

Rheological study of a simulated polymeric gel: shear banding

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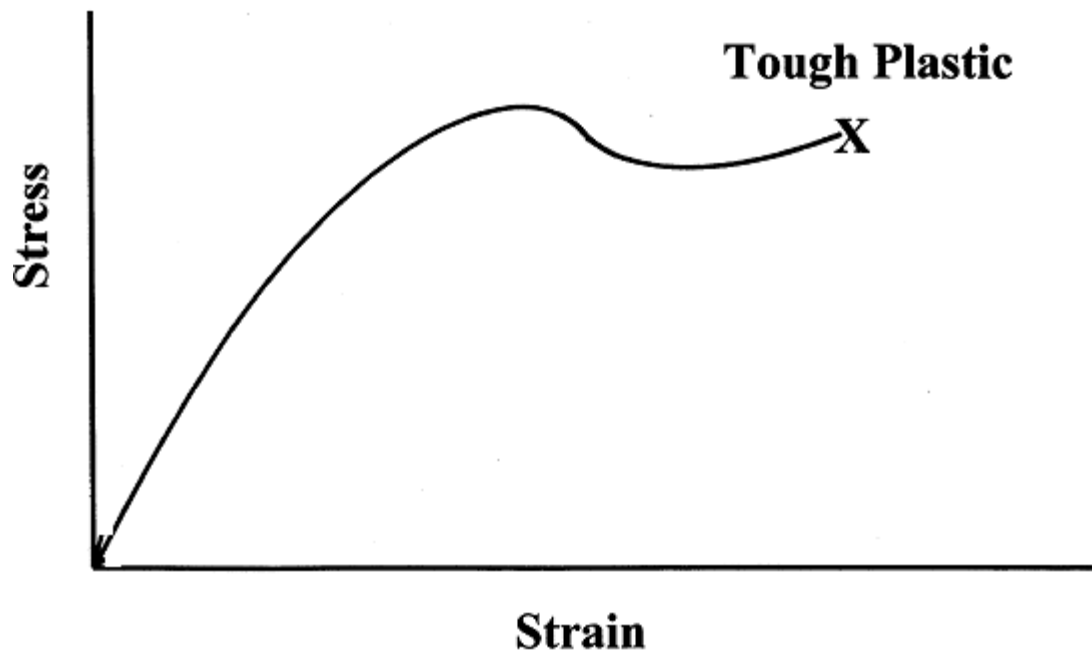
[°] Ben Gurion University of the Negev (Be'er Sheva, Israel)

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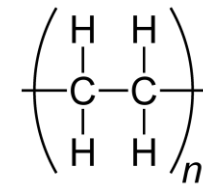


Polymers

- Long-chain molecules of high molecular weight



polyethylene



[Introduction to Physical
Polymer Science,
L. Sperling (2006)]

Motivation of research



Polymer science

Polymer chemistry
(synthesis)

