} \right) , \tag{7}$$

$$e^{-\lambda} \left(\frac{1}{r} \frac{d\nu}{dr} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -\frac{8\pi G}{c^4} \left(\epsilon + \frac{Q^2(r)}{8\pi r^4} \right) . \tag{8}$$

Table 1. Results

f	M $(M_\odot)$	M_b $(M_\odot)$	M_e $(M_\odot)$	Ratio	Charge ($\times 10^{17}$ C)
0.0	1.428	1.428	0	0%	0
0.0001	1.439	1.436	0.00250	0.17%	260.95
0.0008	2.548	2.166	0.38227	15%	3488.01
0.001	4.156	2.905	1.250	30.07%	6727.17

The solution for the metric function $\lambda(r)$ is given by

$$e^{-\lambda} = 1 - \frac{Gm(r)}{rc^2} + \frac{GQ^2}{r^2c^4} \, . \tag{9}$$

The first two terms on the right-hand-side of Eq. (9) describe electrically neutral stars, while the third term originates from the net electric charge distribution inside the star. From Eqs. (7)–(9) one finds that the mass contained in a spherical shell of radius, $m(r)$, is given by

$$\frac{dm(r)}{dr} = \frac{4\pi r^2}{c^2}\epsilon + \frac{Q(r)}{c^2 r}\frac{dQ(r)}{dr} \, . \tag{10}$$

The first term on the right-hand-side is the standard result for the gravitational mass of electrically uncharged stars, while the second term accounts for the mass change that originates from the electric field. Finally, the Tolman-Oppenheimer-Vokoff (TOV) equation generalized to electrically charged stars follows as

$$\frac{dp}{dr} = -\frac{2G\left(m(r) + \frac{4\pi r^3}{c^2}\left(p - \frac{Q^2(r)}{4\pi r^4 c^2}\right)\right)}{c^2 r^2\left(1 - \frac{2Gm(r)}{c^2 r} + \frac{GQ^2(r)}{r^2c^4}\right)}(p+\epsilon) + \frac{Q(r)}{4\pi r^4}\frac{dQ(r)}{dr} \, . \tag{11}$$

To solve the TOV equation, we need to specify the charge distribution inside the star. Here, we will follow the approach of Ref. 100, 104 and assume that the charge distribution is proportional to the energy density, $j^0(r) = f \times \epsilon$, where f is a constant which essentially controls the amount of net electric charge carried by the star. Moreover, we choose as initial and boundary conditions of the problem the following values, $\epsilon(0) = 1550$ MeV/fm^3, $Q(0) = m(0) = \lambda(0) = \nu(0) = 0$, and $p(R) = 0$. In this study we focus on the difference between the mass-energy coming from baryonic matter, here represented by the polytropic EoS, and from the electric field. To this aim we rewrite Eq. (10) as (b=baryonic matter, e=electric field)

$$\frac{dm(r)}{dr} = \frac{dm_b}{dr} + \frac{dm_e}{dr} \, , \quad \text{where} \;\; \frac{dm_b}{dr} = \frac{4\pi r^2}{c^2}\epsilon \, , \;\; \frac{dm_e}{dr} = \frac{Q(r)}{c^2 r}\frac{dQ(r)}{dr} \, . \tag{12}$$

Table 6 summarizes the properties of several electrically charged sample stars. Figures 8 and 9 show, for two selected values of f, how much of the star's total (gravitational) mass exists as baryonic mass (m_b) and how much mass-energy (m_e) is associated with the electric field energy. For a strongly charged star, $f = 0.001$, we find that 30% of the star's mass (1.25 $M_\odot$) is actually electrostatic energy.

Acknowledgments

The research of F. Weber is supported by the National Science Foundation under Grant PHY-0457329, and by the Research Corporation.

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