NUMERICAL STUDIES OF COMPACT STARS

Fridolin Weber
Department of Physics
SDSU

Outline

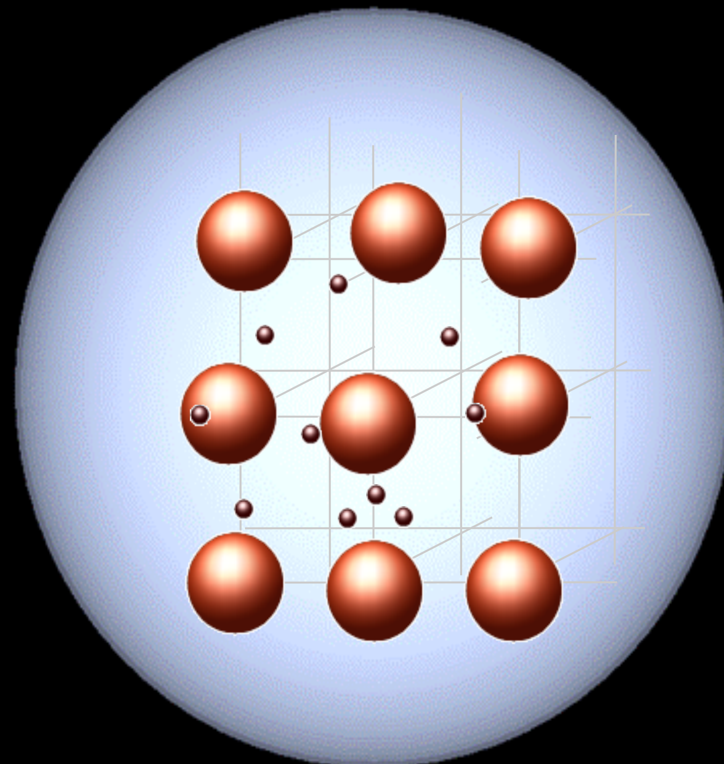
- Compact star basics
- The building blocks of ultra-dense matter
- Signals of "exotic" matter in cores of neutron stars
- Cas A
- Reheating of magnetars (AXPs, SGRs)
- Pycnonuclear reactions
- Ultra-high electric fields & differential rotation
- Unusually small Compact central objects (CCOs)
- Summary

Compact Star Basics

1. White Dwarfs
2. “Neutron” Stars
3. Low-Mass Black Holes

Earth

White Dwarf



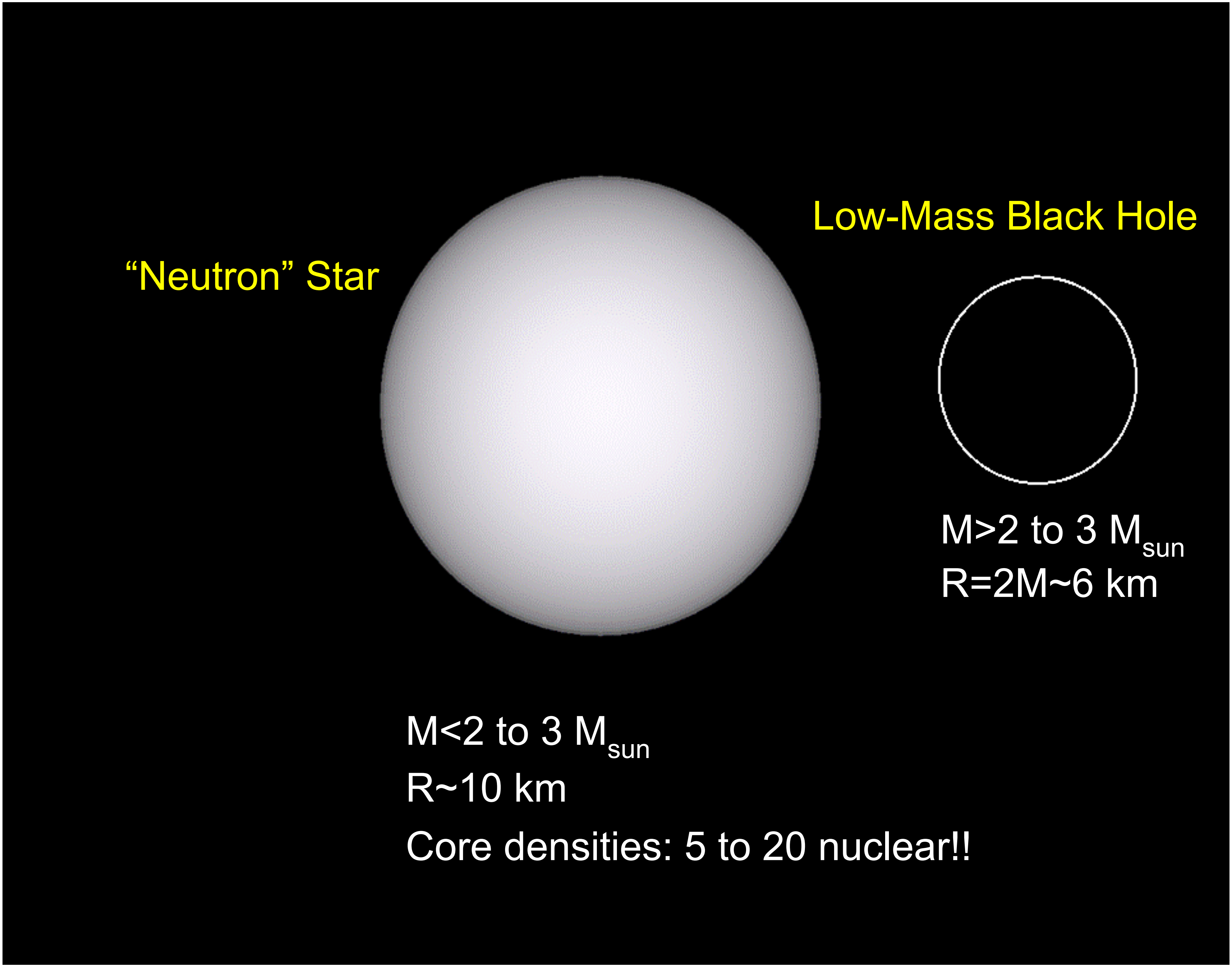
$M \sim 1.4\ M_{sun}$, $R \sim 10^4$ km, $\varepsilon_c \sim 10^6$ g/cm^3

A star (PTF 11kly) in the Pinwheel Galaxy undergoing a Type-I Supernova Event

August 22, 2011

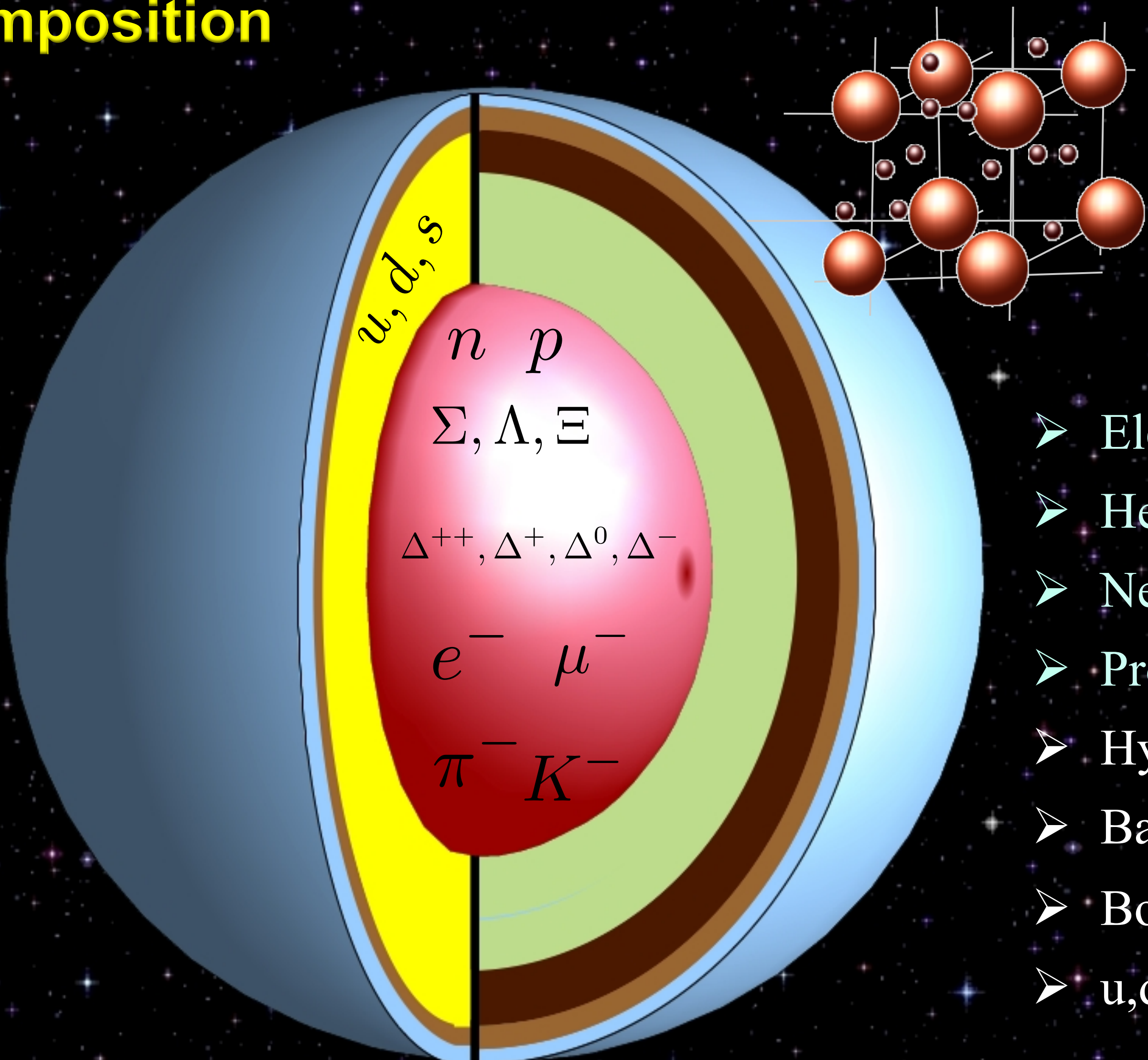
August 23, 2011

August 24, 2011



Low-Mass Black Hole
"Neutron" Star
M>2 to 3 M$_{sun}$
R=2M~6 km
M<2 to 3 M$_{sun}$
R~10 km
Core densities: 5 to 20 nuclear!!

Composition



- Electron gas
- Heavy atomic nuclei
- Neutrons (superfluid)
- Protons (superconducting)
- Hyperons
- Baryon resonances
- Boson condensates
- u,d,s quarks (supercond.)

Radius ~ 10 to 14 km, Mass ~ 1 to 2 M_{sun}

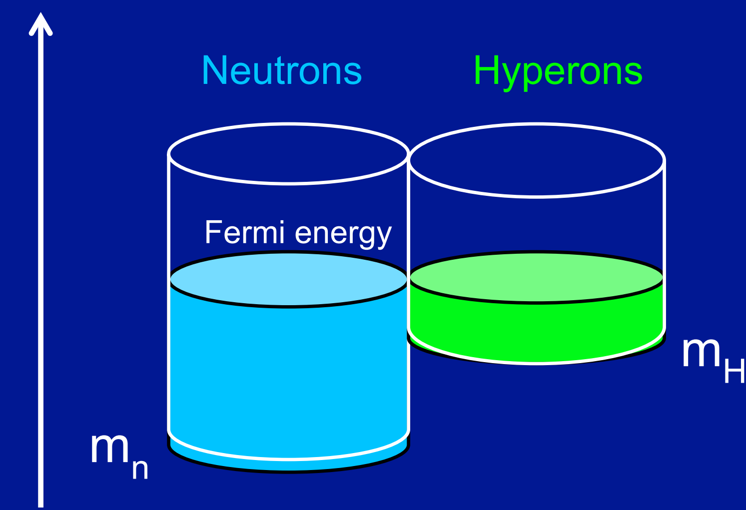
F. Weber (SDSU, 2010)

The Building Blocks I

- **Hyperons: ∑, Λ, Ξ**
- **Delta particle: Δ**

Ambartsumyan & Saakyan (1960)

$$E^2 = m^2 + p^2$$



Threshold density

$$p_F > \sqrt{m_H^2 - m_n^2} \simeq 3 \text{ fm}^{-1} \Rightarrow \rho > 2\rho_0$$

Applies to **free** particles!