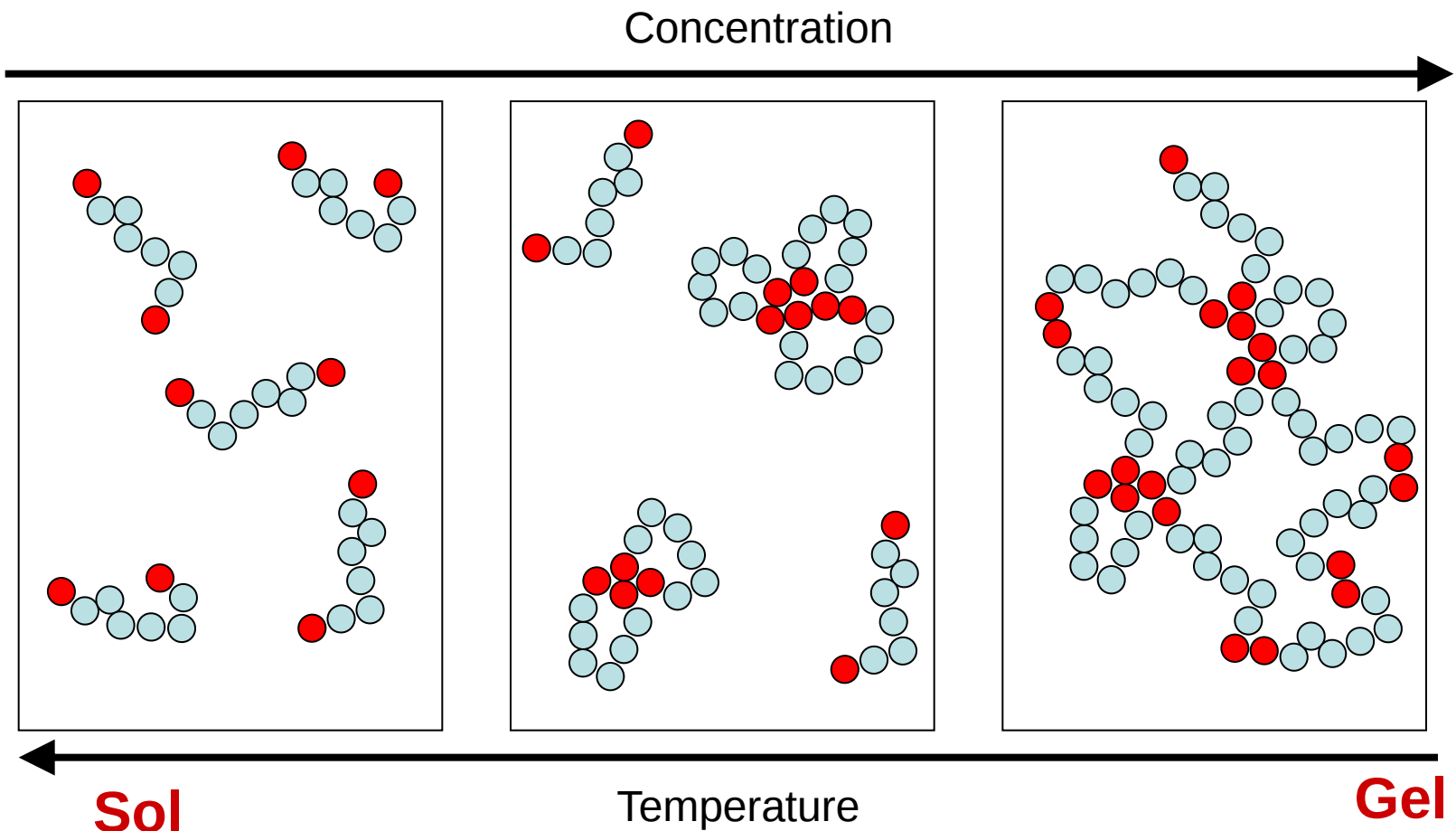
Polymer physics

Polymer rheology

Introduction: polymeric gels

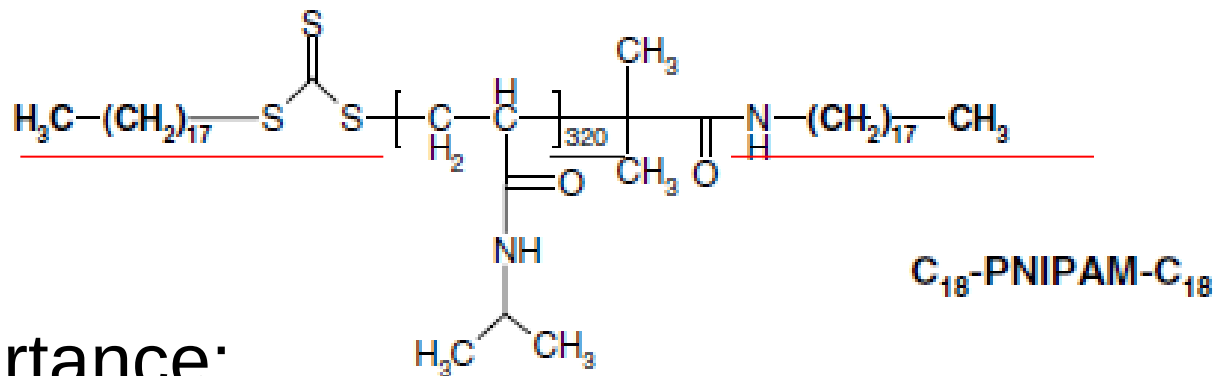
Polymeric gels

Reversible junctions between endgroups
(telechelic polymers)



