Hilbert-Huang Transform Analysis of Global Precipitation Datasets Since 1900

Sam Shen 沈善普 and David New 牛大卫

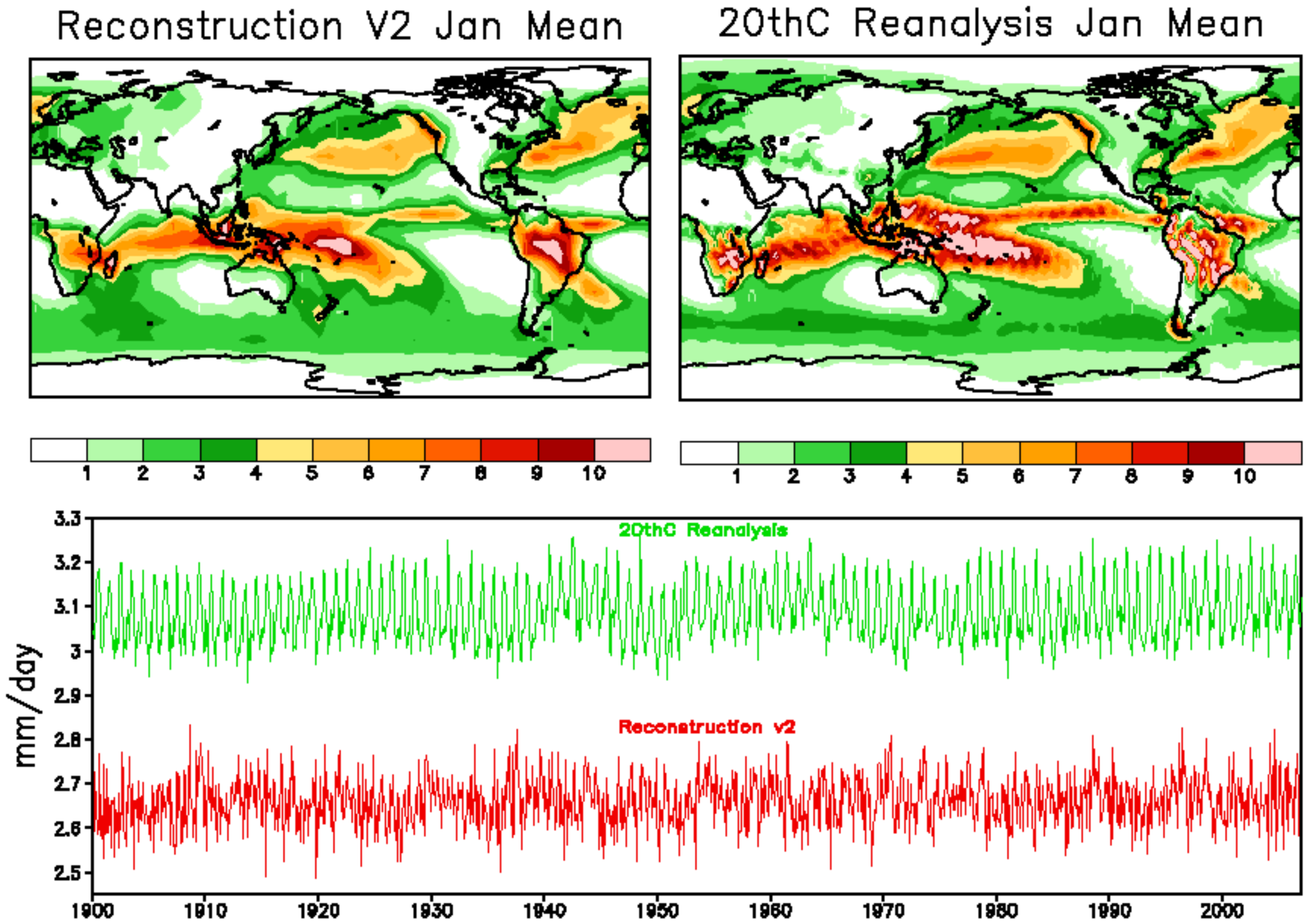
San Diego State University, USA

and

Thomas Smith, Phillip Arkin, and Li Ren

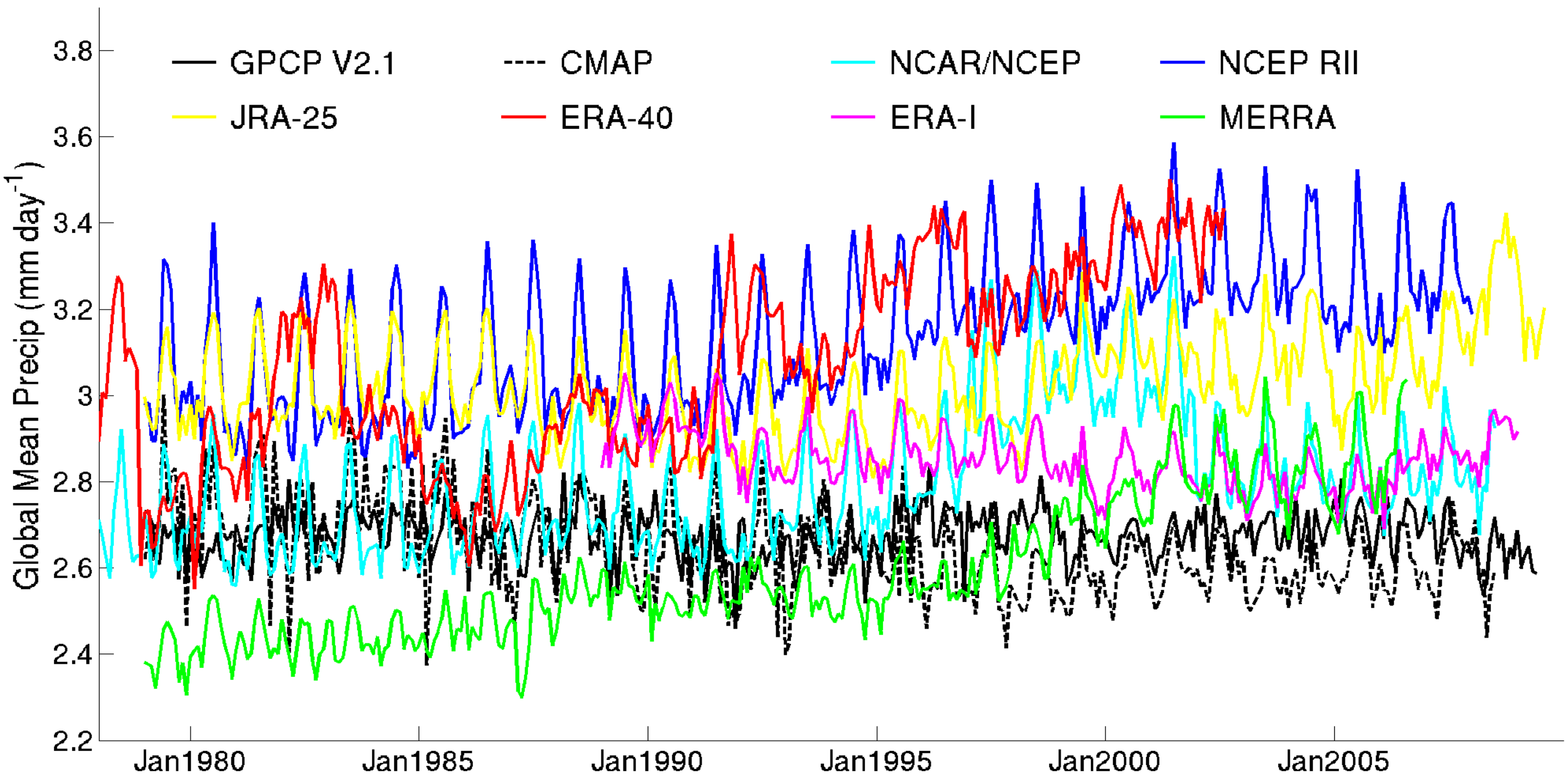
NOAA Cooperative Institute for Climate and Satellites and University of Maryland, USA

Email: shen@math.sdsu.edu



Reconstruction V2 Jan Mean
20thC Reanalysis Jan Mean
1 2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10
20thC Reanalysis
Reconstruction v2
mm/day
3.3
3.2
3.1
3
2.9
2.8
2.7
2.6
2.5
1900 1910 1920 1930 1940 1950 1960 1970 1980 1990 2000