Example of a model lagrangian for neutron star matter

$$
\begin{aligned}
\mathcal{L} = &\sum_B \bar{\psi}_B \left(i\partial\!\!\!/ - m_B\right)\psi_B + \frac{1}{2}\left(\partial^\mu\sigma\partial_\mu\sigma - m_\sigma^2\sigma^2\right) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}m_\omega^2\omega^\nu\omega_\nu + \frac{1}{2}\left(\partial^\mu\boldsymbol{\pi}\cdot\partial_\mu\boldsymbol{\pi} - m_\pi^2\boldsymbol{\pi}\cdot\boldsymbol{\pi}\right) \\
&-\frac{1}{4}\mathbf{G}^{\mu\nu}\cdot\mathbf{G}_{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}^\mu\cdot\boldsymbol{\rho}_\mu - \sum_B\Big(g_{\sigma B}\bar{\psi}_B\sigma\psi_B + g_{\omega B}\bar{\psi}_B\omega\!\!\!/\psi_B + \frac{f_{\omega B}}{4m_B}\bar{\psi}_B\sigma^{\mu\nu}F_{\mu\nu}\psi_B + \frac{f_{\pi B}}{m_\pi}\bar{\psi}_B\gamma^5\partial\!\!\!/\boldsymbol{\tau}\cdot\boldsymbol{\pi}\psi_B \\
&+g_{\rho B}\bar{\psi}_B\gamma^\mu\boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu\psi_B + \frac{f_{\rho B}}{4m_B}\bar{\psi}_B\sigma^{\mu\nu}\boldsymbol{\tau}\cdot\mathbf{G}_{\mu\nu}\psi_B\Big) - \frac{1}{3}m_N b_N\left(g_{\sigma N}\sigma\right)^3 - \frac{1}{4}c_N\left(g_{\sigma N}\sigma\right)^4 + \sum_L \bar{\psi}_L\left(i\partial\!\!\!/ - m_L\right)\psi_L
\end{aligned}
$$

- Equations of motion for baryon and meson field operators
- Chemical equilibrium
- Electric charge neutrality (local, global)

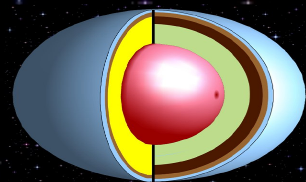
Equation of state: **P(ε, T, . . .)**

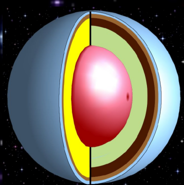
Equations of motion for baryon and meson field operators

$$
\begin{aligned}
(i\gamma^\mu \partial_\mu - m_B)\,\psi_B &= g_{\sigma B}\sigma\psi_B + \Big(g_{\omega B}\gamma^\mu\omega_\mu + \frac{f_{\omega B}}{4m_B}\sigma^{\mu\nu}F_{\mu\nu}\Big)\psi_B \\
&\quad + \Big(g_{\rho B}\gamma^\mu\ \boldsymbol{\tau}\cdot\boldsymbol{\rho}_\mu + \frac{f_{\rho B}}{4\,m_B}\sigma^{\mu\nu}\boldsymbol{\tau}\cdot\mathbf{G}_{\mu\nu}\Big)\psi_B + \frac{f_{\pi B}}{m_\pi}\gamma^\mu\gamma^5(\partial_\mu\boldsymbol{\tau}\cdot\boldsymbol{\pi})\psi_B \\
(\partial^\mu\partial_\mu + m_\sigma^2)\sigma &= -\sum_B g_{\sigma B}\bar{\psi}_B\psi_B - m_N b_N\, g_{\sigma N}\,(g_{\sigma N}\sigma)^2 - c_N\, g_{\sigma N}\,(g_{\sigma N}\sigma)^3\ , \\
\partial^\mu F_{\mu\nu} + m_\omega^2\,\omega_\nu &= \sum_B \Big(g_{\omega B}\bar{\psi}_B\gamma_\nu\psi_B - \frac{f_{\omega B}}{2m_B}\partial^\mu\left(\bar{\psi}_B\sigma_{\mu\nu}\psi_B\right)\Big)\,, \\
\left(\partial^\mu\partial_\mu + m_\pi^2\right)\boldsymbol{\pi} &= \sum_B \frac{f_{\pi B}}{m_\pi}\partial^\mu\left(\bar{\psi}_B\gamma_5\gamma_\mu\boldsymbol{\tau}\psi_B\right)\ , \\
\partial^\mu\mathbf{G}_{\mu\nu} + m_\rho^2\,\boldsymbol{\rho}_\nu &= \sum_B \Big(g_{\rho B}\bar{\psi}_B\boldsymbol{\tau}\gamma_\nu\psi_B - \frac{f_{\rho B}}{2m_B}\partial^\mu\left(\bar{\psi}_B\boldsymbol{\tau}\sigma_{\mu\nu}\psi_B\right)\Big)\,,
\end{aligned}
$$

$(B = n, p, \Sigma, \Lambda, \Xi, \Delta)$

Model Composition of a $M=1.7\ M_{sun}$ Neutron Star

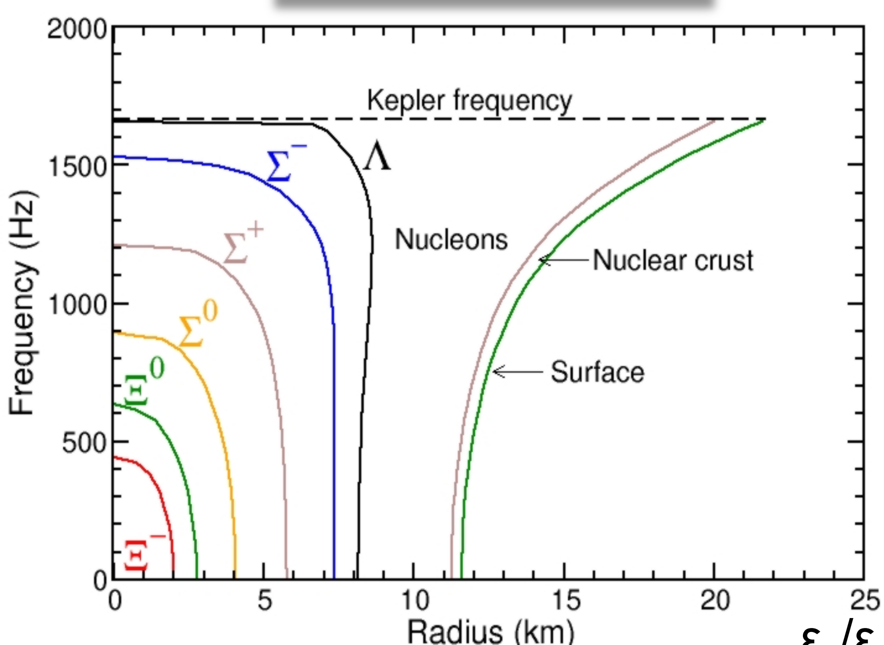


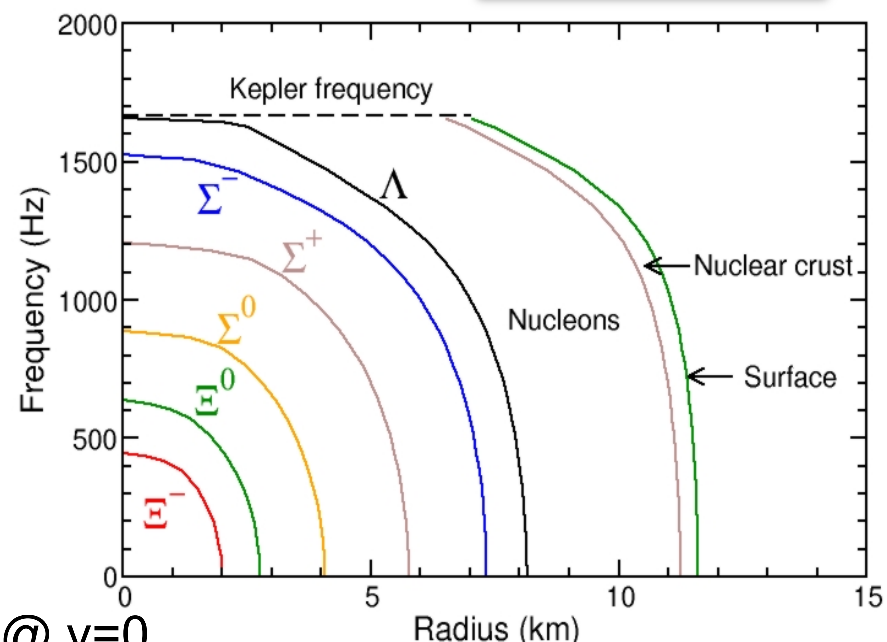


Equatorial direction

$\varepsilon_c/\varepsilon_0=3$ @ $v=v_K$

Polar direction



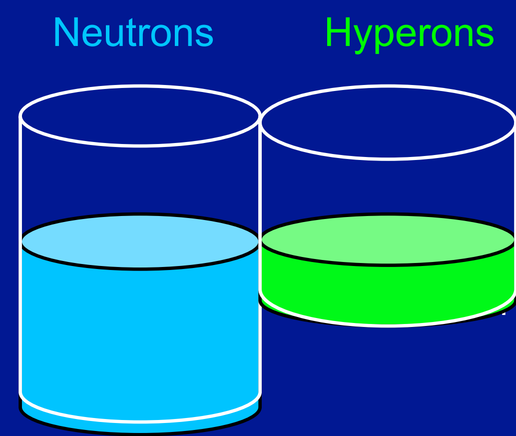


$\varepsilon_c/\varepsilon_0=7$ @ $v=0$

P. Rosenfield (2007)

The Building Blocks II

- Baryons: Σ, Λ, Ξ, Δ

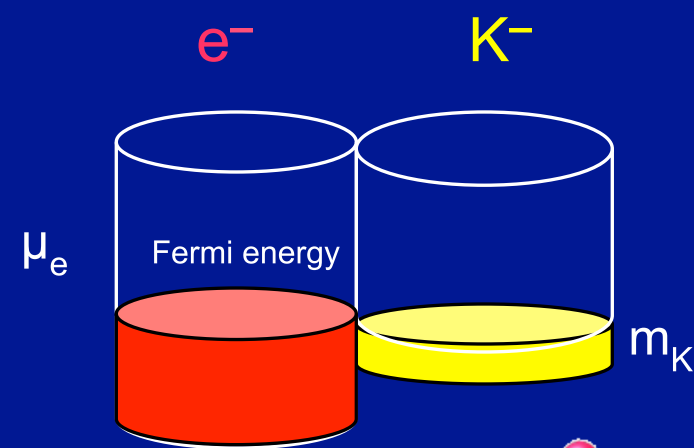


- **Boson condensates:**

$$e^{-} \rightarrow \pi^{-} + \upsilon_e$$

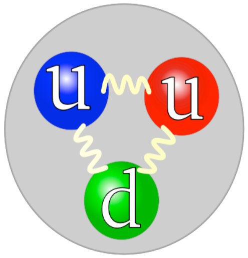
$$e^{-} \rightarrow K^{-} + \upsilon_e$$

Brown & Weise, 1976
Kaplan & Nelson, 1986
Politzer & Wise, 1991
Brown et al., 1992
Waas, Rho, Weise, 1997
Schaffner-Bielich, 1998
Mao, 1999

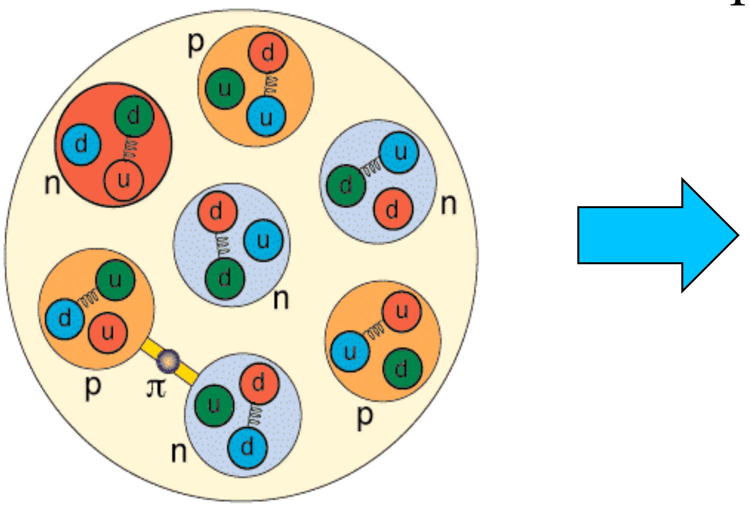


$$\mu^e = m_{\mathrm{K}} \quad \Rightarrow \quad \rho > 3 - 5\rho_0$$

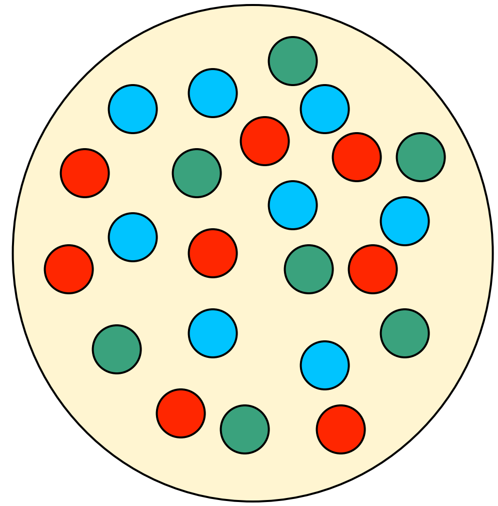
The Building Blocks III - Quarks



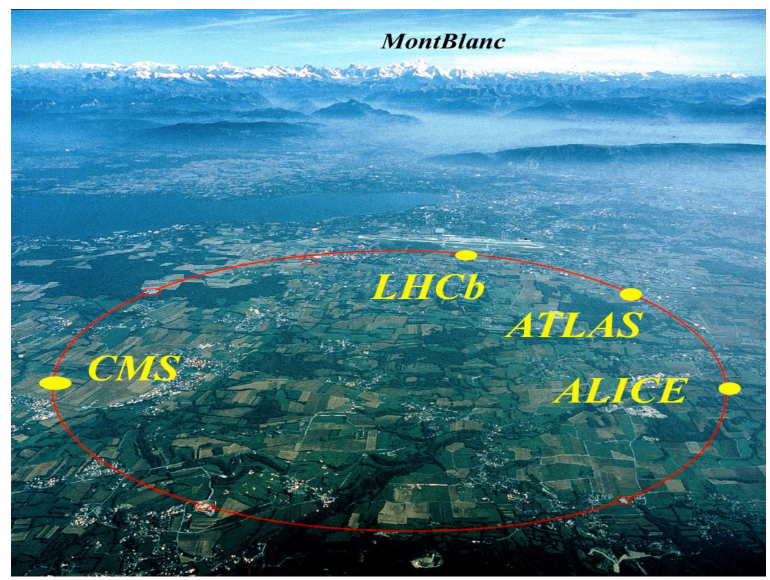
Transition density?
Transition temperature?