Polymeric gels

- Examples
 - PEO (polyethylene glycol) chains terminated by hydrophobic moieties
 - Poly-(N-isopropylacrylamide) (PNIPAM)

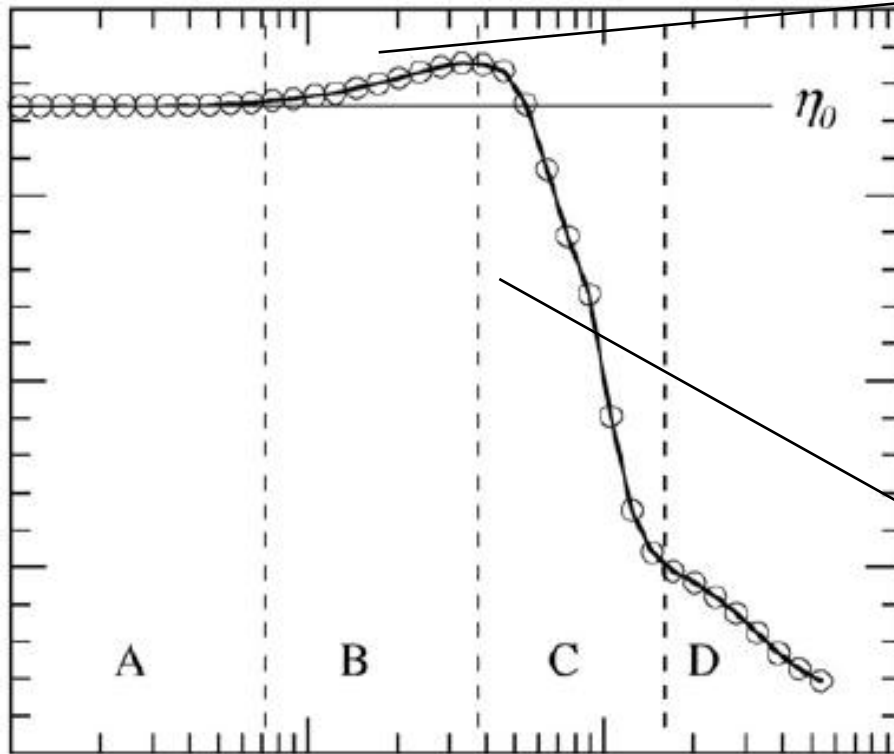


- Importance:
 - laxatives, skin creams, tooth paste, paintball fill, preservative for objects salvaged from underwater, eye drops, print heads, spandex, foam cushions,...
 - cytoskeleton