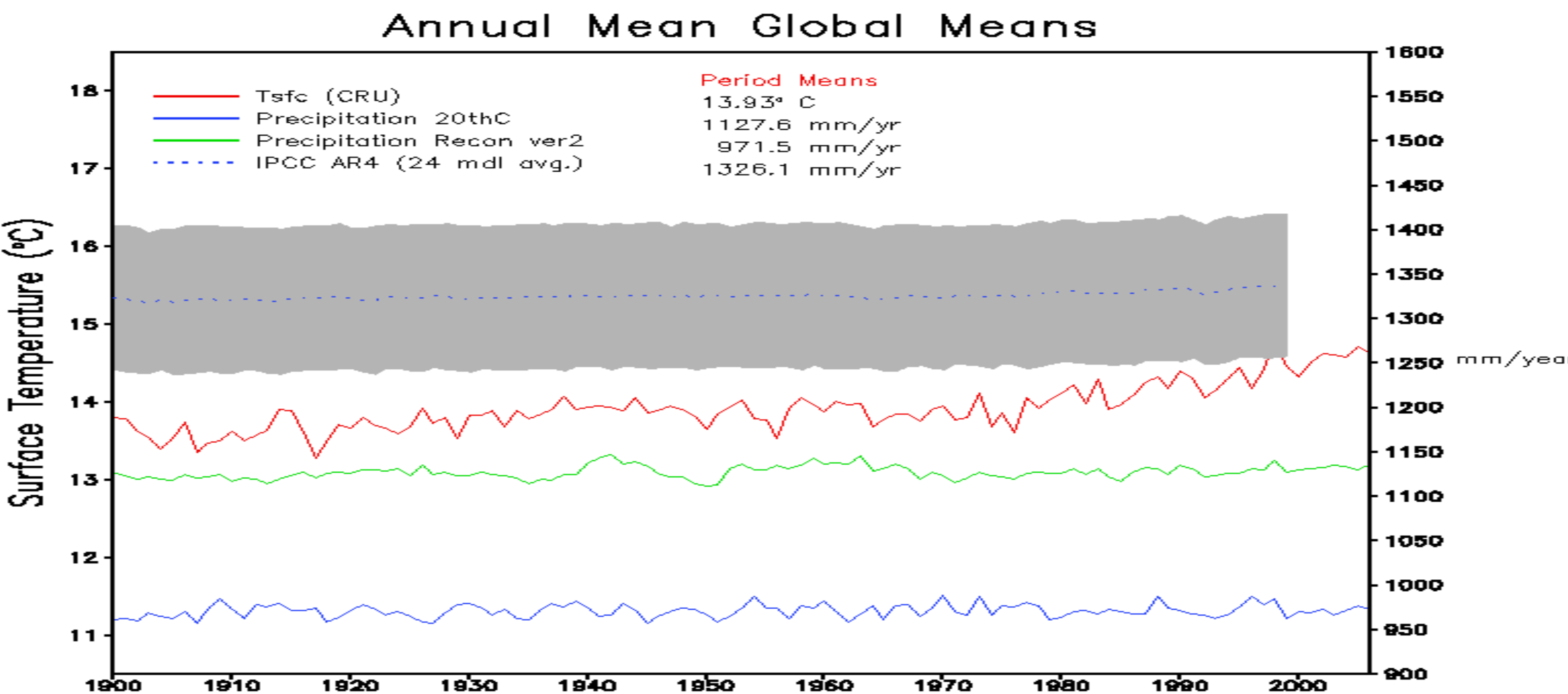
Global annual precipitation from models and observations



- Datasets based on observations (GPCP, CMAP) give 2.6-2.7 mm/day (AR4 range is about 3.2-3.9 mm/day)
- Data assimilation products average about 3 mm/day; also have larger mean annual cycle and greater interannual variability than GPCP/ CMAP

Global Mean Precipitation

- Lowest (blue) curve (2.66 mm/day) is reconstruction mean (where totals are obtained by adding GPCP climatology)
- Green curve (3.09 mm/day) is from 20th Century reanalysis
- Upper (blue dotted) curve (3.63 mm/day) is mean of 24 model simulations from AR4; gray area is ±1 standard deviation of the model means
- Red is global mean temperature (from CRU)



Scientific issues:

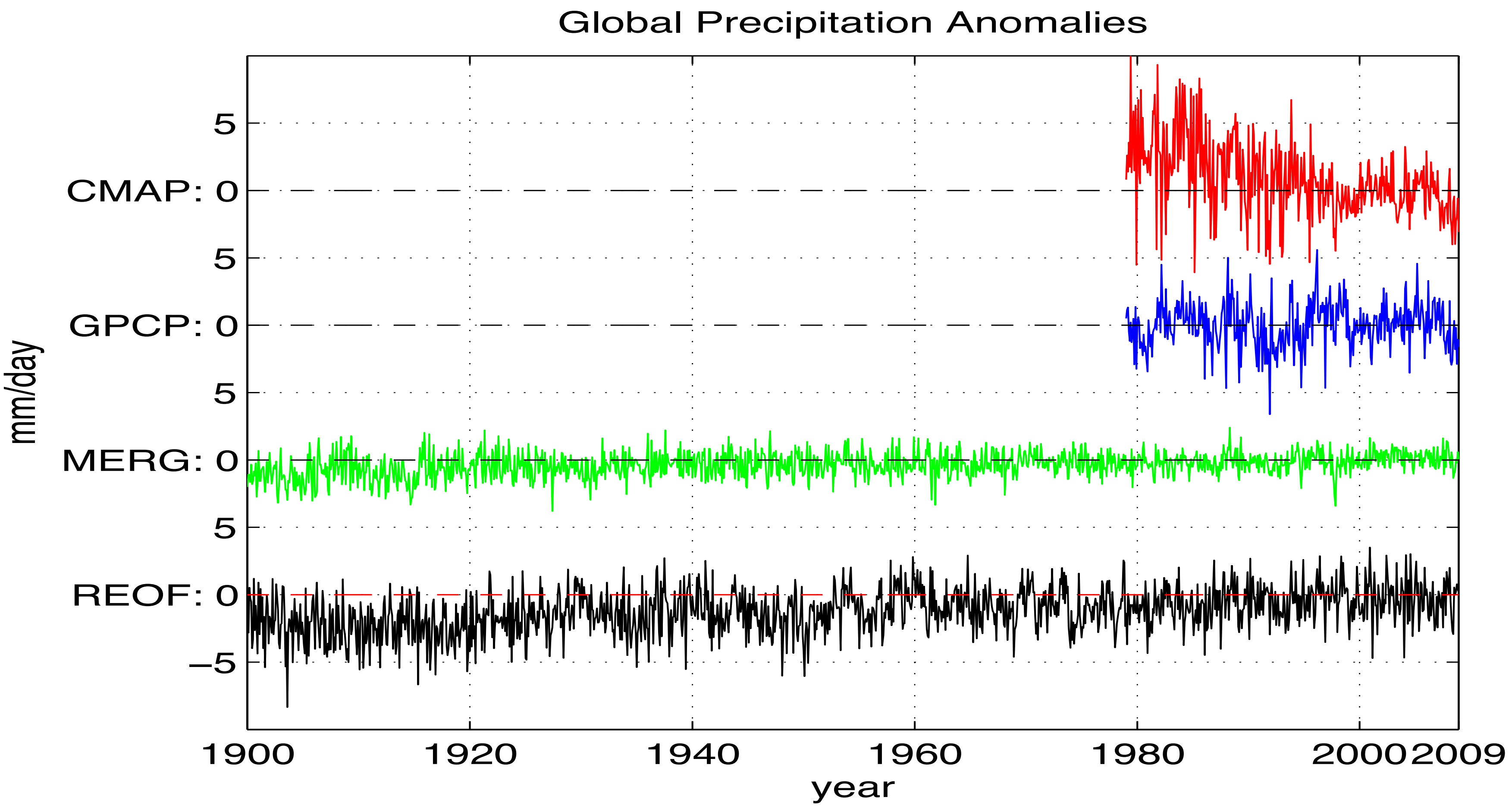
There is a consistent discrepancy between the "observed" and climate model precipitations.

The model precipitation is about 15%-30% higher than the "observed".

HHT is used as one of the tools to resolve this issue.