Quarks inside of neutrons and protons



Deconfined quarks and gluons
(quark matter)

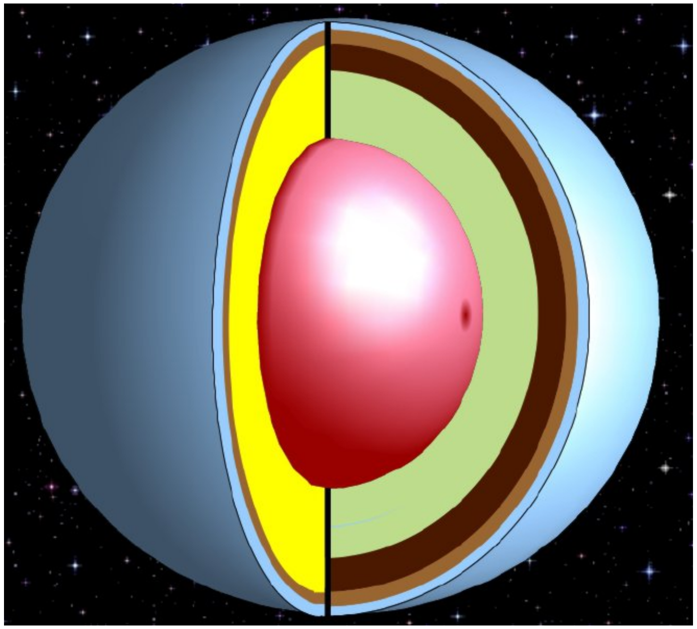


The Large Hadron Collider (LHC) at CERN, Switzerland

The Building Blocks III

- **Quarks: u, d, s, c, t, b**

Ivanenko & Kurdgelaidze, 1965
Fritzsch, Gell-Mann & Leutwyler, 1973
Collins & Perry, 1975
Baym & Chin; Keister & Kisslinger, 1976
Chapline & Nauenberg, 1977



Gibbs condition (2 conserved charges!)

$$P_H(\mu^e, \mu^n) = P_Q(\mu^e, \mu^n) \Rightarrow \rho > 2 - 3\rho_0$$

Possible existence of:

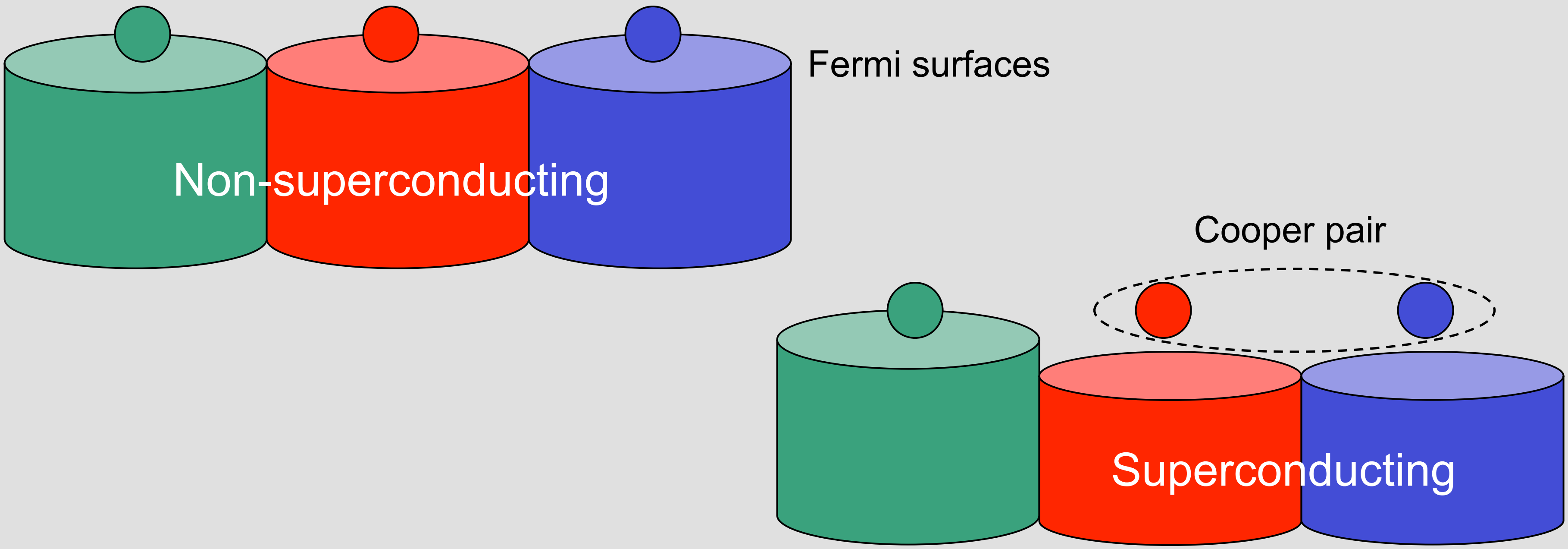
- Mixed phase of quarks and hadrons
- Quark drops, quark rods, quark slabs
- Pure quark matter in cores of neutron stars

Discovery of color superconductivity
Alford, Rajagopal, Wilczek (1998);
Rapp, Shuryak, Schaefer, Velkovsky (1998)

CFL, 2SC, gCFL, LOFF, …

Color Superconductivity

flavors: f=u, d , s   colors: a=r, g, b



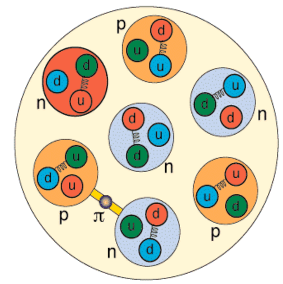
$$\langle \psi^a_{f_a} C\gamma_5 \psi^b_{f_b} \rangle \sim \Delta \cdot \left(\delta^a_{f_a}\delta^b_{f_b} - \delta^a_{f_b}\delta^b_{f_a} \right)$$

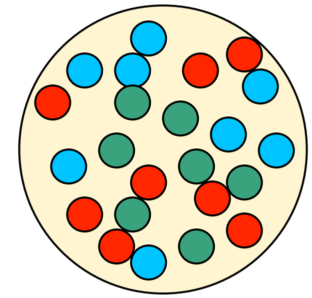
100 MeV

Quark field operators

Color-Flavor Locking (CFL)

Modeling the Quark-Hadron Phase Transition





Hadronic matter:

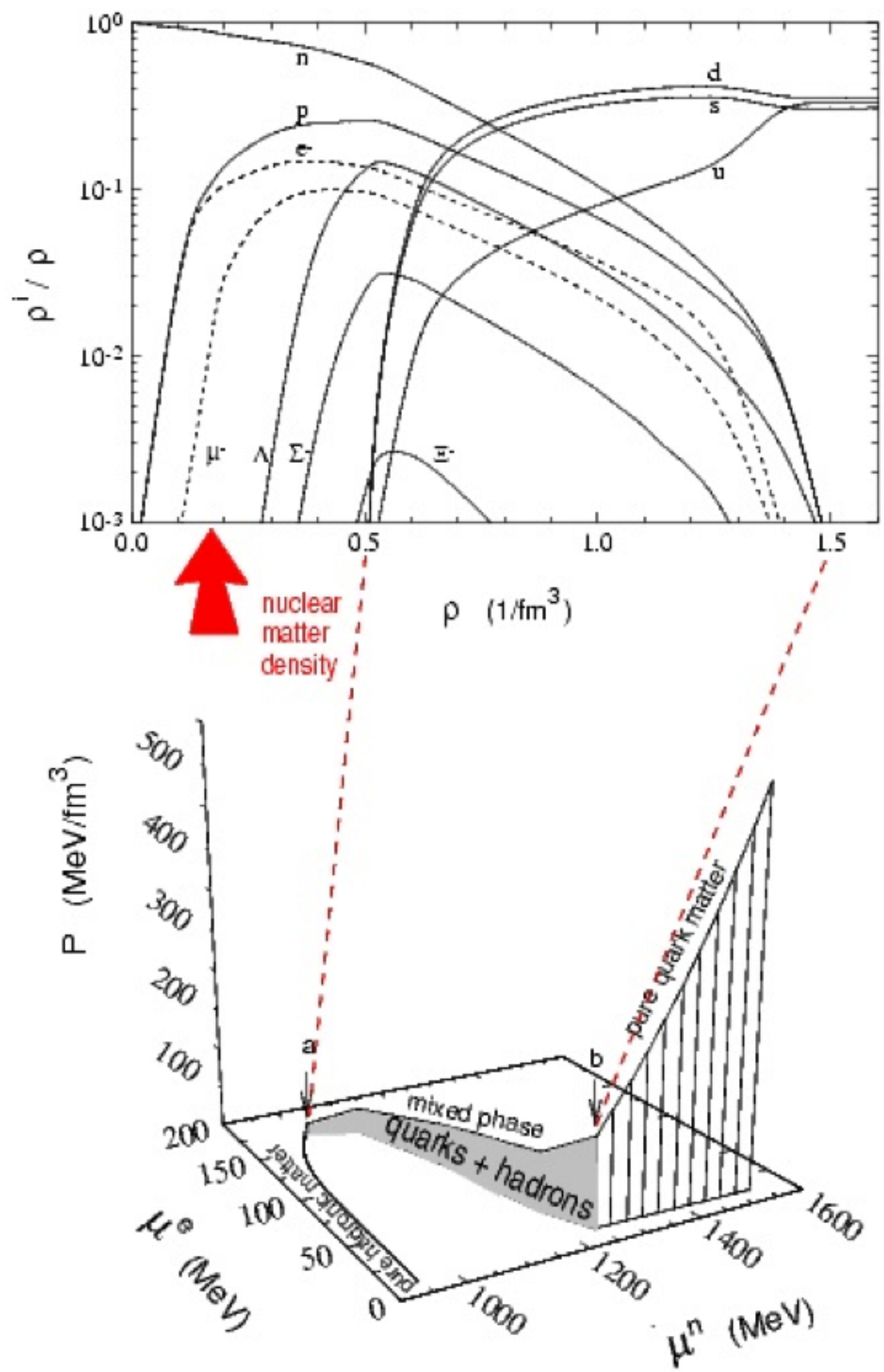
$$L = \Psi_B (i\gamma^\mu \partial_\mu - m_B)\, \Psi_B \; + \; \text{mesons } (\sigma,\omega,\pi,\rho, \ldots)$$

Quark matter:

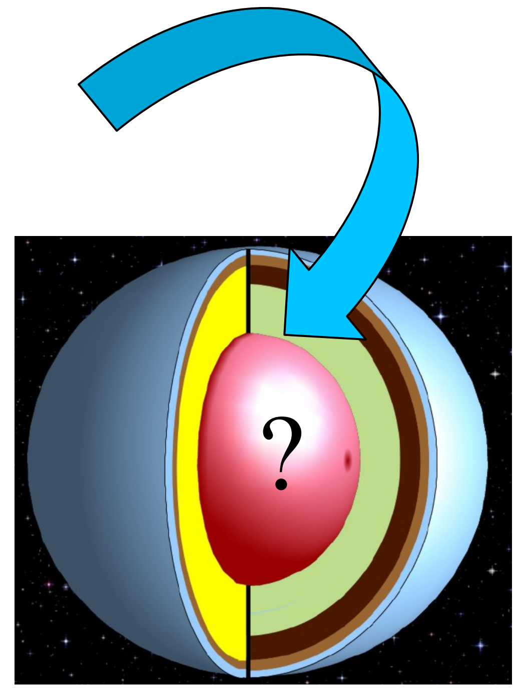
$$L = \Psi^a{}_f (i\gamma_\mu D^\mu{}_{ab} - m_f)\, \Psi^b{}_f \; - \; F^i{}_{\mu\nu} F_i{}^{\mu\nu} / 4$$

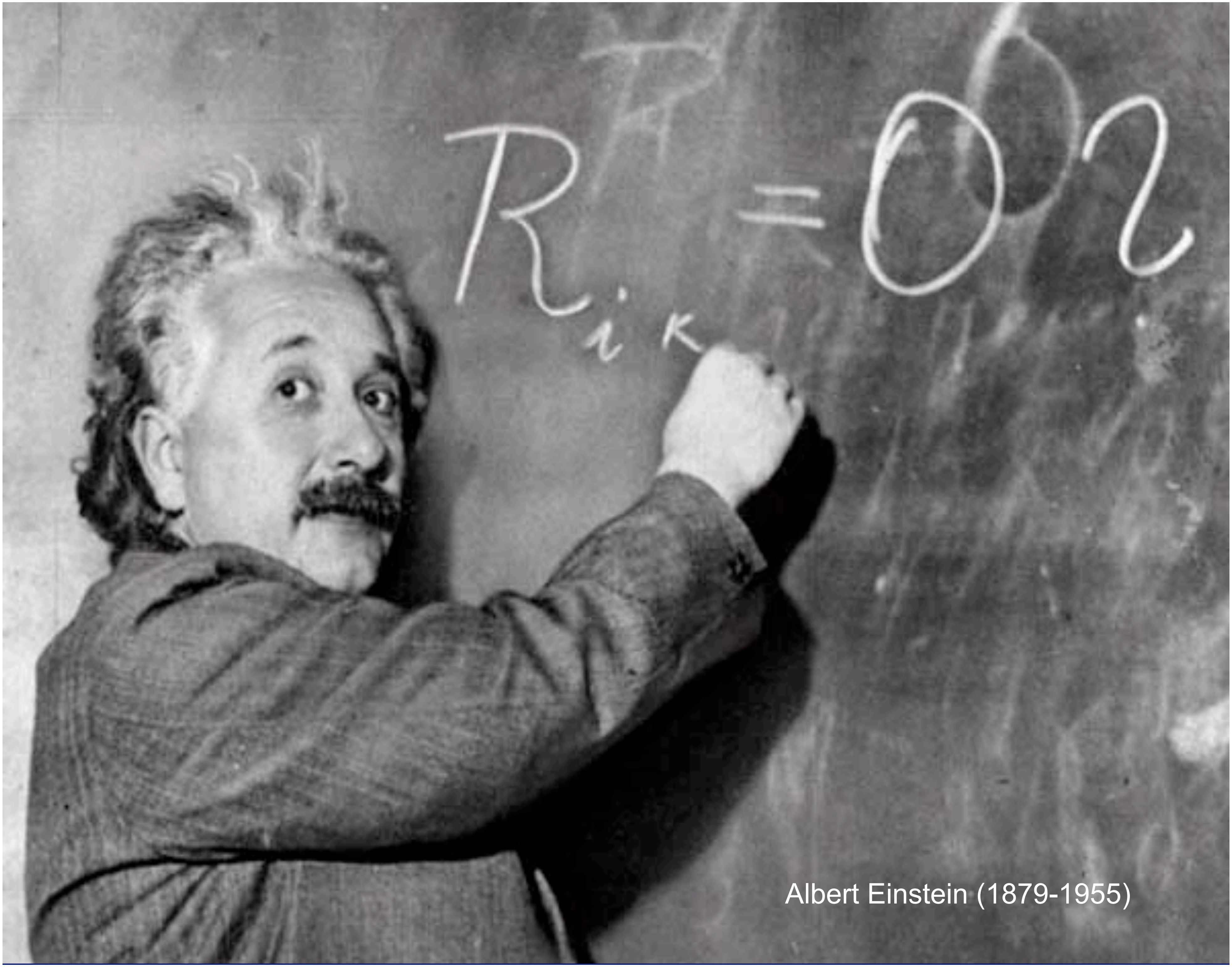
$$P_{Hadronic}(\{\psi^H\},\mu^n,\mu^e) = P_{Quark}(\{\psi^q\},\mu^n,\mu^e)$$

Model
quark-hadron
composition



Associated
equation of
state





Albert Einstein (1879-1955)

Einstein's Field Equations for Rotating Compact Objects

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R - g^{\mu\nu}\Lambda = 8\pi T^{\mu\nu}(\epsilon, P(\epsilon))$$

