Solving Navier-Stokes Equations with Mimetic Operators

Here I present a new scheme for solving Navier-Stokes equations using mimetic difference operators. These operators can be constructed to high orders of accuracy and maintain the physical properties of the problems under consideration. We demonstrate the effectiveness of our scheme by modeling a lock release in 3D Cartesian coordinates, then extend our techniques to 3D curvilinear grids. The resulting scheme allows for simple and efficient computation of fluid processes on curvilinear grids, which allows us to solve problems in more complex regions while minimizing the restrictions of finite difference methods.

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The Navier-Stokes Equation

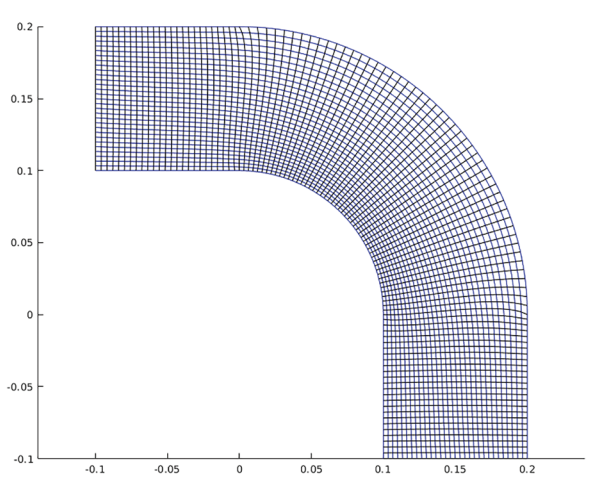
$$\frac{\partial \boldsymbol{u}}{\partial t} + \hat{\boldsymbol{u}} \cdot (\nabla \hat{\boldsymbol{u}}) = -\frac{1}{\rho}\nabla \hat{p} + \frac{1}{Re}\nabla^2 \hat{\boldsymbol{u}} + \frac{1}{Fr^2}\hat{S}$$

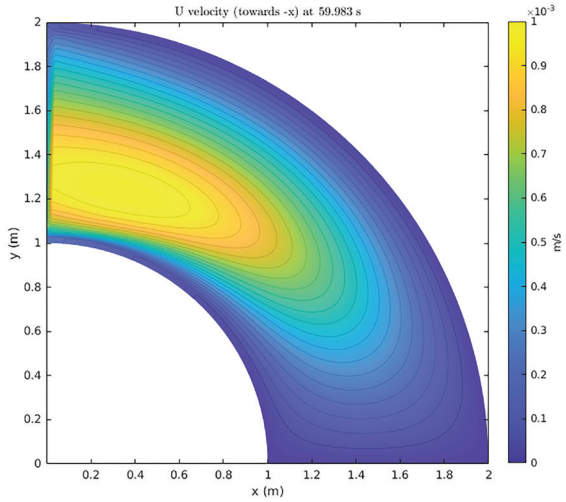
In curvilinear coordinates, are very difficult to solve

$$\frac{\partial}{\partial \tau}\left(\frac{\rho \boldsymbol{u}}{J}\right) + \boldsymbol{u}\frac{\partial}{\partial \xi^i}\left(\frac{\rho \boldsymbol{g}^i}{J}\boldsymbol{u}\right) = -\frac{\boldsymbol{g}^i}{\rho J}\frac{\partial p}{\partial \xi^i} + \frac{1}{Re}\frac{\partial}{\partial \xi^i}\left(\frac{\mu \boldsymbol{g}^i \cdot \boldsymbol{g}^k}{J}\frac{\partial \boldsymbol{u}}{\partial \xi^k}\right) + \frac{S}{Fr^2\, J}$$

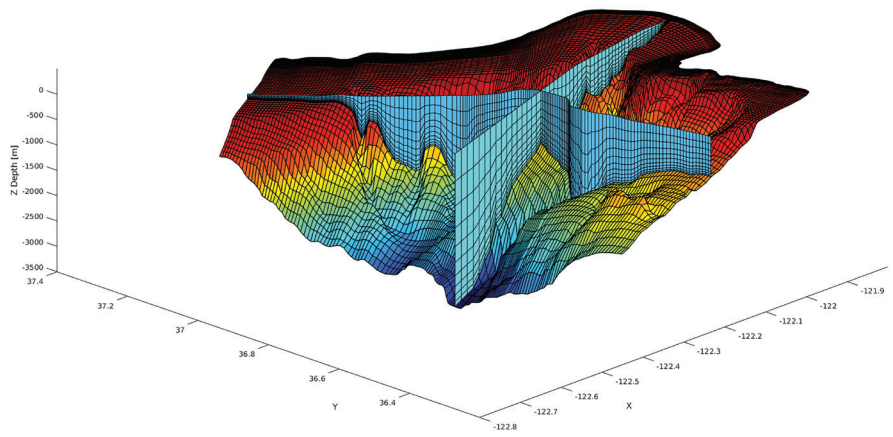
Mimetic curvilinear operators G, D, L removes difficult part

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \tilde{\boldsymbol{G}}\boldsymbol{u} = -\frac{1}{\rho}\tilde{\boldsymbol{D}}p + \frac{1}{Re}\tilde{\boldsymbol{L}}\boldsymbol{u} + \frac{S}{Fr^2}$$





No special requirement is needed for solving fluid problems on curvilinear grids.Mimetic curvilinear operators perform the appropriate transforms from physical to logical grids.



Nearshore coastal bathymetry can be very intense, and curvilinear grids are used to better capture the effects of those features. Monterey Bay Canyon, (shown here), is notoriously difficult to model due to its steep canyons and near vertical walls. Using curvilinear operators, the complex grid is transformed into a logical cube where the calculus operations are performed.