Visco-elastic properties

• $\eta \dot{\gamma} = \text{stress}$

Viscosity η



Shear rate $\dot{\gamma}$



Hybrid MD/MC simulation of a polymeric gel

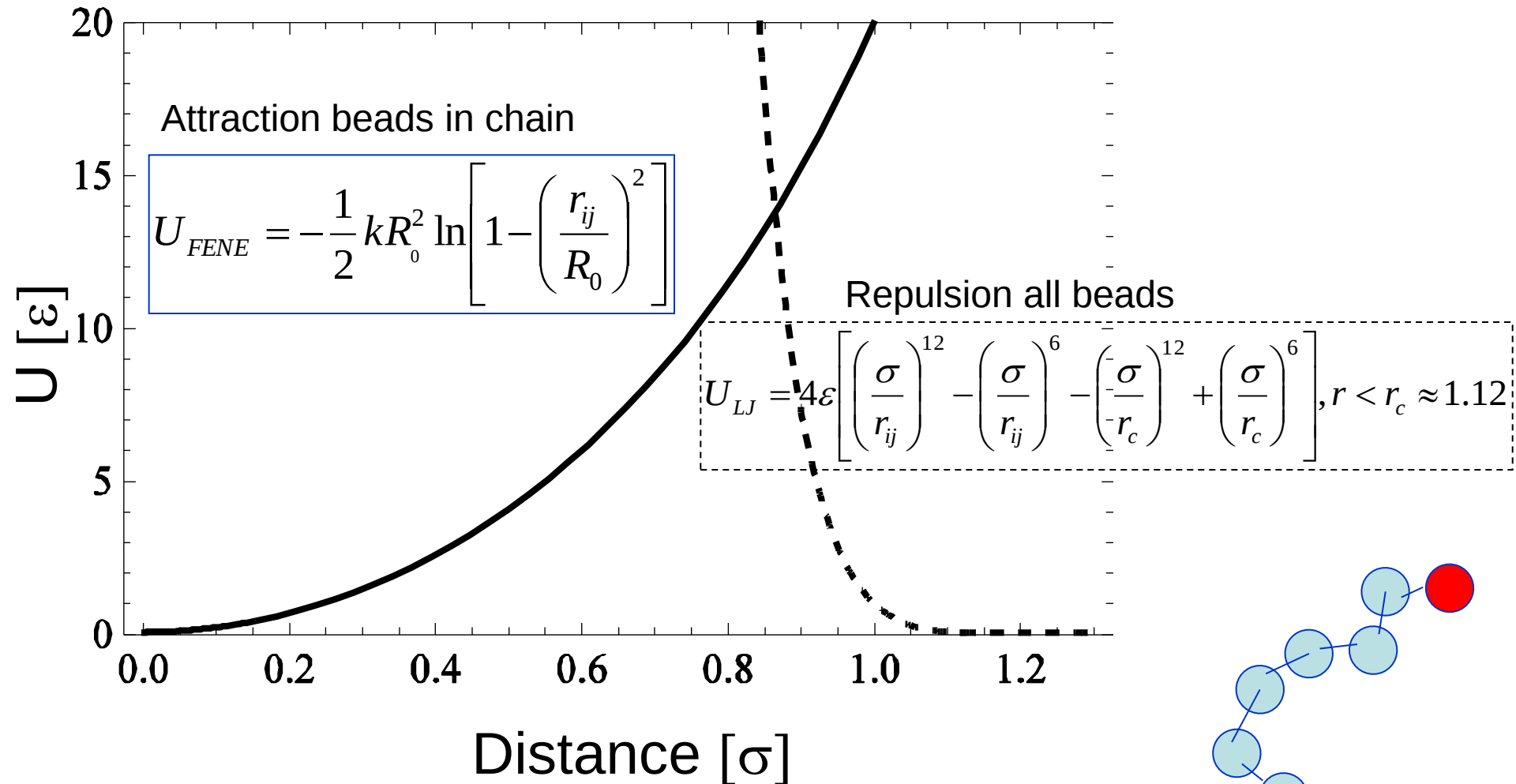
Molecular dynamics simulation

ITERATE

- Give initial positions, choose short time Δt
- Get forces $F = -\frac{dU}{d\vec{r}}$ and acceleration $a=F/m$
- Move atoms
- Move time $t = t + \Delta t$