- Metric: $ds^2 = -e^{-2\nu} dt^2 + e^{2(\alpha+\beta)} r^2 \sin^2\theta \, (d\varphi - N^{\varphi} dt)^2 + e^{2(\alpha-\beta)} (dr^2 + r^2 d\theta^2)$

- Christoffel symbols:
  $\Gamma^{\sigma}{}_{\mu\nu} = g^{\sigma\lambda} (\partial_\nu g_{\mu\lambda} + \partial_\mu g_{\nu\lambda} - \partial_\lambda g_{\mu\nu}) / 2$

- Riemann tensor:
  $R^{\tau}{}_{\mu\nu\sigma} = \partial_\nu \Gamma^{\tau}{}_{\mu\sigma} - \partial_\sigma \Gamma^{\tau}{}_{\mu\nu} + \Gamma^{\kappa}{}_{\mu\sigma} \Gamma^{\tau}{}_{\kappa\nu} - \Gamma^{\kappa}{}_{\mu\nu} \Gamma^{\tau}{}_{\kappa\sigma}$

- Ricci tensor: $R_{\mu\nu} = R^{\tau}{}_{\mu\sigma\nu} \, g^{\sigma}{}_{\tau}$

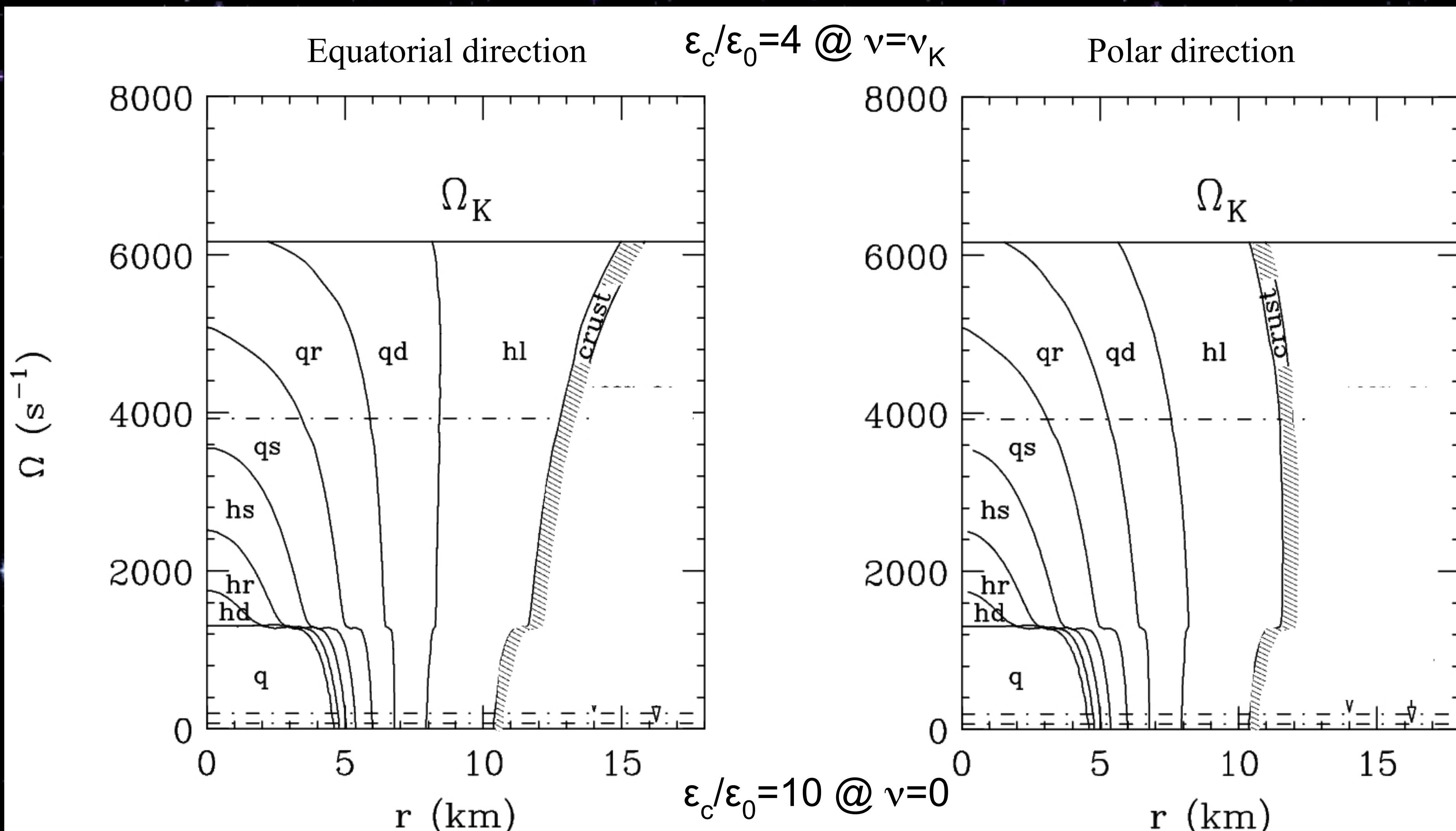
- Scalar curvature: $R = R_{\mu\nu} \, g^{\mu\nu}$

- Kepler frequency: $\Omega_K = r^{-1} e^{\nu-\alpha-\beta} U_K + N^{\varphi}$

- Differential rotation/uniform rotation

→ Stellar properties: M, R_p, R_{eq}, I, z, Ω_K, ω, P, ε, ρ

Model Quark-Hadron Composition of 1.45 M_{sun} Neutron Star



Moment of inertia:

$$I = \frac{1}{\Omega} \int dr\, d\theta\, d\phi\, T_{\phi}{}^{t} \sqrt{-g}$$

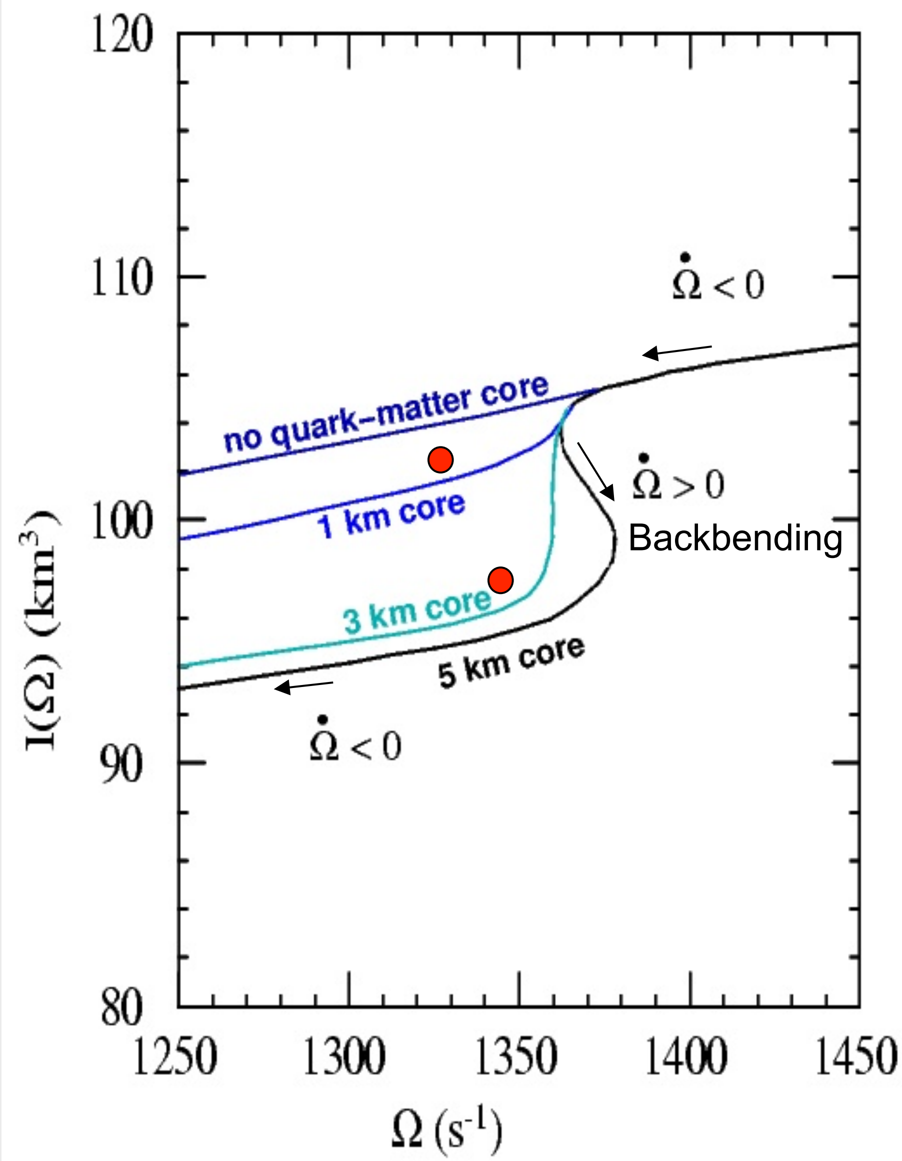
Braking index (n) of a pulsar:

$$n = 3 - \frac{I''\Omega^2 + 3I'\Omega}{I'\Omega + 2I}$$

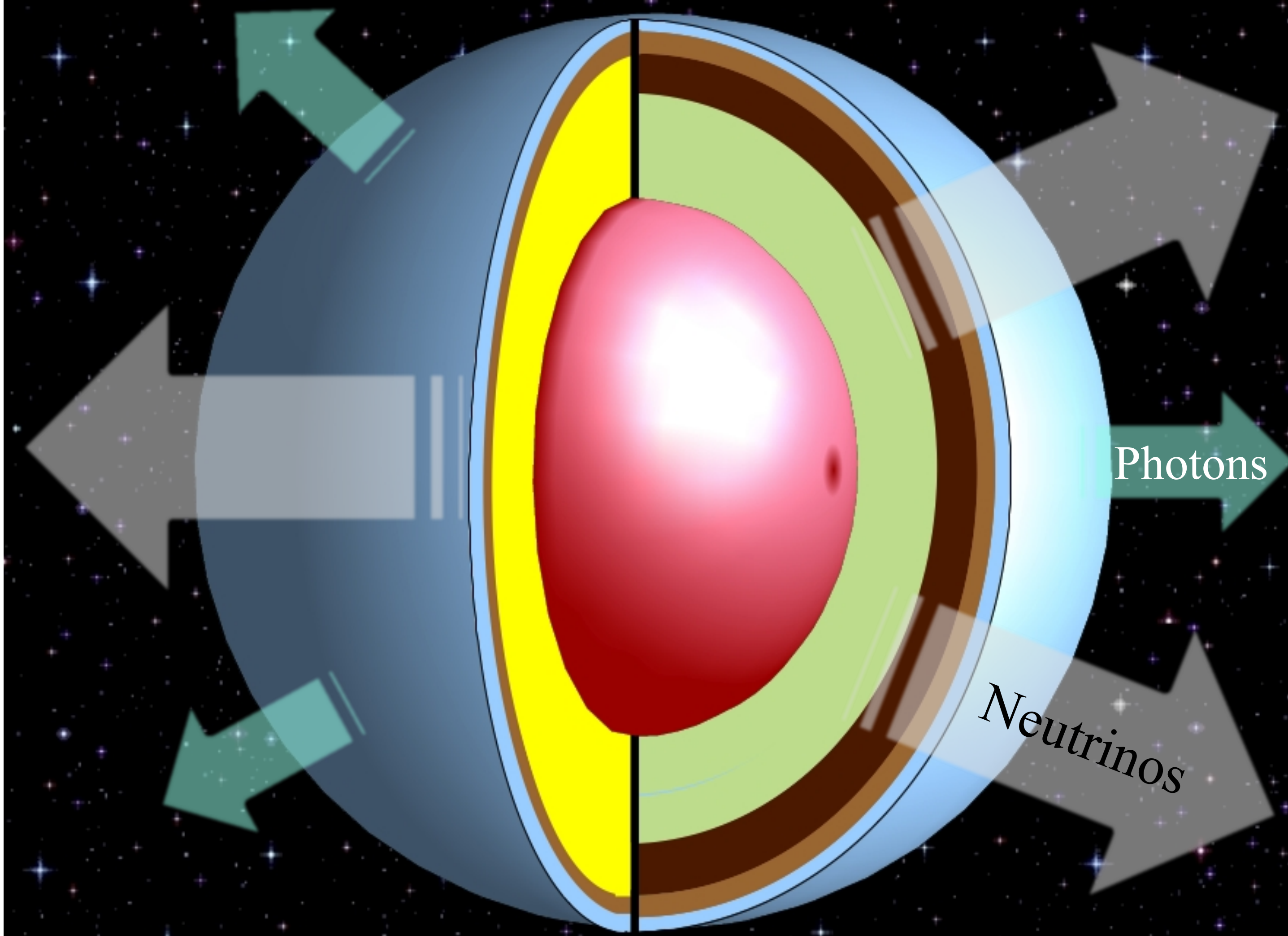
Signals of quark deconfinment:

- Spin-up of isolated rotating neutron stars
- Braking indices of pulsars $-\infty < n < +\infty$

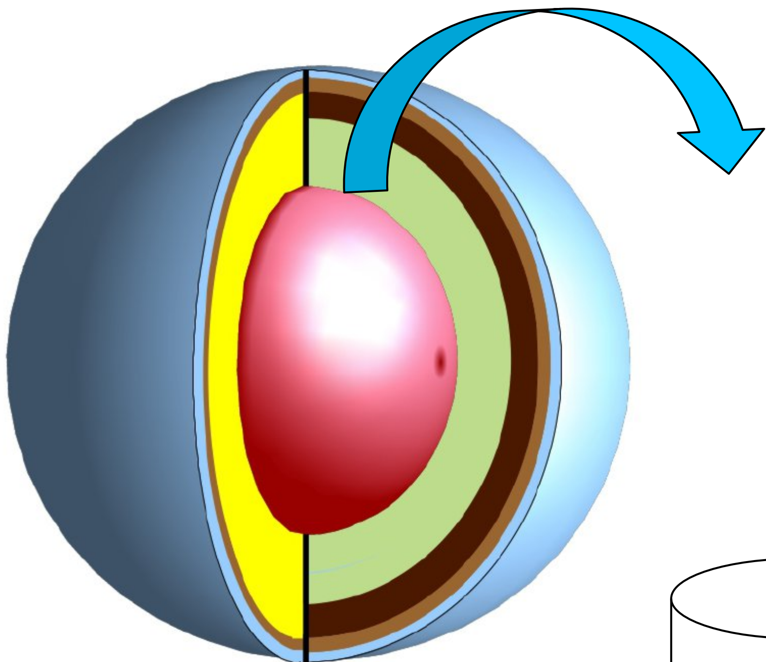
Moment of Inertia



Cooling of Compact Stars

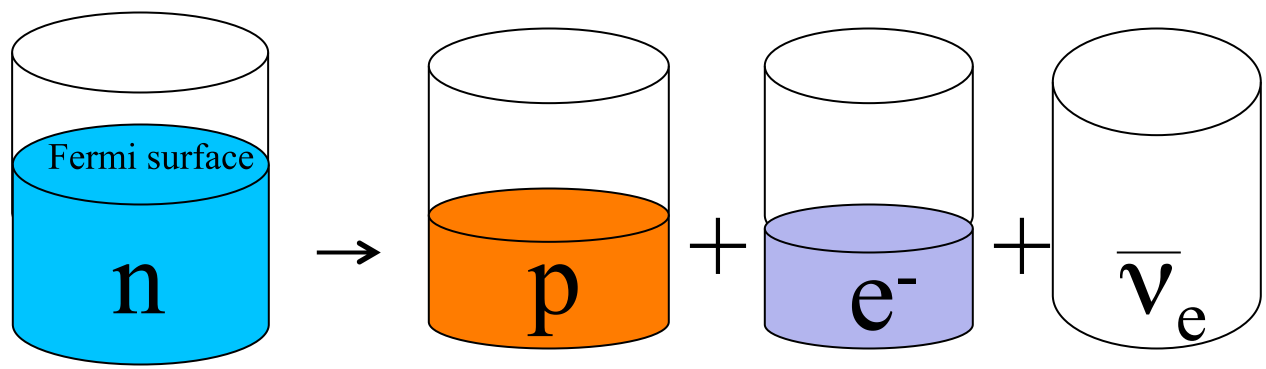


Neutrino-Emitting Particle Reactions inside of Neutron Stars . . .