Four "observed" global average monthly precipitation [Units: mm/day]

- **CMAP: CPC Merged Analysis of Precipitation**
- **GPCP: Global Precipitation Climatology Project**
- **MERG and REOF: Statistical reconstruction by Tom Smith**



Statistics of the four datasets

	Mean [mm/day]	Standard Deviation [mm/day]	Skewness [dimensionless]	Kurtosis [dimensionless]	Trend [(mm/day)/decade]
1979-2008					
MERG	-0.36	0.74	-0.30	1.17	0.1624
REOF	-0.10	1.47	-0.06	-0.01	0.3194
GPCP	-0.01	1.73	-0.10	0.69	0.0958
CMAP	0.87	2.81	0.43	0.42	-1.5153
1900-2008					
MERG	-0.36	0.94	-0.21	0.16	0.0895
REOF	-0.49	1.62	-0.10	-0.11	0.1126

Introduction to HHT (Hilbert-Huang Transform)

David Hilbert (1862-1943)

Norden Huang at NASA/GSFC and NCU/Taiwan

Introduction to HHT (Hilbert-Huang Transform)

- Original papers of the HHT method

1. Huang, N. E., Long, S. R.and Shen, Z. 1996: The mechanism for frequency downshift in nonlinear wave evolution. Adv. Appl. Mech., 32, 59-111.
2. Huang, M. L. Wu, S. R. Long, S. S. Shen, W. D. Qu, P. Gloersen, and K. L. Fan (1998), The empirical mode decomposition and the Hilbert spectrum for nonlinear and non-stationary time series analysis. Proc. Roy. Soc. Lond., 454A, 903-993.
3. Huang, N. E., Z. Shen, and S. R. Long (1999), A New View of Nonlinear Water Waves – The Hilbert Spectrum, Ann. Rev. Fluid Mech., 31, 417-457.

- Book: NE Huang and SSP Shen, Hilbert-Huang Transform and Its Applications, World Scientific, 2005.

Hilbert Transform, phase, and frequency of data

For any data $x(t)$, *Hilbert transform is defined as*

$$y(t) = H[x(t)] = \frac{1}{\pi} \wp \int_{\tau} \frac{x(\tau)}{t-\tau} d\tau .$$

If $x(t) = \cos t$, *then* $H[x](t) = \sin t$ *and* $x + iy = e^{it}$ *with a phase angle* $\theta = t$ *and frequency* $\omega = d\theta / dt = 1.$

In general, for what $x(t)$ *such that* $H[x](t)$ *is complex conjugate:*

$$z(t) = x(t) + i\, y(t) = a(t)\, e^{i\theta(t)},$$

with

$$a(t) = \left(x^2 + y^2\right)^{1/2}, \quad \theta(t) = \tan^{-1} \frac{y(t)}{x(t)}, \quad \omega = \frac{d\theta}{dt}.$$

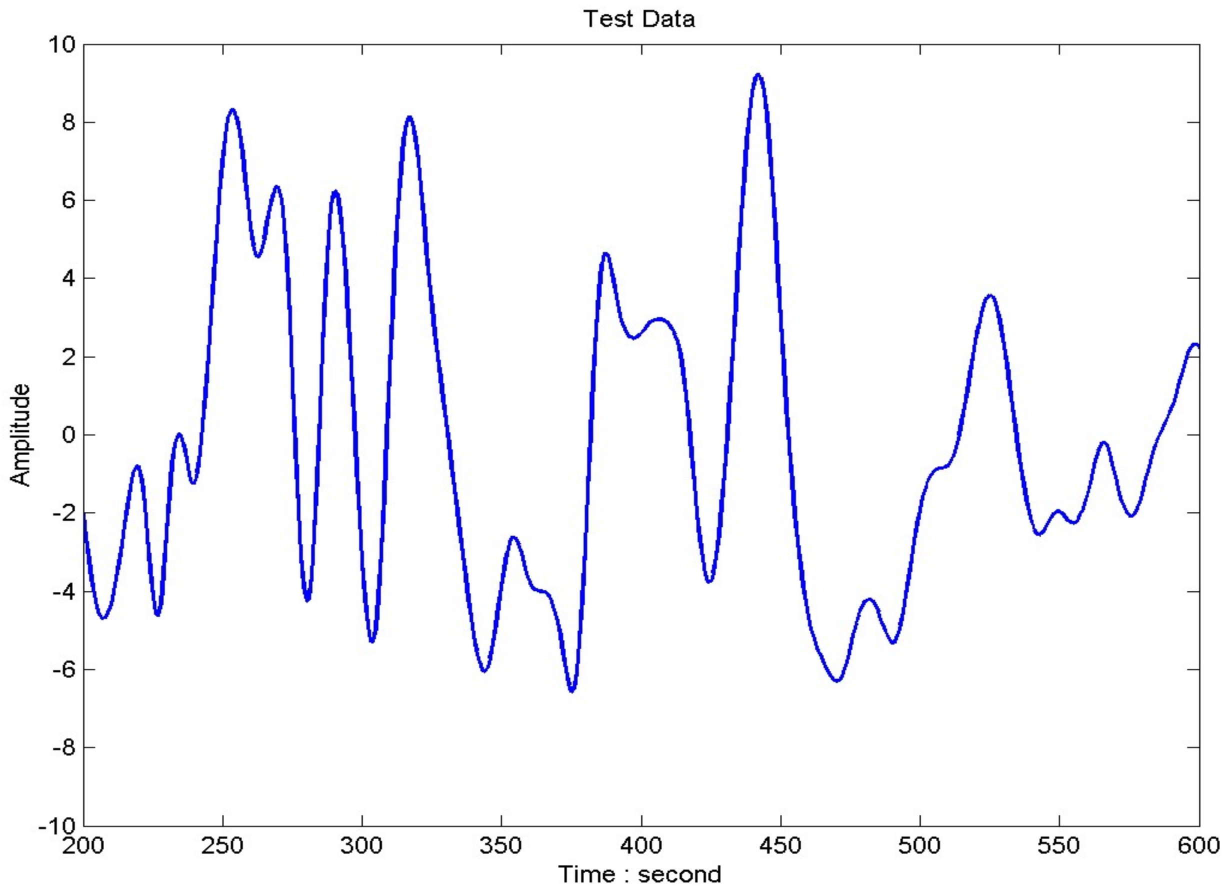
Introduction to HHT (Hilbert-Huang Transform)

- What functions have mathematically uniquely defined and physically meaningful frequency?

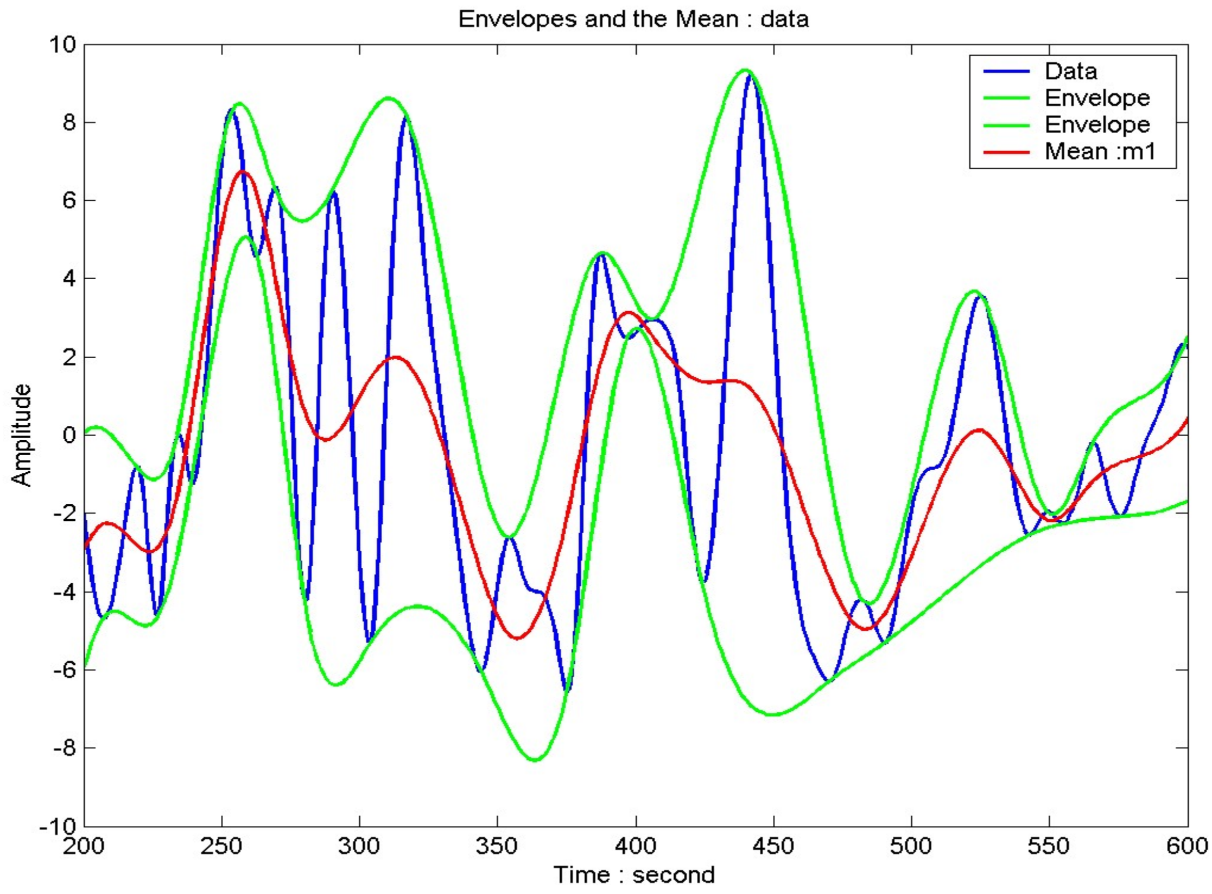
- Answer: Intrinsic mode functions (IMFs) calculated from empirical model decomposition procedures, aka sifting procedures developed by NE Huang.