Bead-spring model

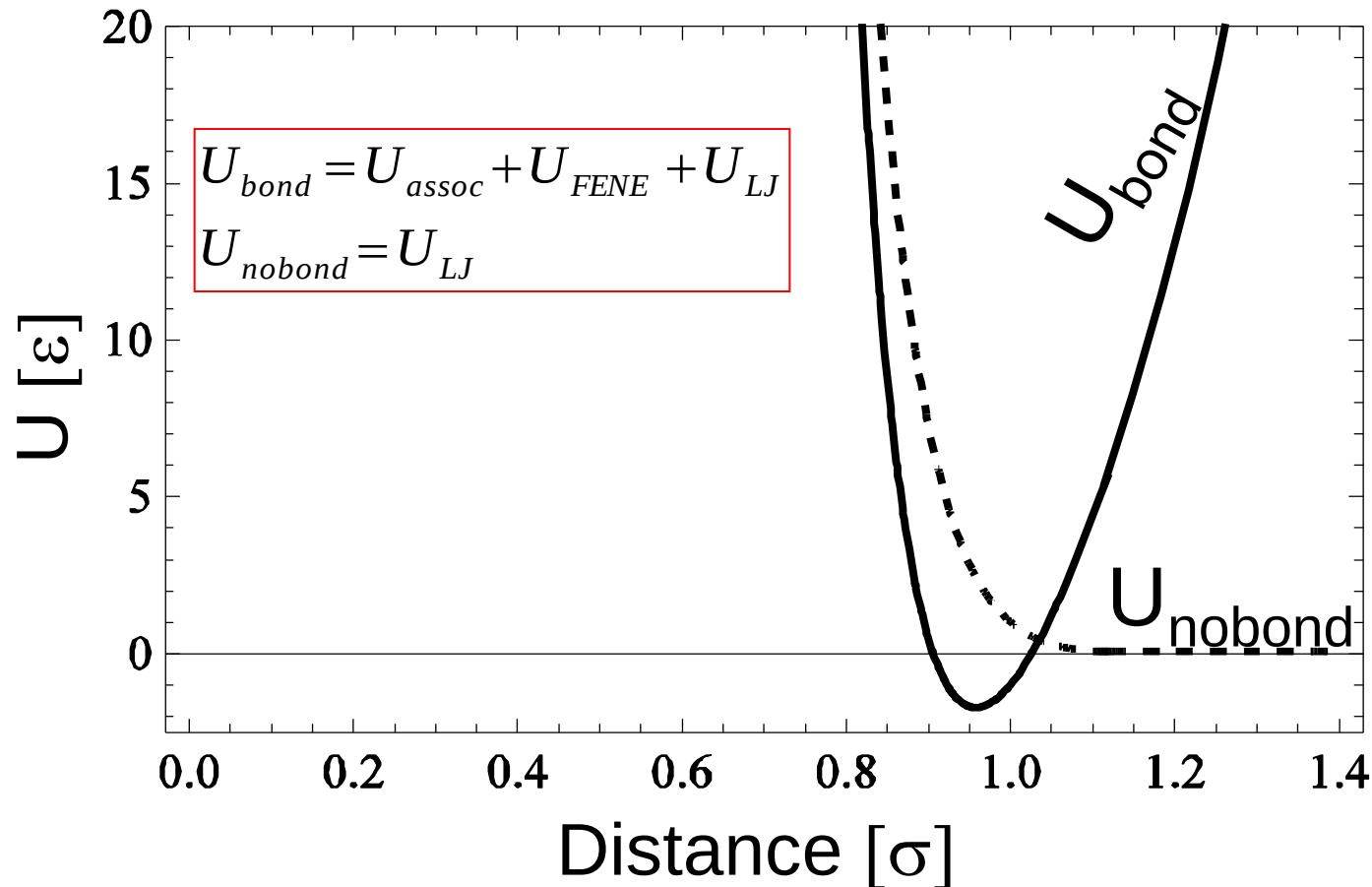
[K. Kremer and G. S. Krest.
J. Chem. Phys 1990]



- Temperature control through coupling with heat bath

Associating polymer

- Junctions between end groups : FENE + Association energy
- Dynamics ...