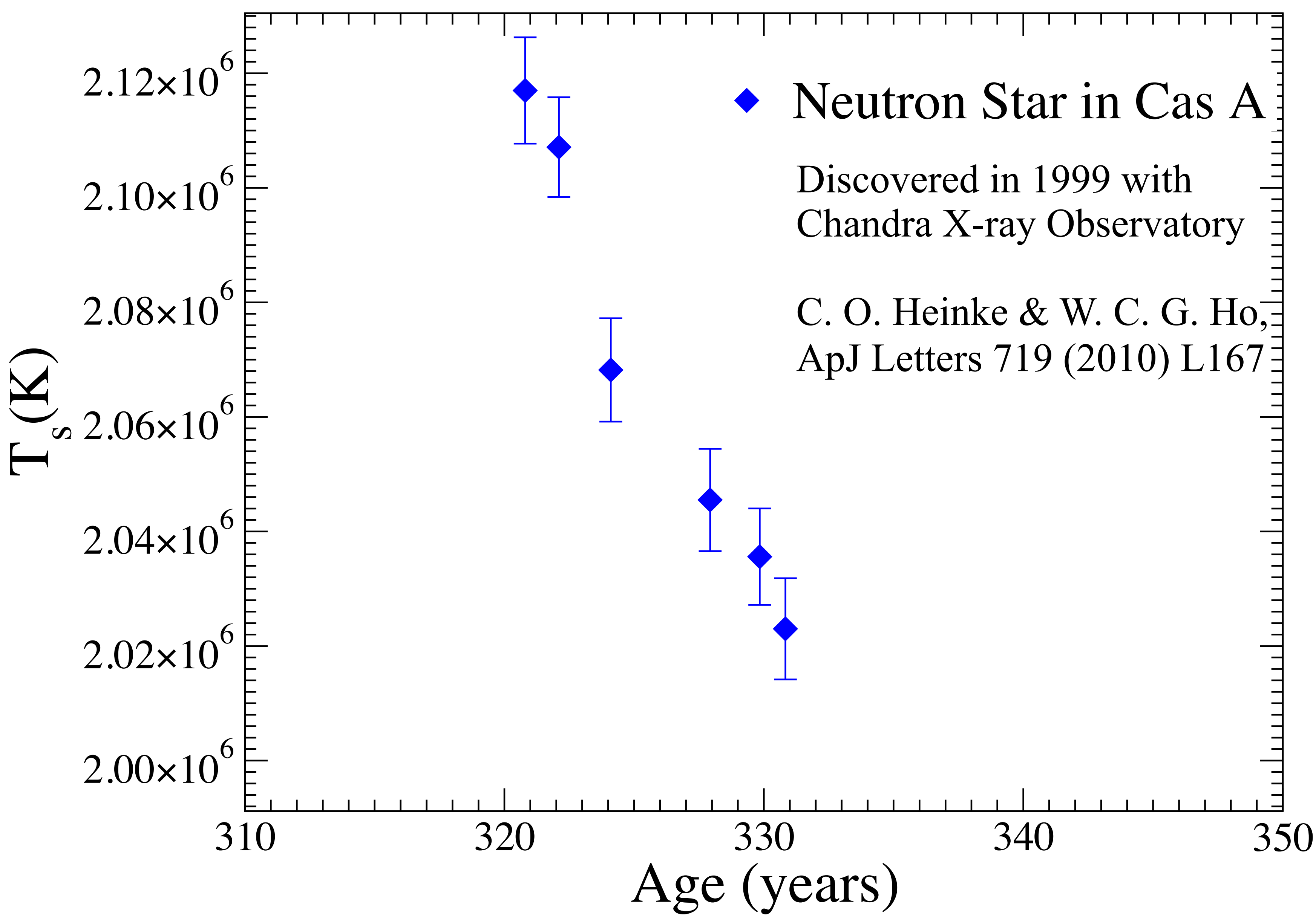
Direct Urca Process

$$n \longrightarrow p + e^{-} + \bar{\nu}_e$$



Neutron Star Cooling I

Direct Urca	n → p+e+ν	fast
Modified Urca	n+n → n+p+e+ν	slow
	p+n → p+p+e+ν	slow
Bremsstrahlung	n+n →n+n+ν+ν	slow
π^- condensate	n+< π^-> → n+e+ν	fast
K^- condensate	n+<K^-> → n+e+ν	fast
Cooper pair formation	n+n →[nn] + ν+ν	slow


Neutron Star in Cas A
Discovered in 1999 with
Chandra X-ray Observatory
C. O. Heinke & W. C. G. Ho,
ApJ Letters 719 (2010) L167
T_s (K)
2.12×10^6
2.10×10^6
2.08×10^6
2.06×10^6
2.04×10^6
2.02×10^6
2.00×10^6
310
320
330
340
350
Age (years)

Cooling of rotating neutron stars …
Thermal energy transport in GR

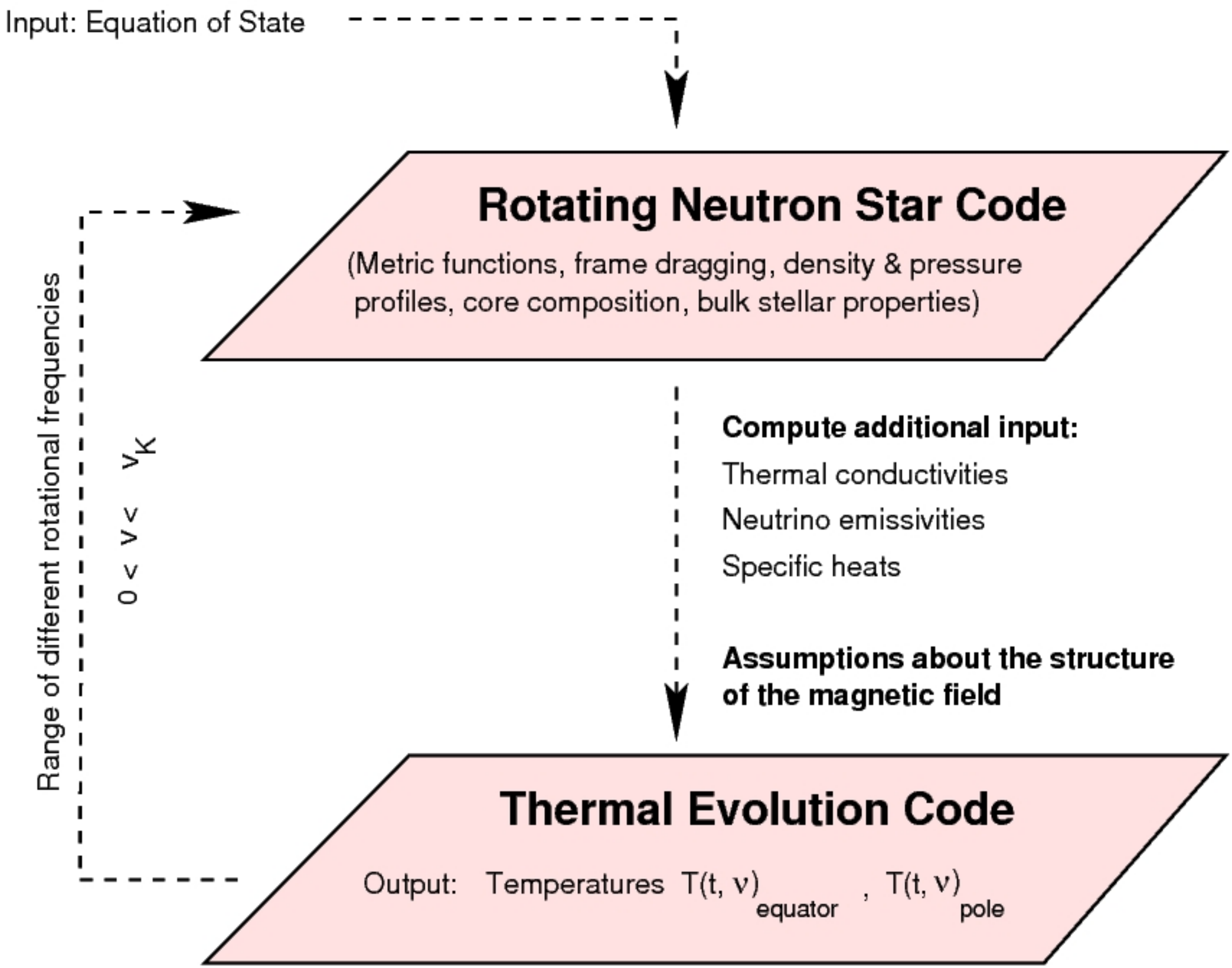
$$\partial_t \tilde{T} = -\frac{1}{\Gamma^2} e^{2\nu} \frac{\epsilon}{C_V} - r \sin\theta \, U e^{\nu+\gamma-\xi} \frac{1}{C_V} \left(\partial_r \Omega + \frac{1}{r} \partial_\theta \Omega \right)$$

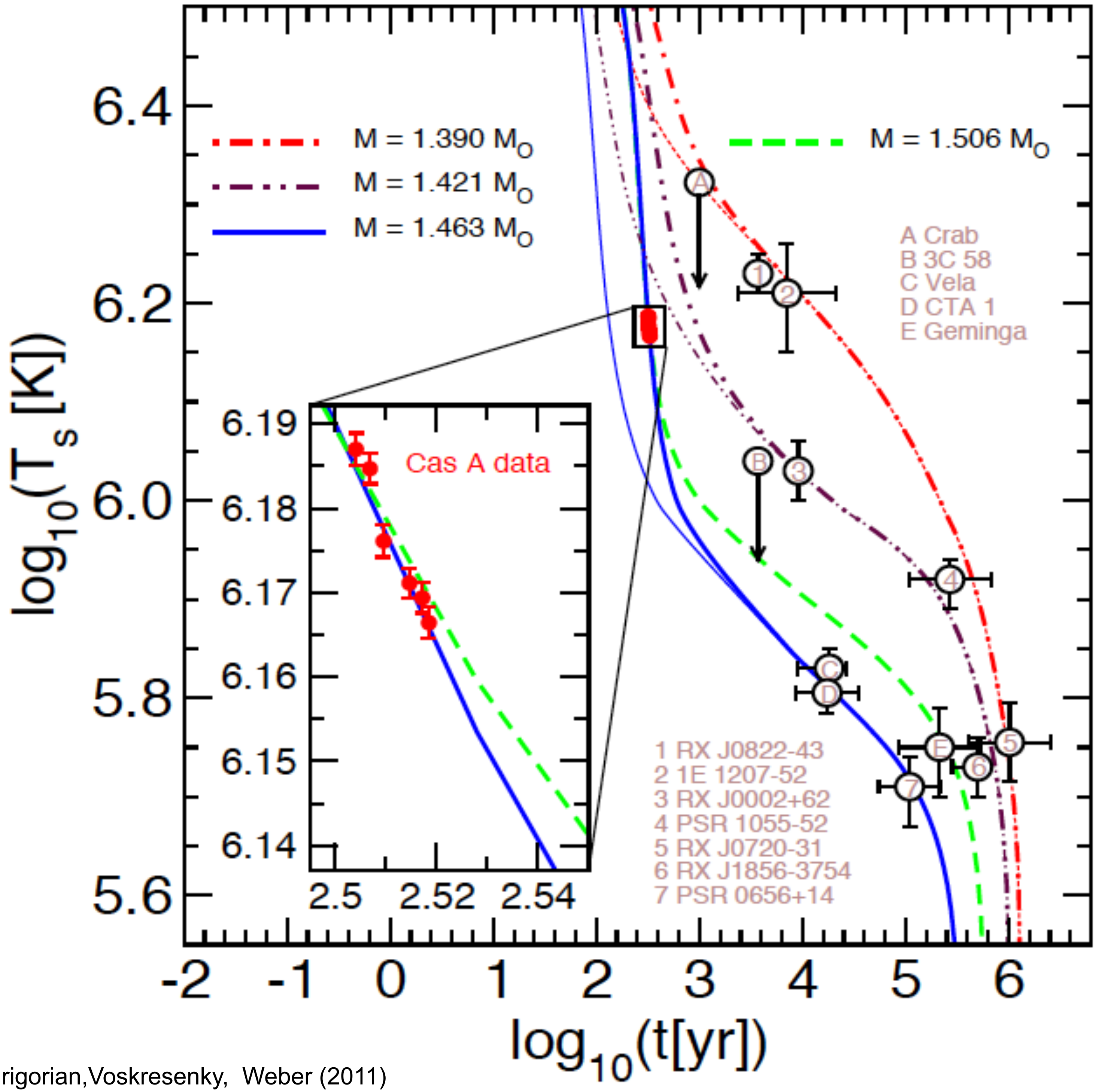
$$+ \frac{1}{r^2 \sin\theta} \frac{1}{\Gamma} e^{3\nu-\gamma-2\xi} \frac{1}{C_V} \left(\partial_r \left(r^2 \kappa \sin\theta \, e^{\gamma} \left(\partial_r \tilde{T} + \Gamma^2 U e^{-2\nu+\gamma} \tilde{T} \partial_r \Omega \right) \right) \right.$$

$$\left. + \frac{1}{r^2} \partial_\theta \left(r^2 \kappa \sin\theta \, e^{\gamma} \left(\partial_\theta \tilde{T} + \Gamma^2 U e^{-2\nu+\gamma} \tilde{T} \partial_\theta \Omega \right) \right) \right)$$

. . .to be solved simultaneously with stellar rotation equations.