Empirical Mode Decomposition:

Methodology : Test Data

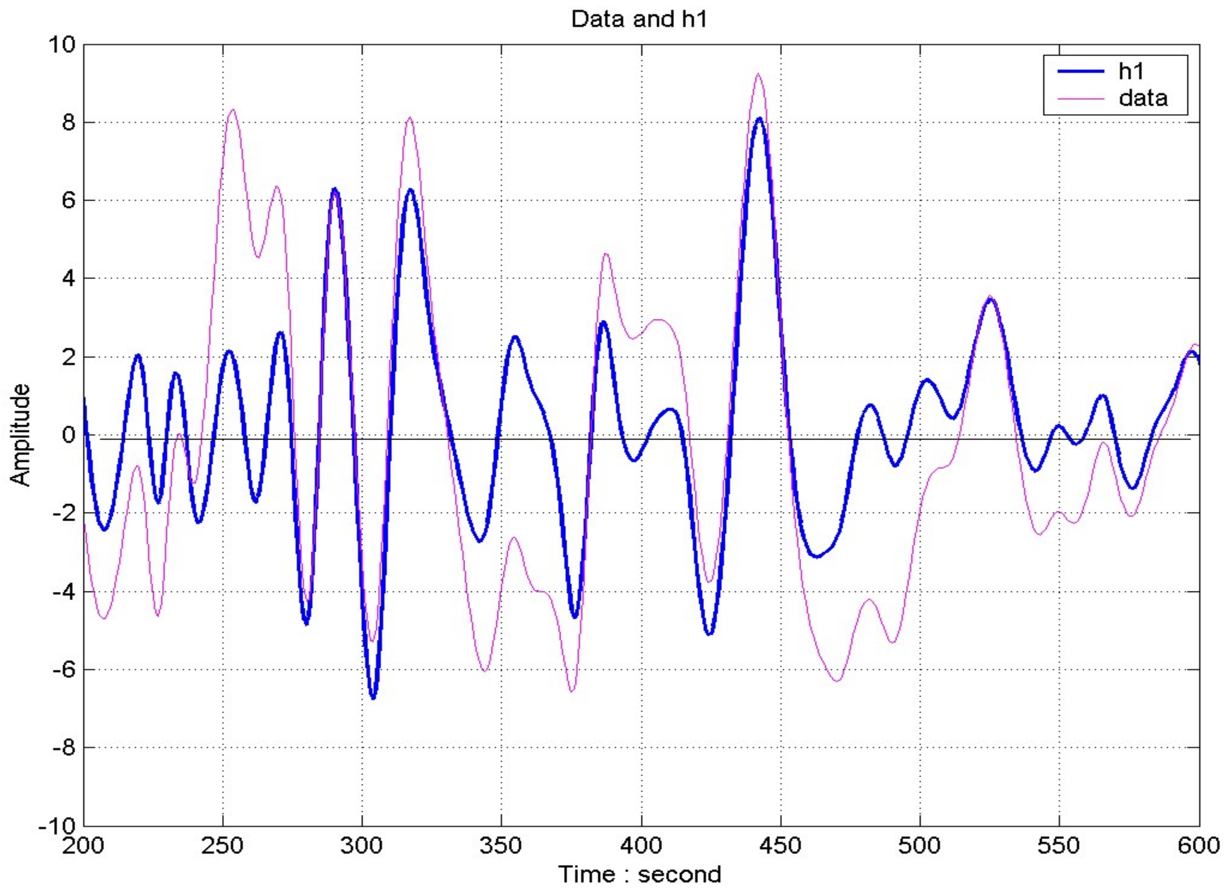


Empirical Mode Decomposition:
Methodology : data and m1

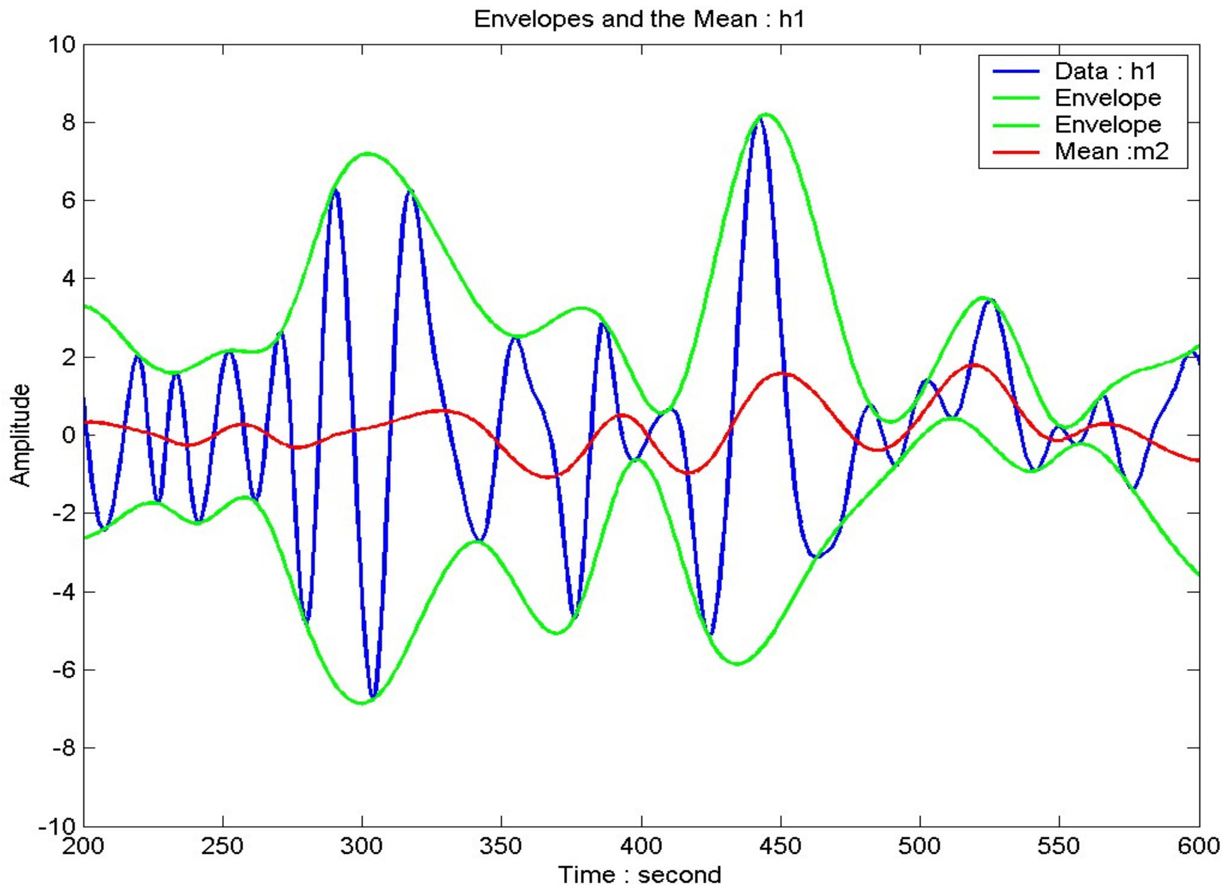


Empirical Mode Decomposition:

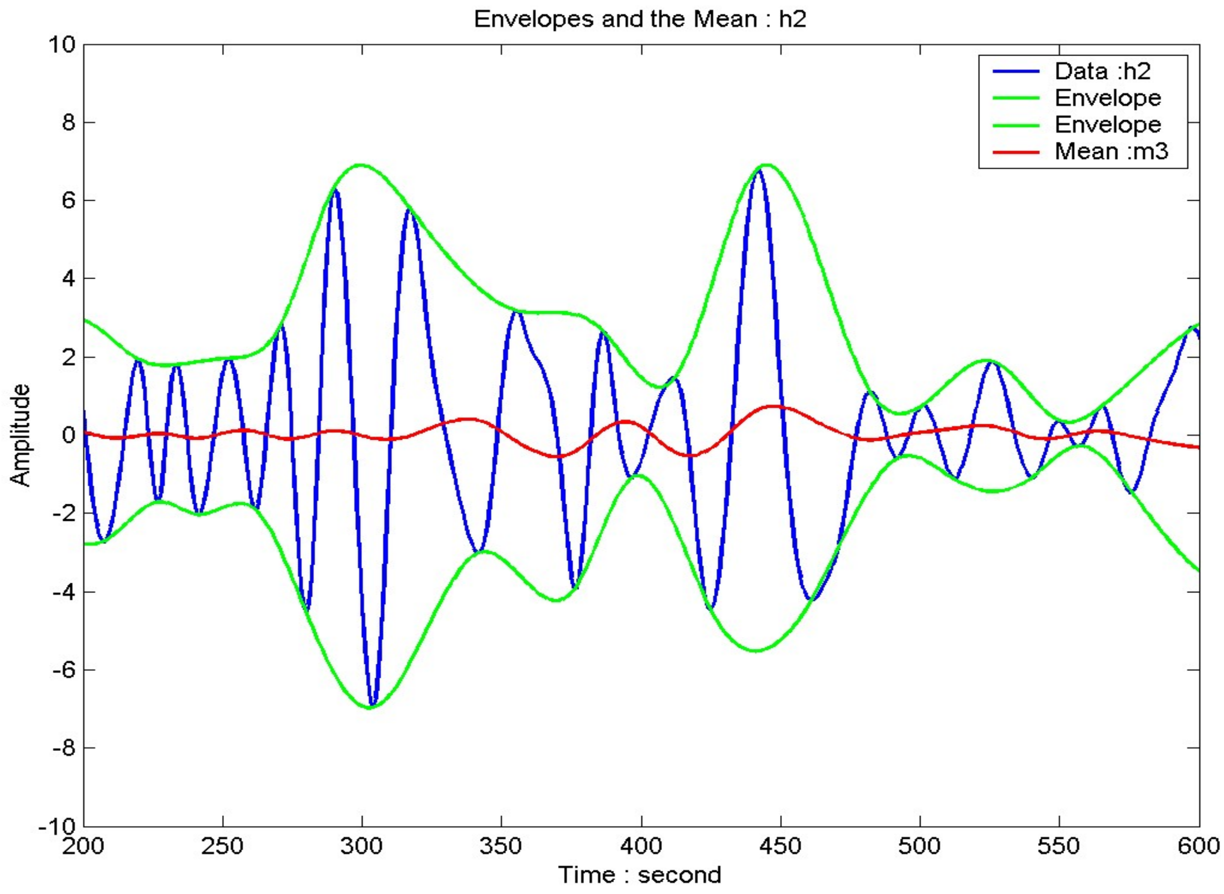
Methodology : data & h1



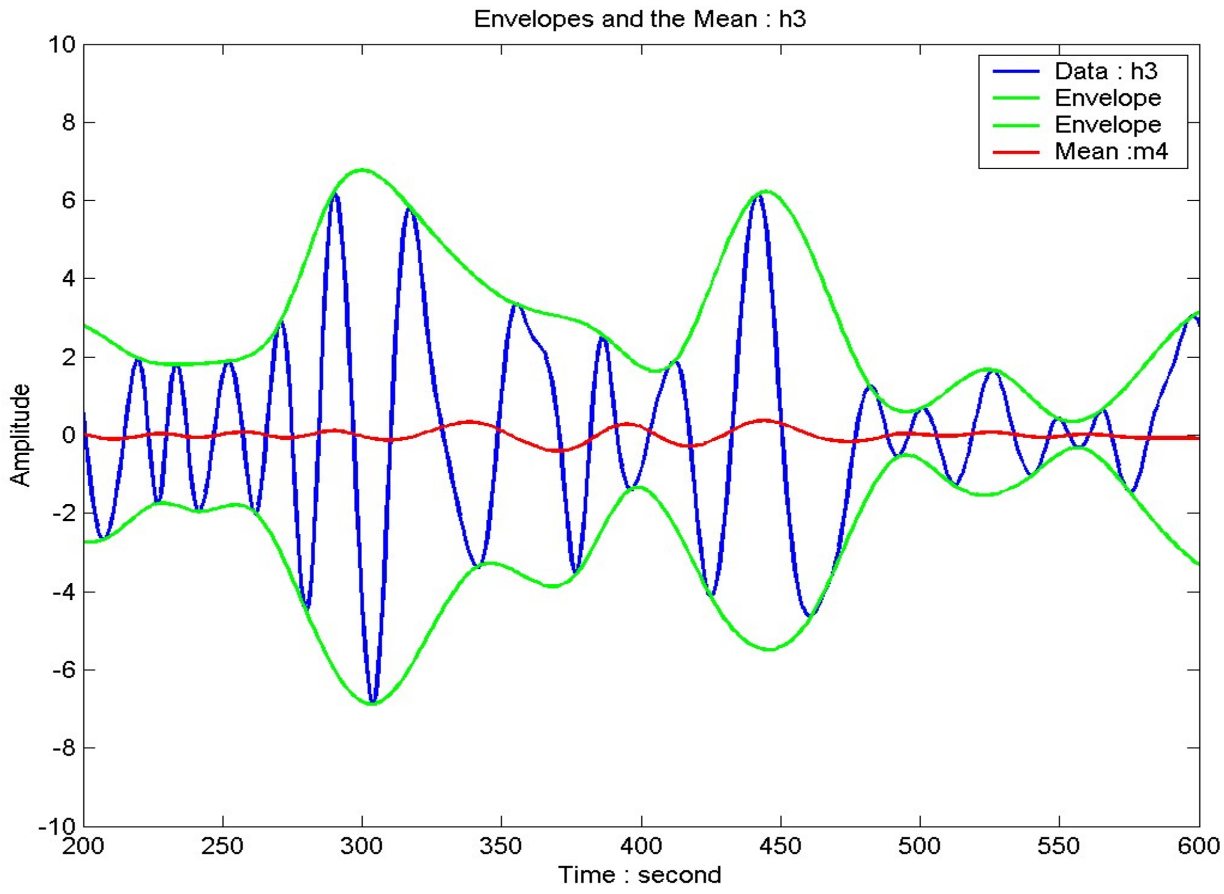
Empirical Mode Decomposition:
Methodology : h1 & m2