For first attempts: M. Stejner, F. Weber, J. Madsen, ApJ 694 (2009) 1019;
R. Negreiros, S. Schramm, F. Weber (2011).

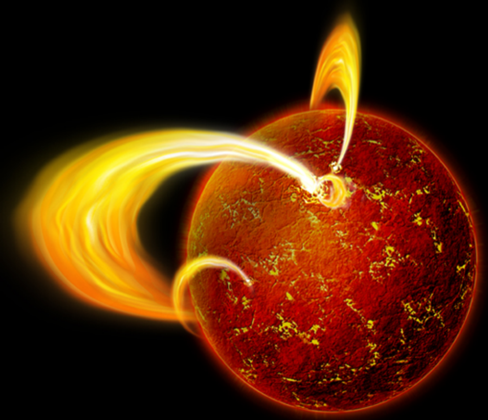

Input: Equation of State
Rotating Neutron Star Code
(Metric functions, frame dragging, density & pressure profiles, core composition, bulk stellar properties)
Range of different rotational frequencies
$0 < \nu < \nu_K$
Compute additional input:
Thermal conductivities
Neutrino emissivities
Specific heats
Assumptions about the structure of the magnetic field
Thermal Evolution Code
Output: Temperatures $T(t, \nu)_{equator}$, $T(t, \nu)_{pole}$



Blaschke, Grigorian,Voskresenky, Weber (2011)

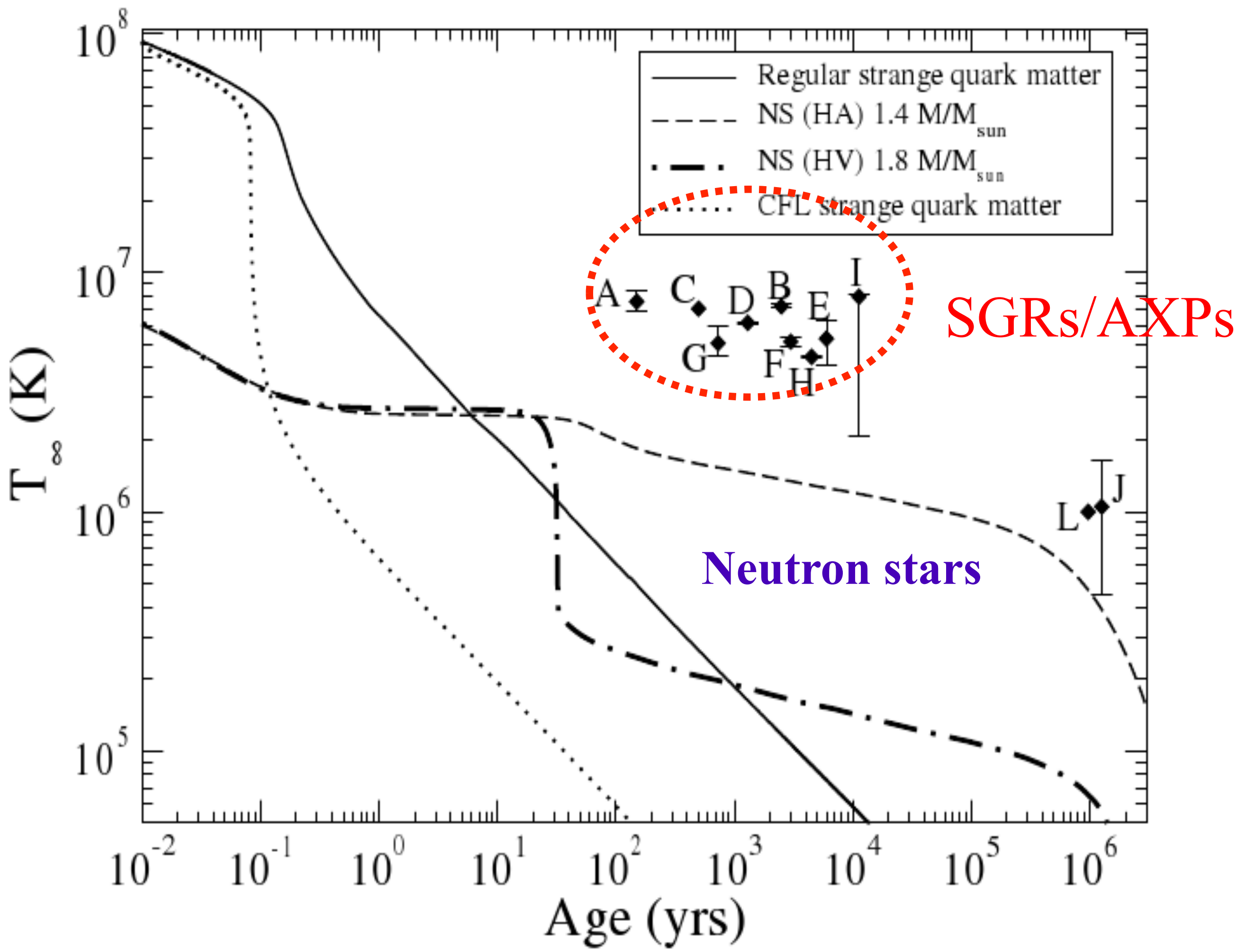
Puzzling new classes of “Neutron” Stars

- **Magnetars (AXPs, SGRs):** Unusually hot objects!
- **Compact Central Objects (CCO’s):** Unusually small?



SGRs and AXPs are unusually hot …

evidence of internal heating?



Negreiros, Niebergal, Ouyed, FW, PRD 81 (2010) 043005

Reheating via Vortex Expulsion

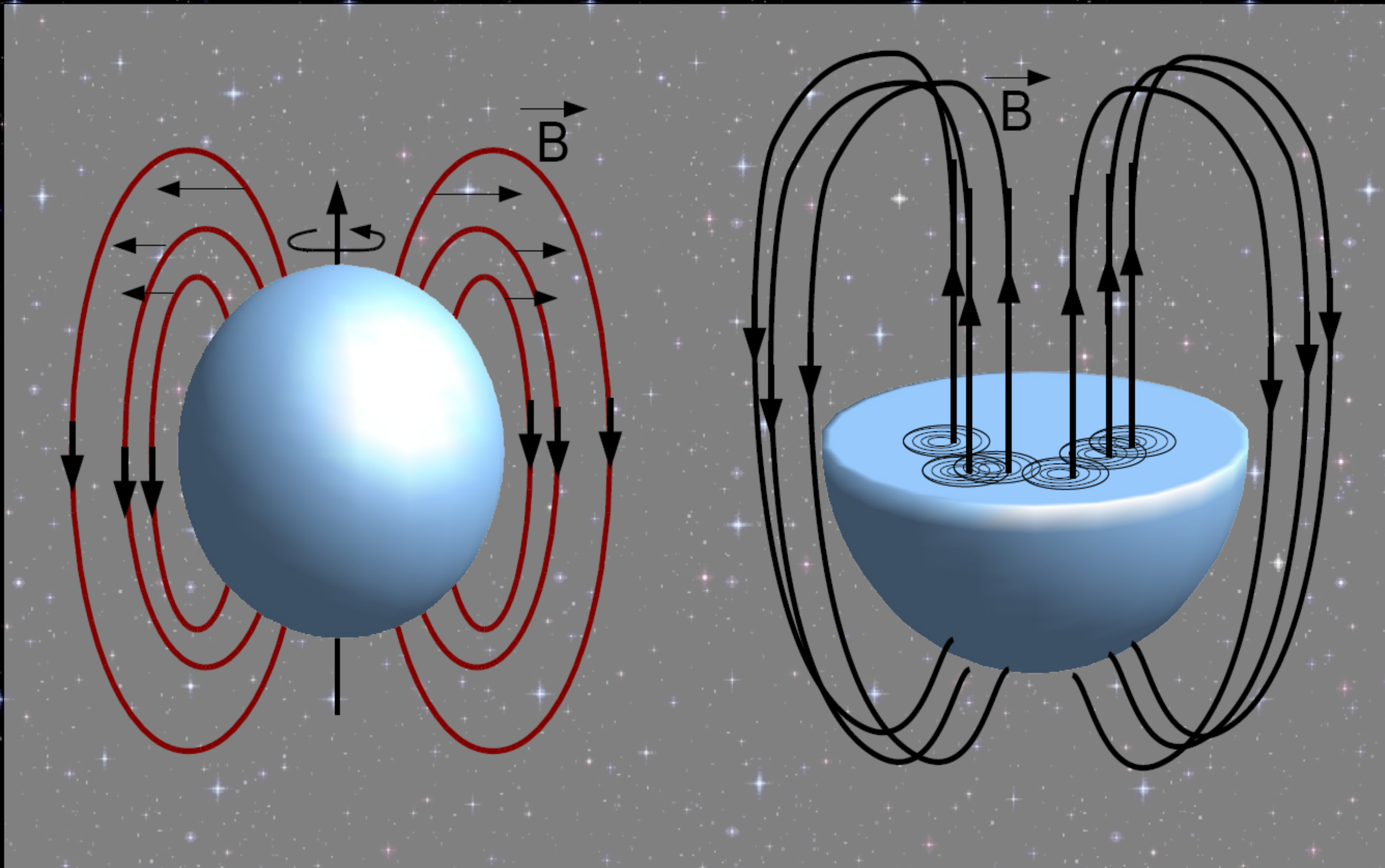
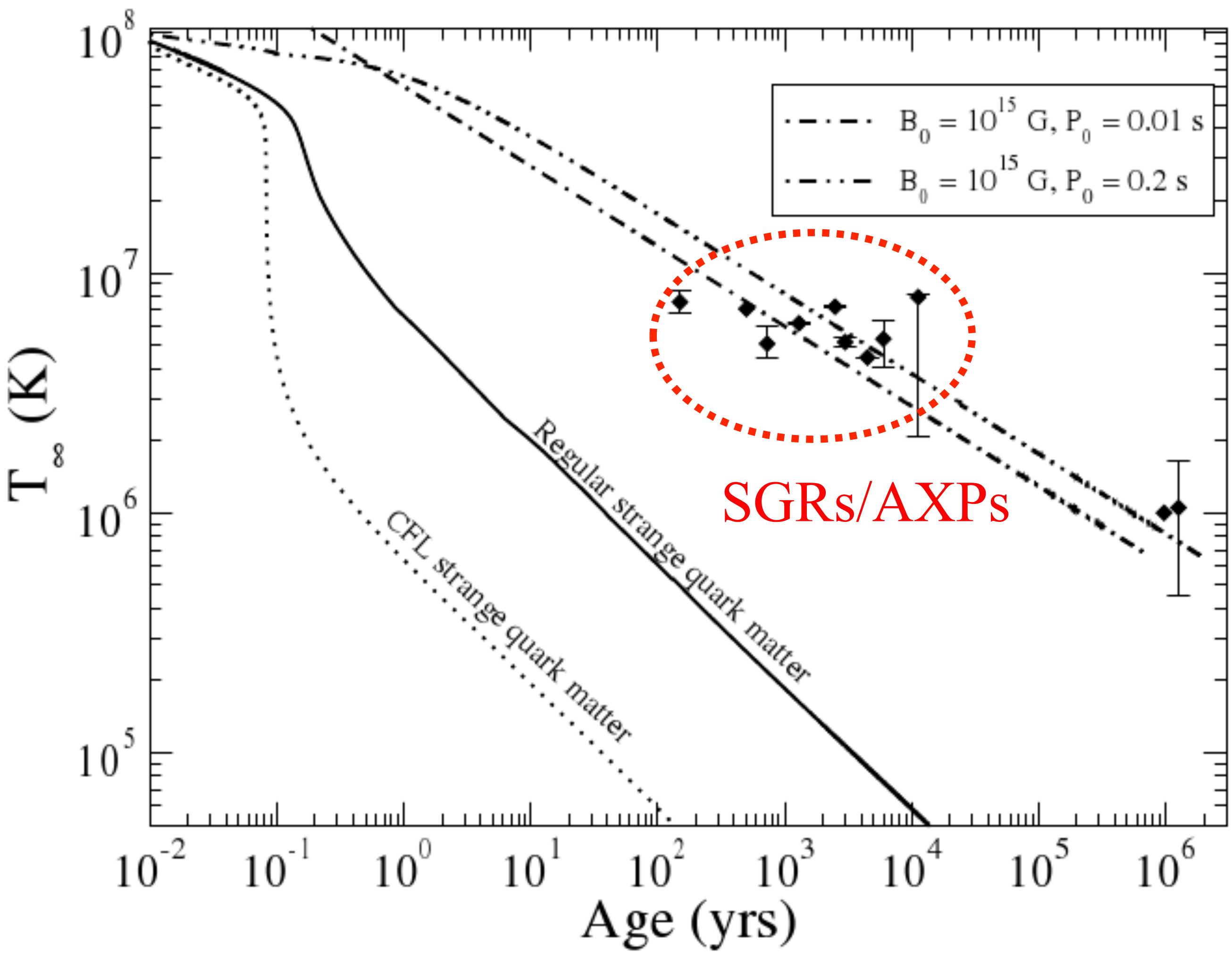