Empirical Mode Decomposition:
Methodology : h2 & m3

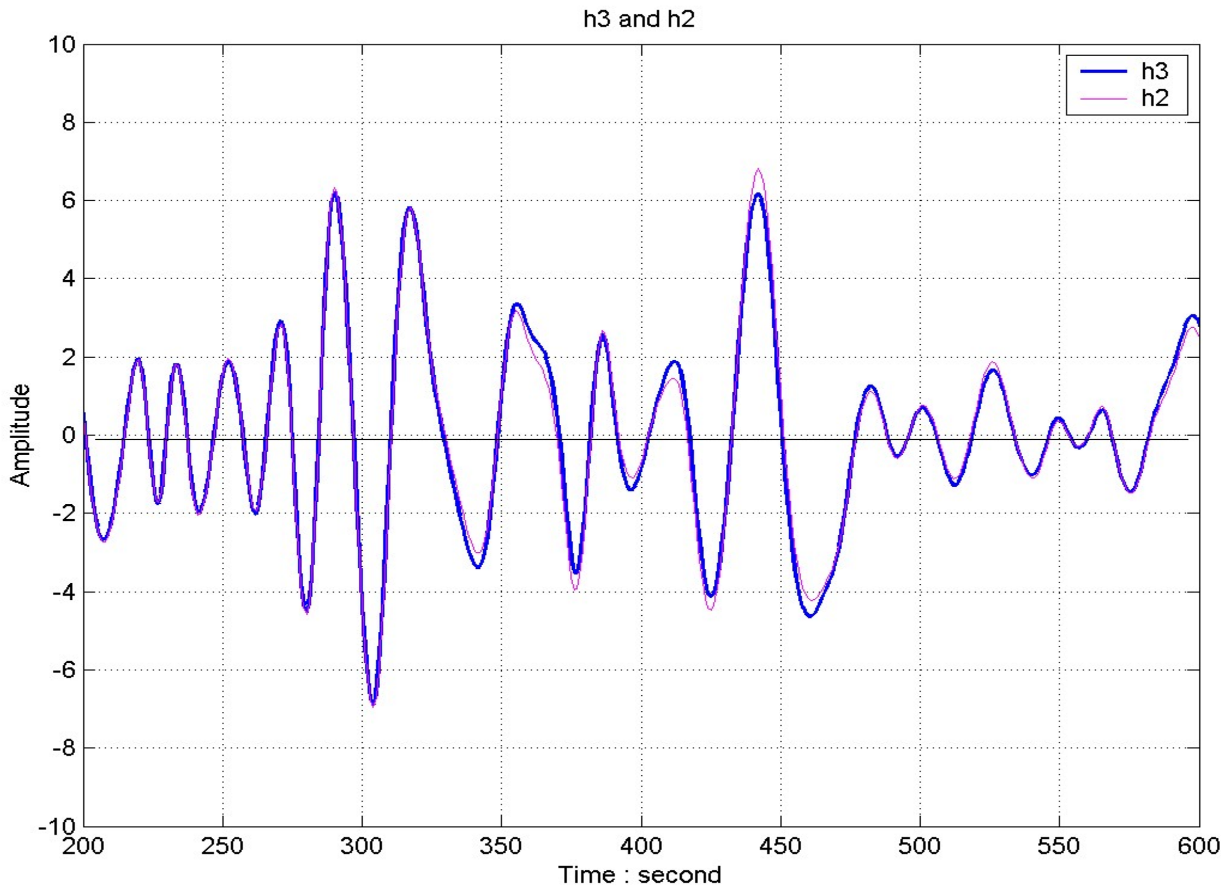


Empirical Mode Decomposition:
Methodology : h3 & m4

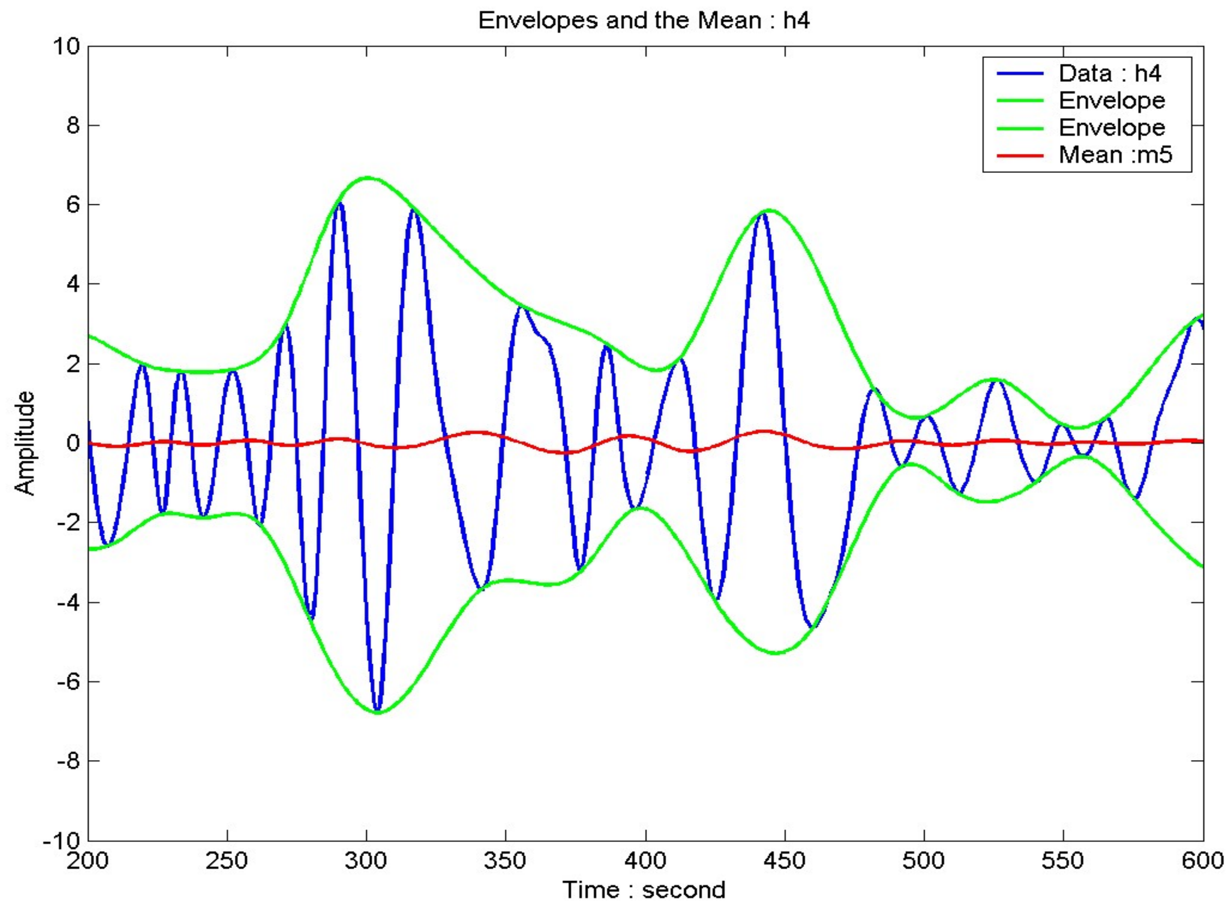


Empirical Mode Decomposition:

Methodology : h2 & h3



Empirical Mode Decomposition:
Methodology : h4 & m5



Empirical Mode Decomposition

Sifting : to get one IMF component

$$x(t) - m_1 = h_1 \ ,$$

$$h_1 - m_2 = h_2 \ ,$$

$$\cdots\cdots$$

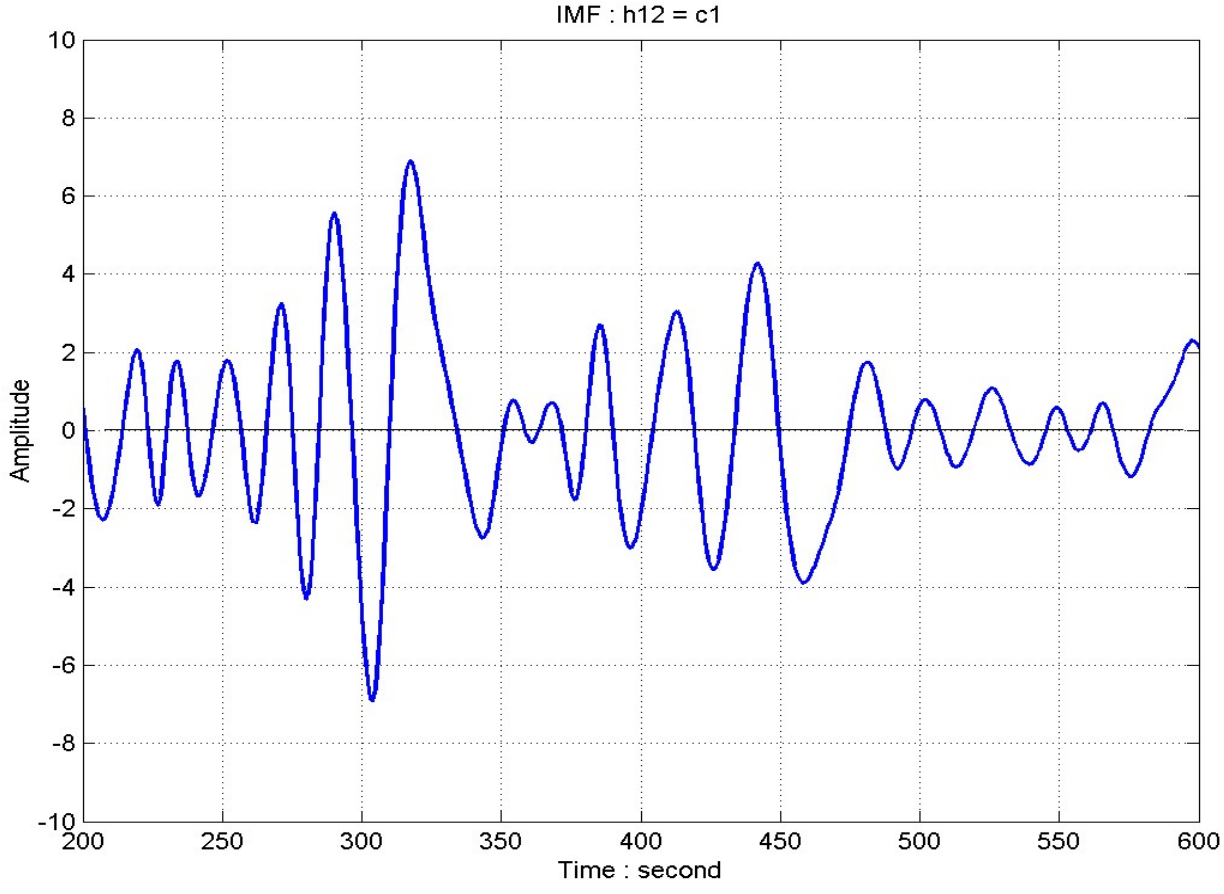
$$\cdots\cdots$$

$$h_{k-1} - m_k = h_k \ .$$

$$\Rightarrow \quad h_k = c_1 \quad .$$

Empirical Mode Decomposition:

Methodology : IMF c1



Definition of the Intrinsic Mode Function

Any function having the same numbers of zero – cros sin gs and extrema , and also having symmetric envelopes defined by local max ima and min ima respectively is defined as an Intrinsic Mode Function (IMF).

Each IMF and its Hilbert Transform yield a meaningful phase and amplitude :

$$\Rightarrow\Rightarrow \quad c(t) = a(t)\,e^{i\theta(t)}$$

Empirical Mode Decomposition

Sifting : to get all the IMF components

$$x(t) - c_1 = r_1 \; ,$$

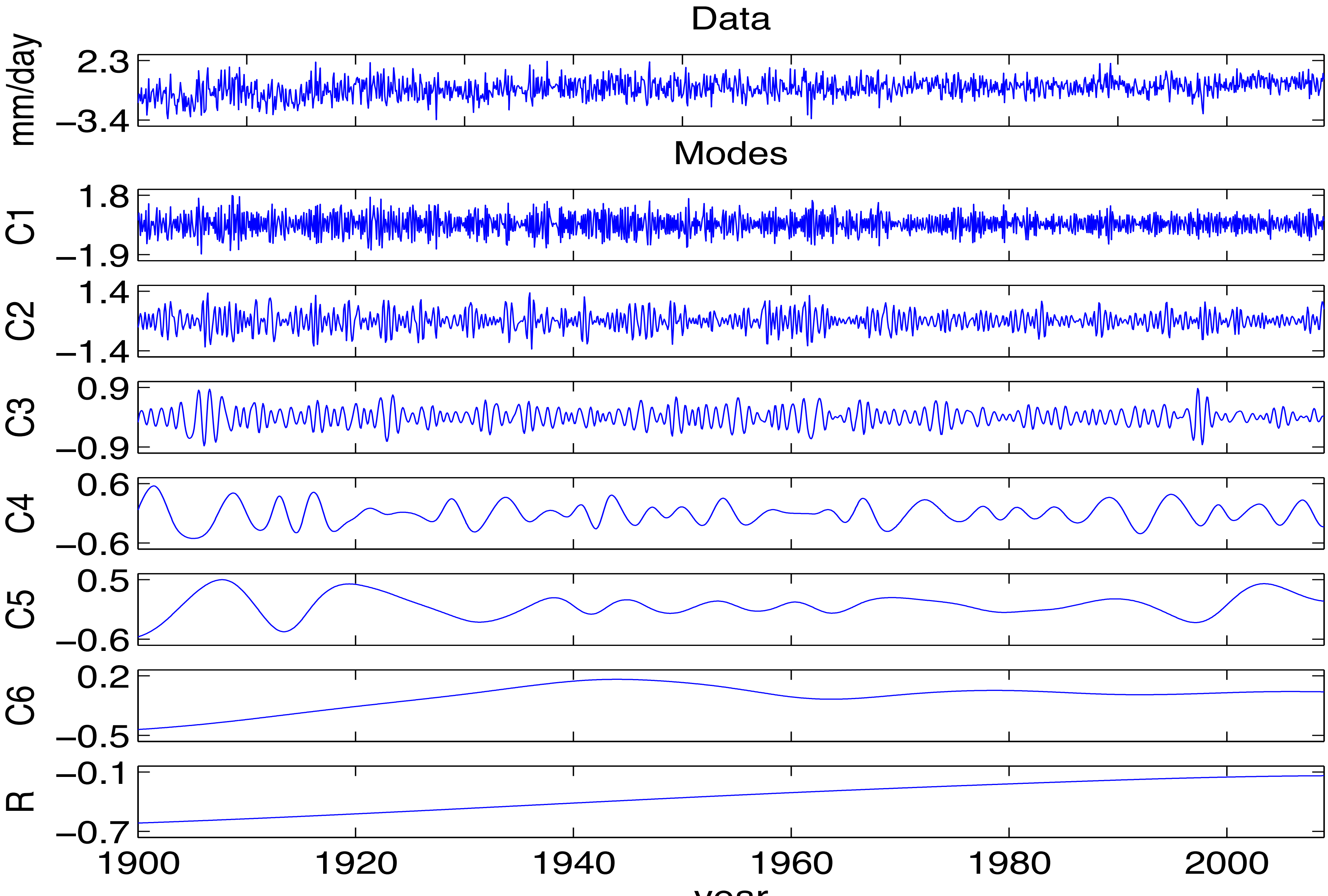
$$r_1 - c_2 = r_2 \; ,$$

$$\cdots$$

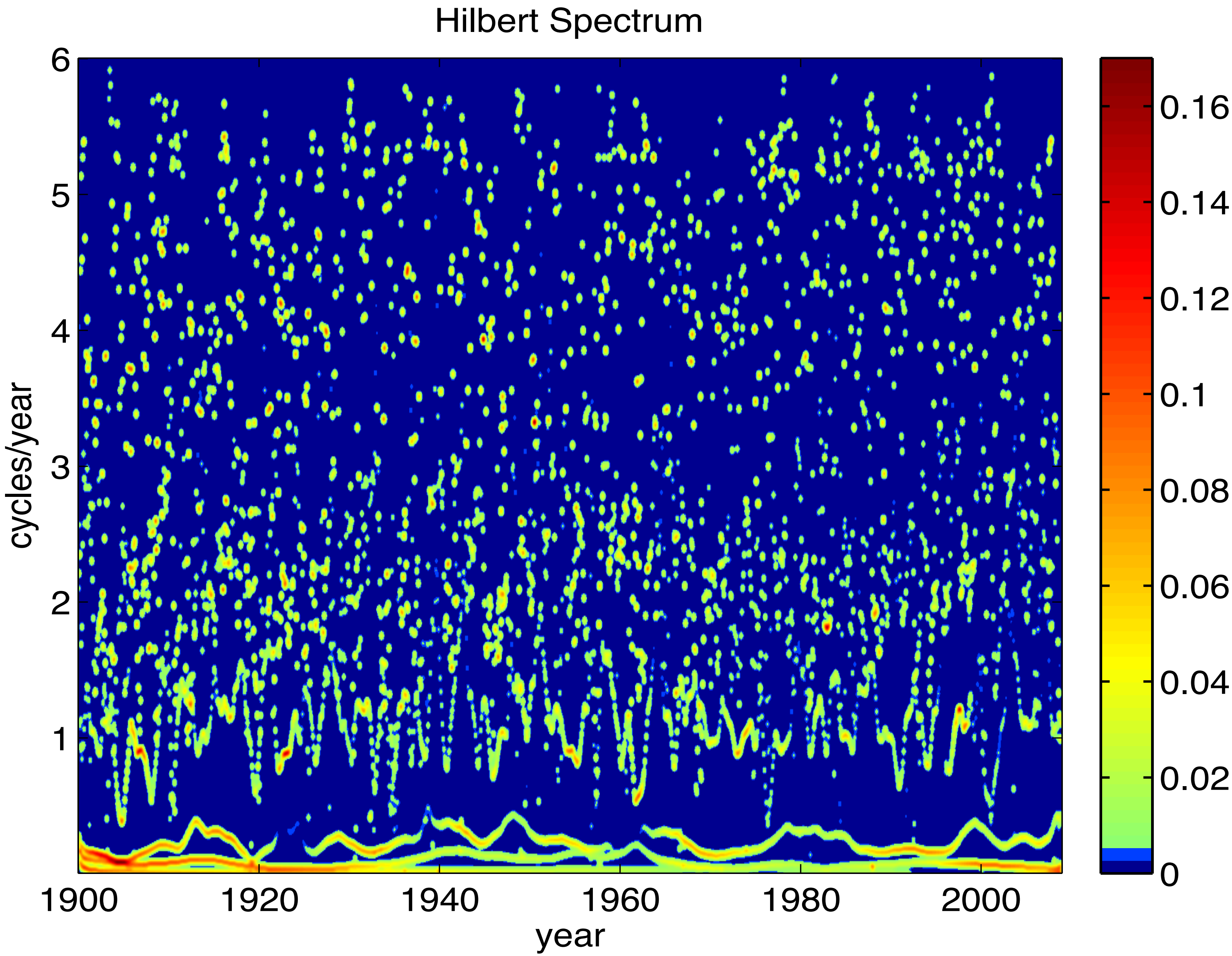
$$r_{n-1} - c_n = r_n \; .$$

$$\Rightarrow x(t) - \sum_{j=1}^{n} c_j = r_n \; , \; x(t) = \left(\sum_{j=1}^{n} c_j \right) + r_n$$