Image Credit: Rodrigo Negreiros

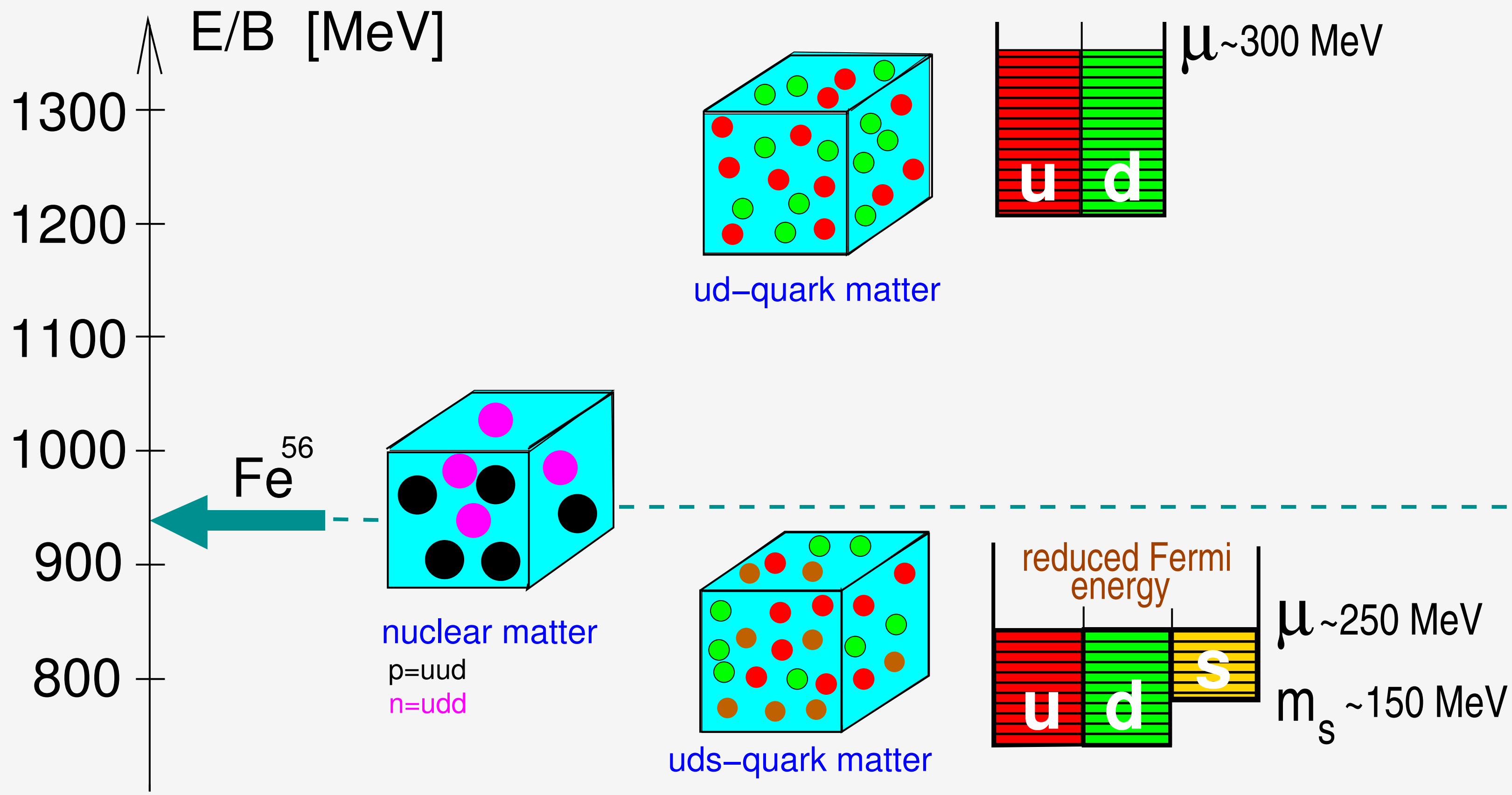
Cooling of Superconducting Strange Stars Experiencing Vortex Expulsion



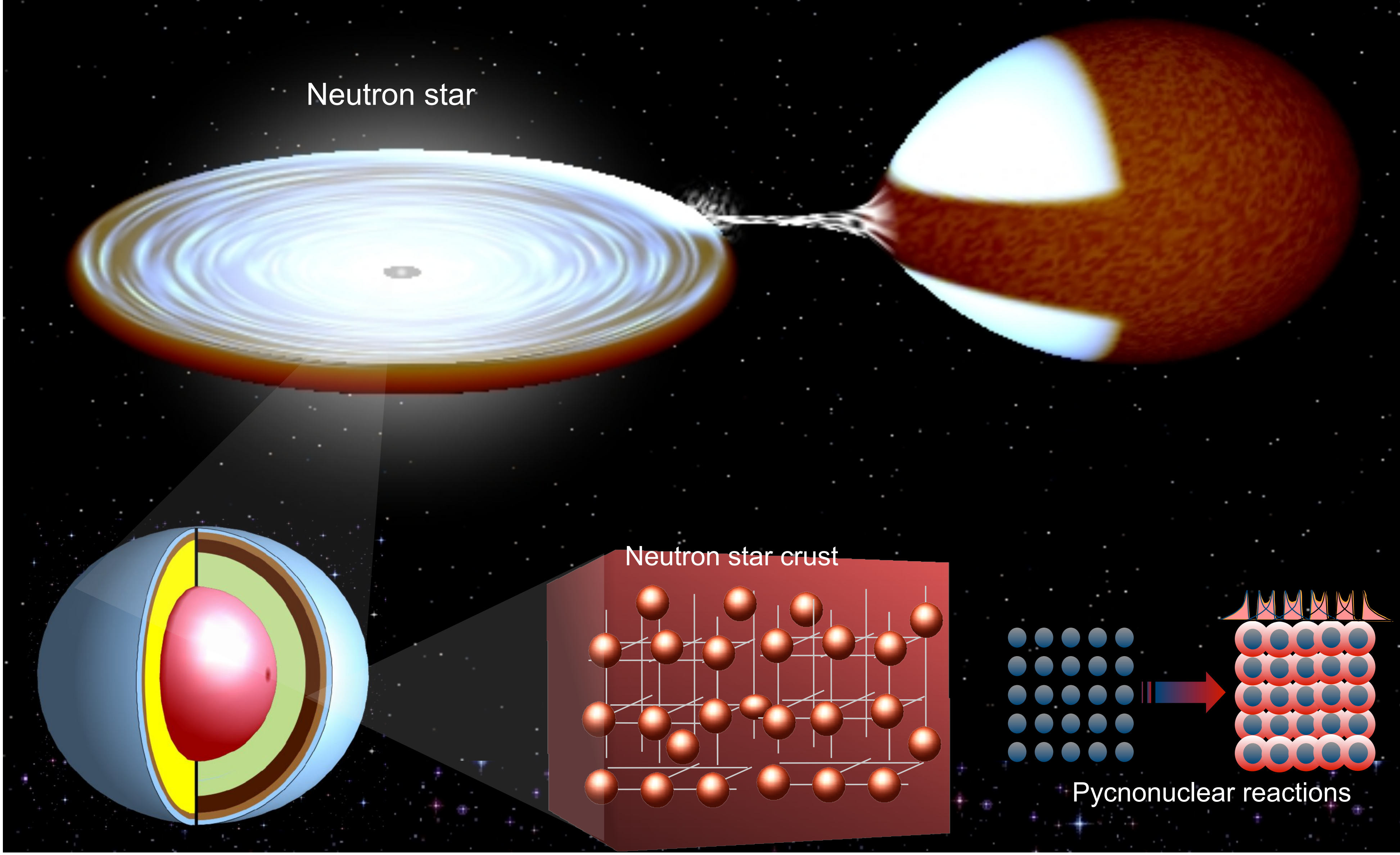
Negreiros, Niebergal, Ouyed, FW, PRD 81 (2010) 043005

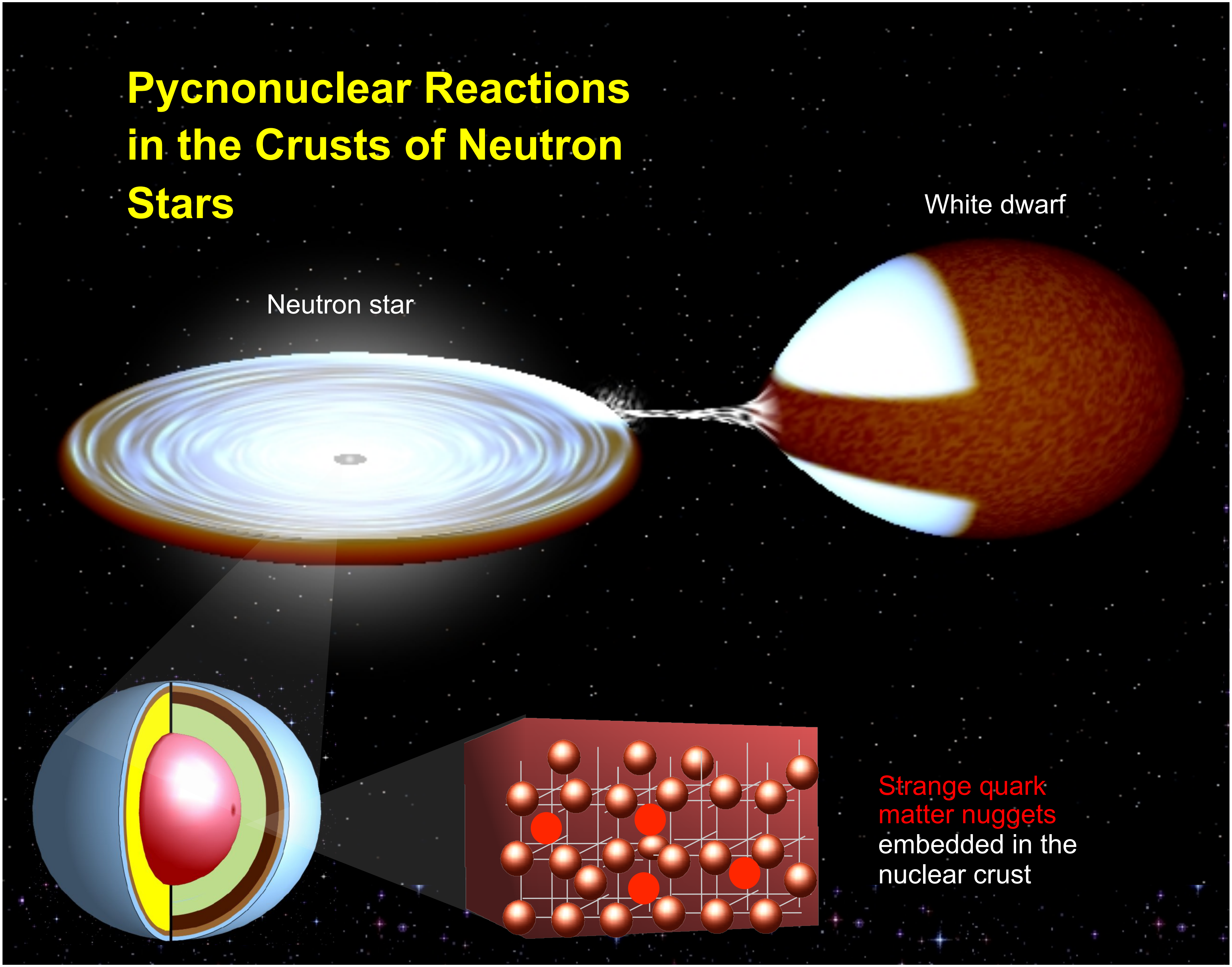
THE TRUE GROUND-STATE MYSTERY

Bodmer (1971), Witten (1984), Jaffe (1986), Terazawa (1989)

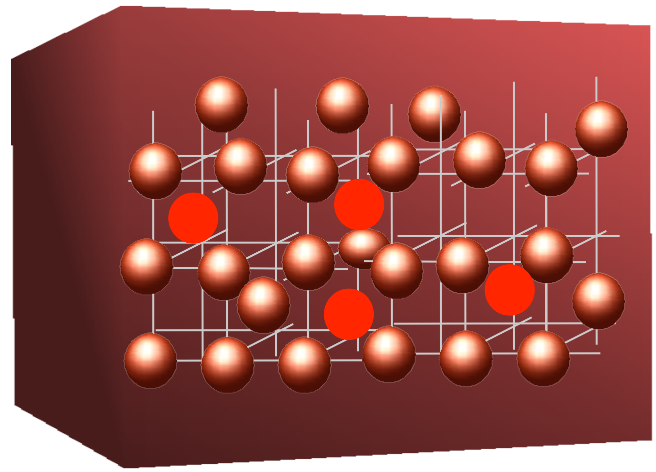


Pycnonuclear Reactions in the Crusts of Neutron Stars




Pycnonuclear Reactions in the Crusts of Neutron Stars
White dwarf
Neutron star
Strange quark matter nuggets embedded in the nuclear crust

Strange Quark Matter Nuggets



- $N_u \sim N_d \sim N_s$
- $A > A_{min}$ (~10 to 100)
- Charge-to-baryon number ratio depends on whether SQM is made of
  - "ordinary" quark matter, $Z \approx 0.1\ (m_{150})^2\ A$, or
  - color superconducting quark matter, $Z \approx 0.3\ m_{150}\ A^{2/3}$

Farhi & Jaffe, PRD 30 (1984) 2379; Berger & Jaffe, PRC 35 (1987) 213; Alcock, Farhi, Olinto, ApJ 310 (1986) 261; Madsen, PRL 87 (2001) 172003