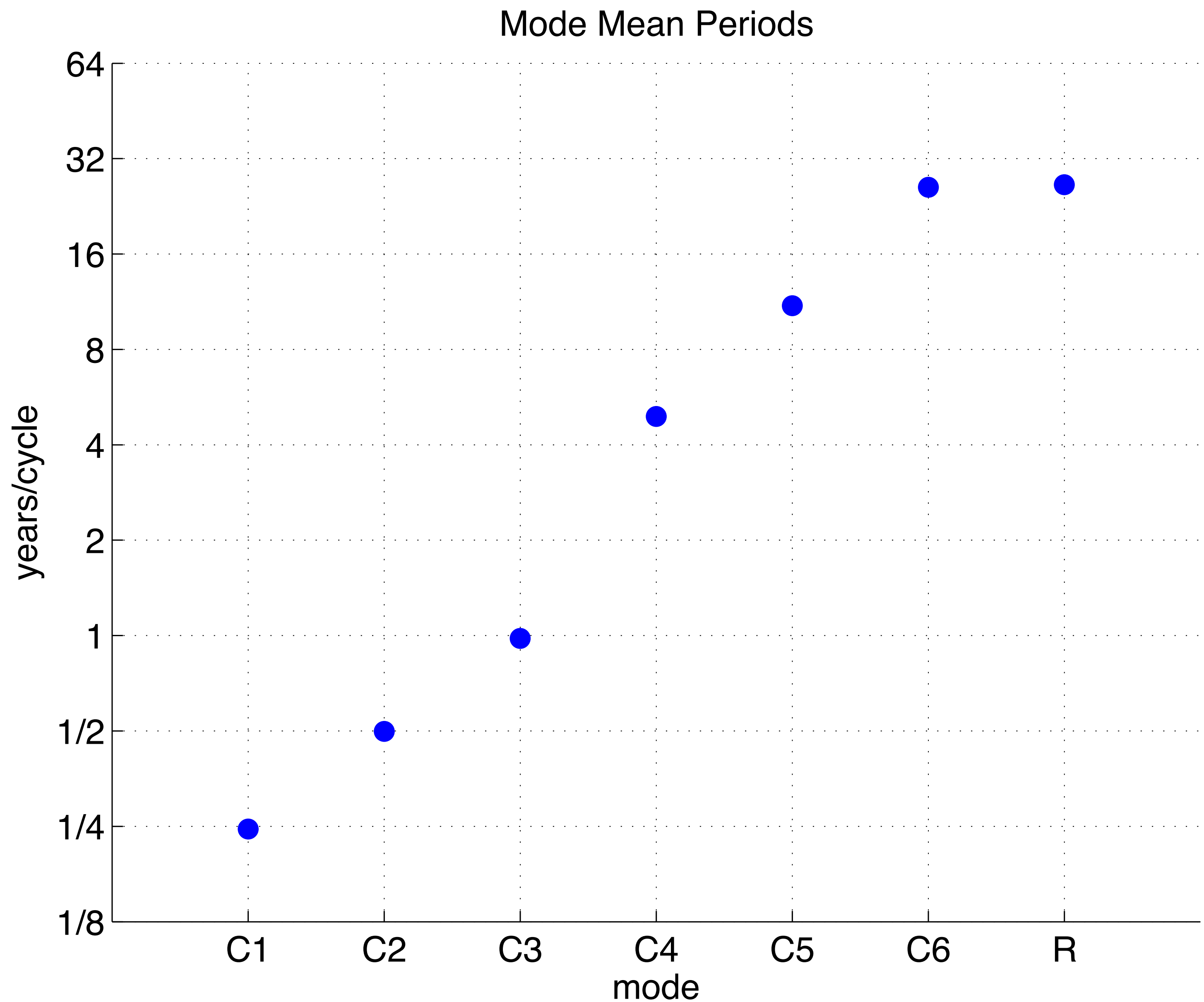
MERG global monthly precipitation since 1900



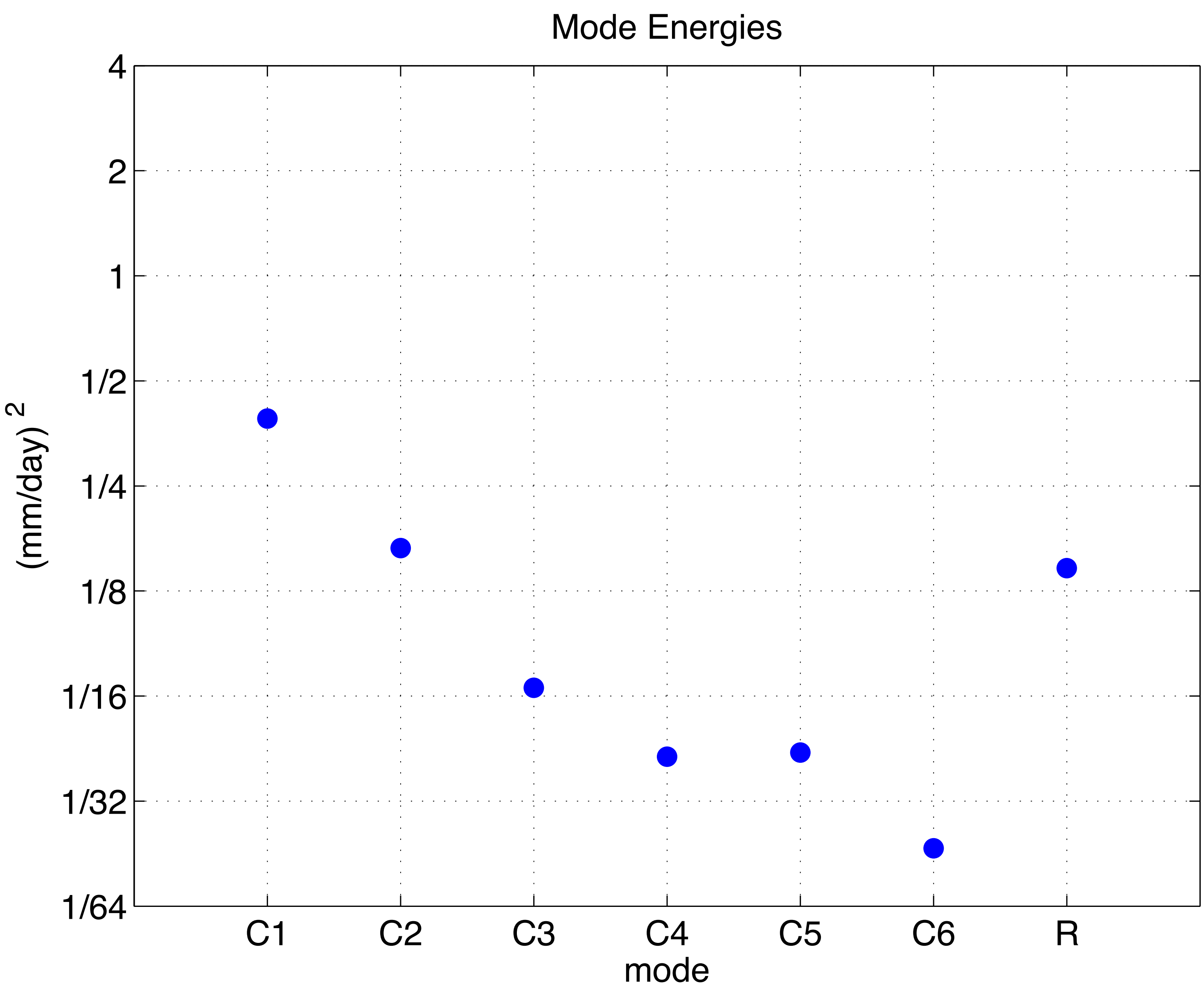
Hilbert spectra of MERG global precipitation



MERG IMF mean period



MERG IMF energy



Conclusions

- **HHT is an effective tool for analyzing nonlinear and non-stationary data.**
- **IMF3 is the nonlinear and non-stationary annual cycle.**
- **MJO, monsoon and PDO simulation improvement seems important in improving the model precipitation and optimal aggregation of observed data.**
- **CMAP has a decreasing trend, while all the other datasets have an increasing trend.**
- **The correlations among the datasets are small, indicating that more accurate algorithms are needed to derive the global precipitation data.**

Ongoing work

- Improving MJO simulation using stochastic parameterization against observed precipitation
  - Better "observations" with minimum errors
- "Modern" precipitation data sets (GPCP, CMAP) useful
  - Estimates of uncertainty
- 20th Century precipitation reconstruction and reanalysis available
  - Super-ensemble reconstruction with an error estimation
  - Testing global models