Madsen, PRL 87 (2001) 172003; Rajagopal & Wilczek, PRL 86 (2001) 3492; Oertel & Urban PRD 77 (2008) 074015

$$R = 3.90\,x\,10^{46}\ \frac{8\,\rho\,A_1\,A_2\,Z_1^2\,Z_2^2}{A_1 + A_2}\ S(E)\ \lambda^{7/4}\ e^{-2.636/\sqrt{\lambda}}\ s^{-1}$$

$$S(E) = \sigma(E)\ E\ e^{2\pi Z_1 Z_2 e^2 \sqrt{\mu/2E}/\hbar}$$

$$\sigma(E) = \frac{\pi\hbar^2}{2\mu E}\sum_{l=0}^{l_{cr}}(2l+1)\,T_l$$

$$T_l = \left(1 + e^{WKB}\right)^{-1}$$

$$WKB = \int_{r_1}^{r_2}\frac{8\mu}{\hbar^2}\left(V_{eff}(r,E) - E\right)dr$$

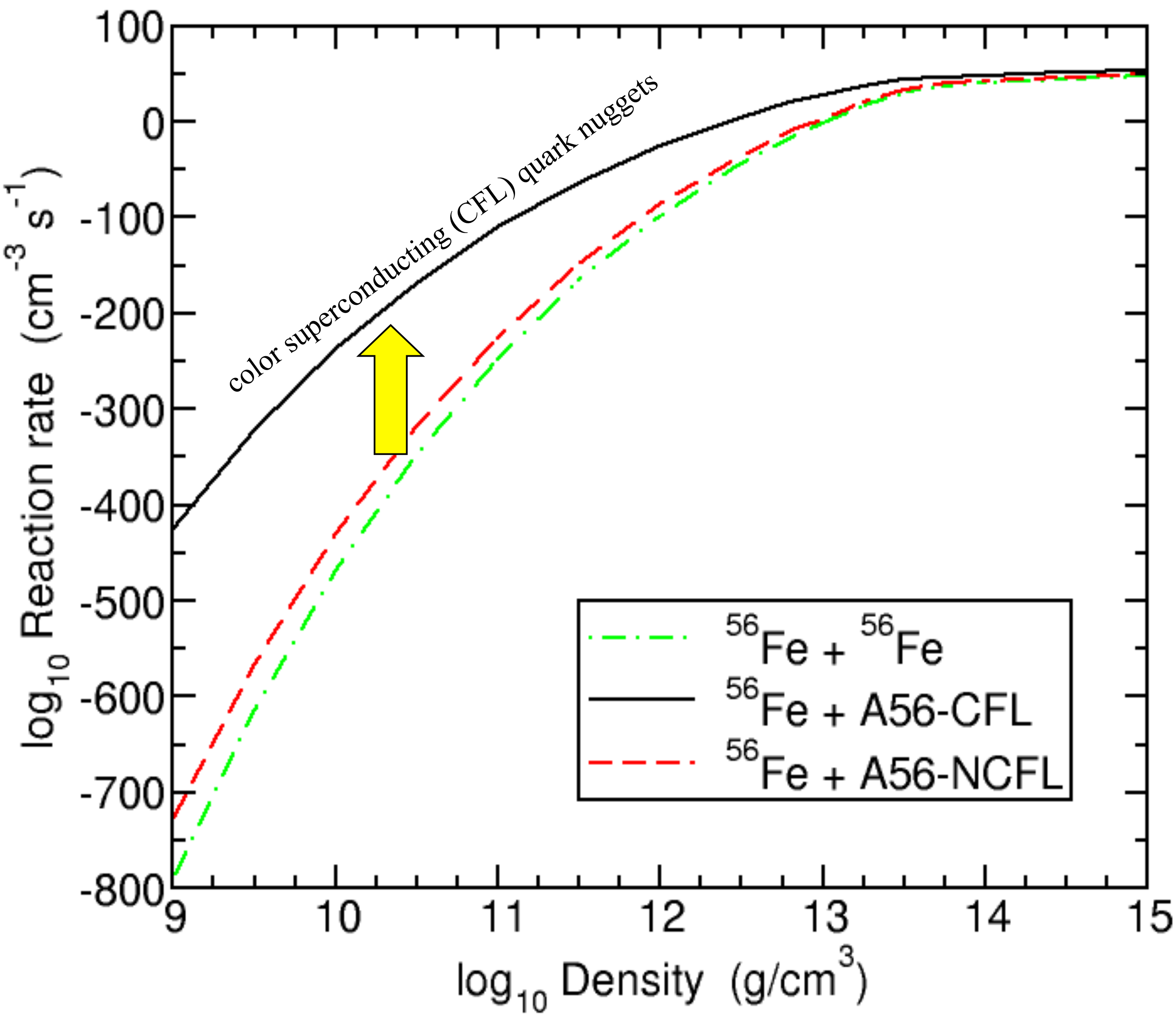
$$V_{eff}(r,E) = V_C(r) + V_N(r,E) + l(l+1)\hbar^2/2\mu r^2$$

$$V_N = \int \rho_1(r)\rho_2(r)V_{NN}(v, \vec{R} - \vec{r}_1 + \vec{r}_2)\,d^3\vec{r}_1\,d^3\vec{r}_2$$

$$V_{NN}(v, \vec{r}) = V_f(\vec{r})\,e^{-4v^2/c^2}$$

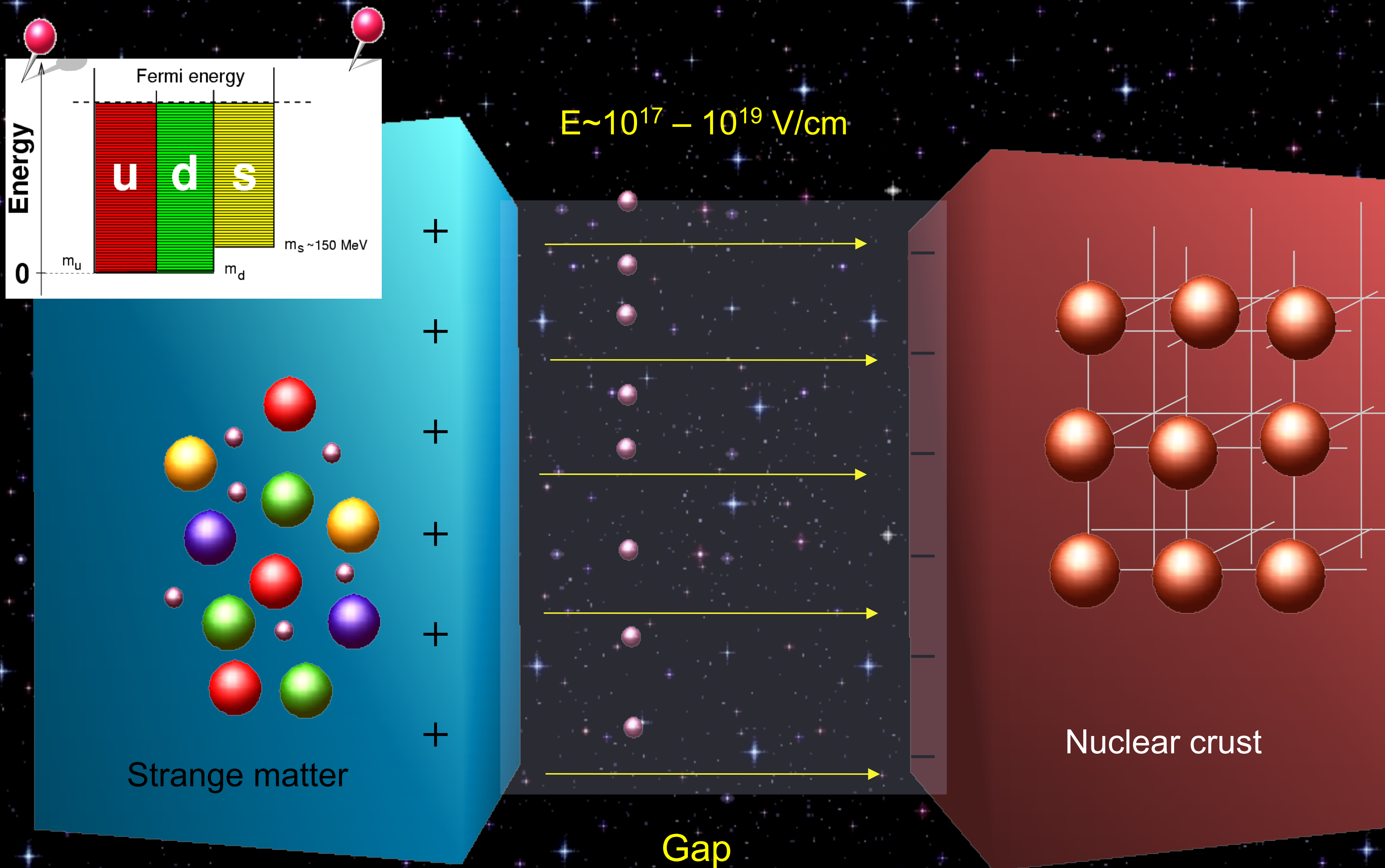
$$V_f(r) = \frac{1}{64\pi a_m^3}V_0\left(1 + \frac{r}{a_m} + \frac{r^2}{3a_m^2}\right)e^{-r/a_m}$$

Impact of quark nuggets on pycnonuclear reaction rates



B. Golf, J. Hellmers, F. Weber, PRC 80 (2009) 015804

NUCLEAR CRUST ON STRANGE MATTER



Electric Charge on Strange Stars …

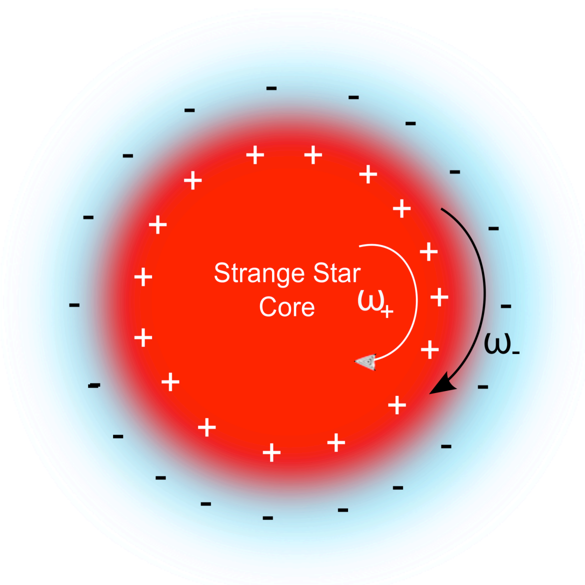
- Alters the energy-momentum tensor:

$$T_\nu{}^\mu = (P+\rho)u_\nu u^\mu + P\delta_\nu{}^\mu + \frac{1}{4\pi}\left(F^{\mu l}F_{\nu l} + \frac{1}{4\pi}\delta_\nu{}^\mu F_{kl}F^{kl}\right)$$

R. Negreiros, FW, M. Malheiro, V. Usov, PRD 80 (2009) 083006

Mass increases by up to 15%, Radius up to 5%

- May be differentially rotating:



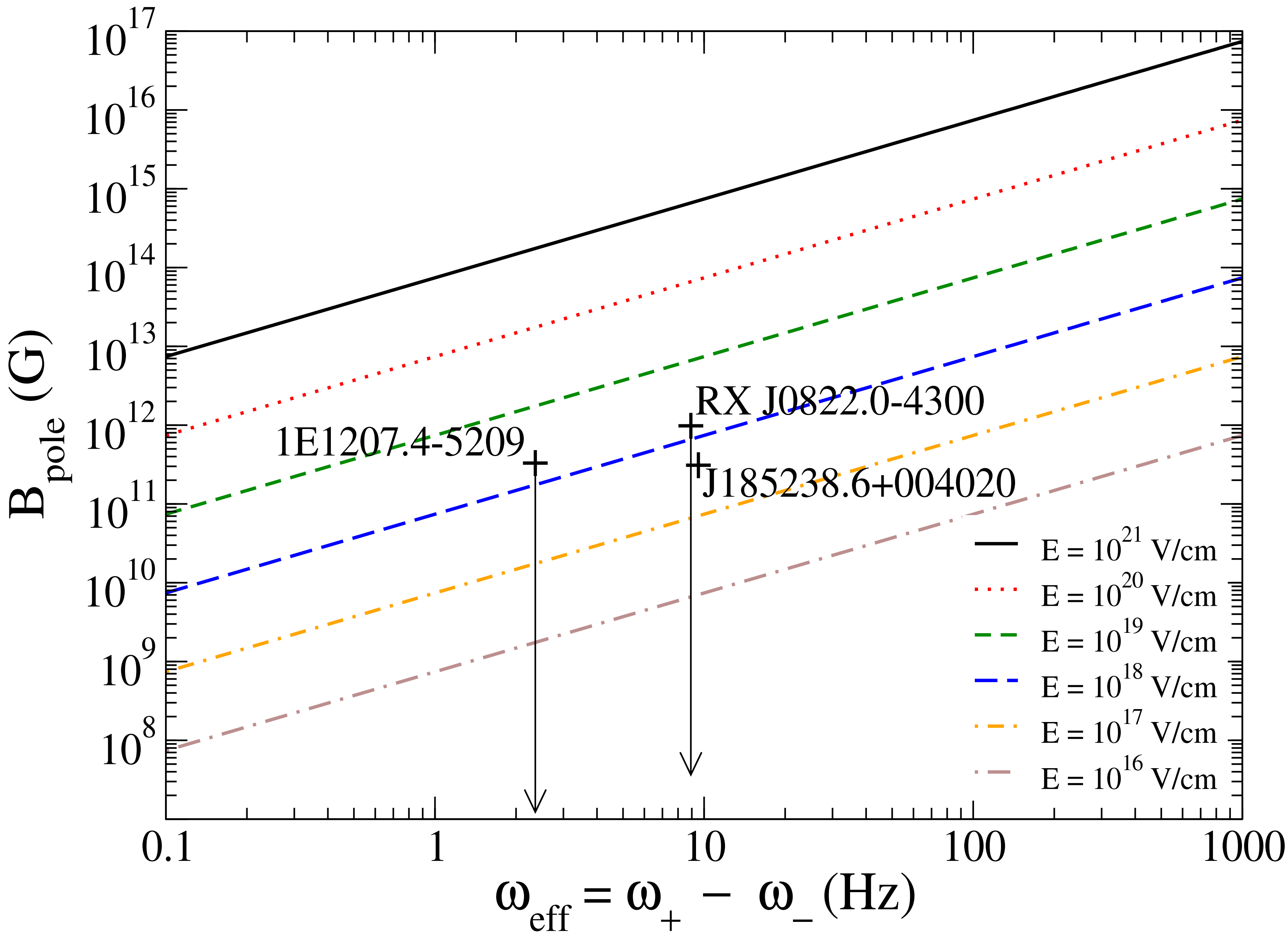
$$I = \sigma(\omega_+ - \omega_-)$$

$$B = \text{const}\ E\,(\omega_+ - \omega_-)\,R$$

Could explain magnetic fields of CCOs

R. Negreiros, I. Mishustin, S. Schramm, FW, PRD 82 (2010) 103010.

Magnetic fields on CCOs generated by differentially rotating electron spheres



R. Negreiros, I. Mishustin, S. Schramm, FW, PRD 82 (2010) 103010.

SUMMARY

- iMSPs & NSs in LMXBs ideal objects to look for phase transitions (e.g., quark re/deconfinement).
- NS in Cas A: predict a mass of 1.46 M_{sun} ; test existence of pion condensates.
- The unusual thermal evolution of magnetars (SGRs, AXPs) can be explained if one assumes that these objects are made of CFL strange quark matter.
- Pycnonuclear reactions in crusts of NSs strongly altered by presence of CFL strange quark matter nuggets. Connection to superbursts?
- Differentially rotating electron spheres on quark stars generate magnetic fields.
- If CCOs are really that small, they should be made of self-bound strange quark matter.

Research on compact stars and relativistic astrophysical phenomena is on its way of providing solid information about the properties of ultra-dense baryonic matter and its associated phase diagram.