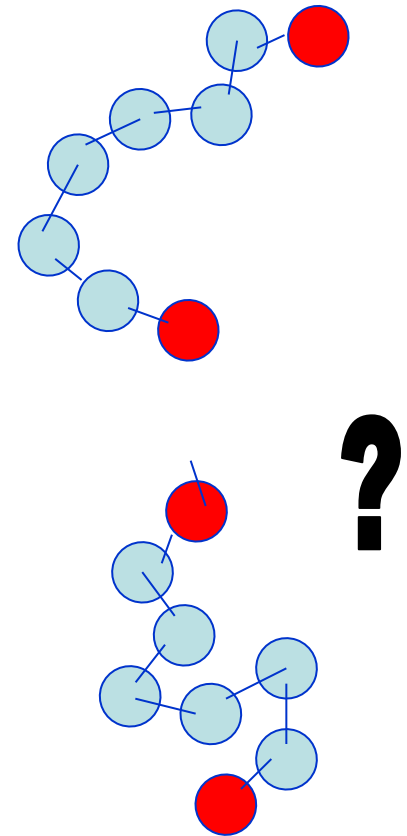
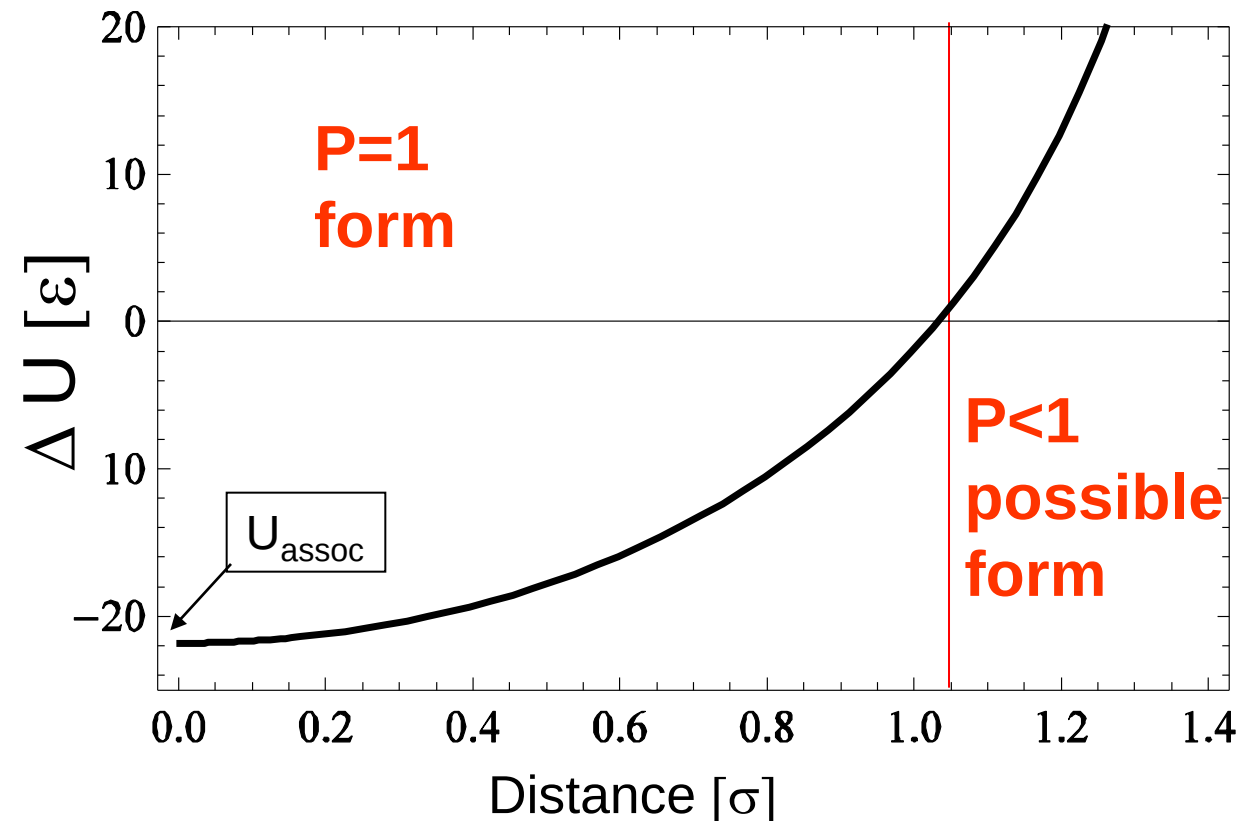


Dynamics of associating polymer (I)

- **Monte Carlo:** attempt to form junction

$$P \sim \exp\left(\frac{-\Delta U}{k_b T}\right)$$

$$\begin{aligned} \Delta U &= U_{\text{possible new}} - U_{\text{old}} \\ &= U_{\text{assoc}} + U_{\text{FENE}} \end{aligned}$$

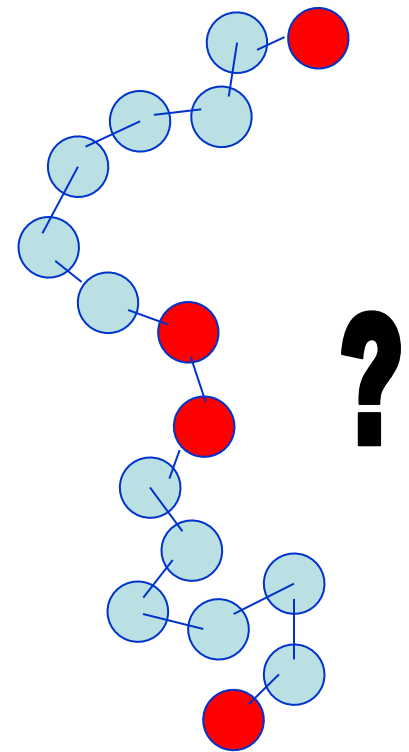
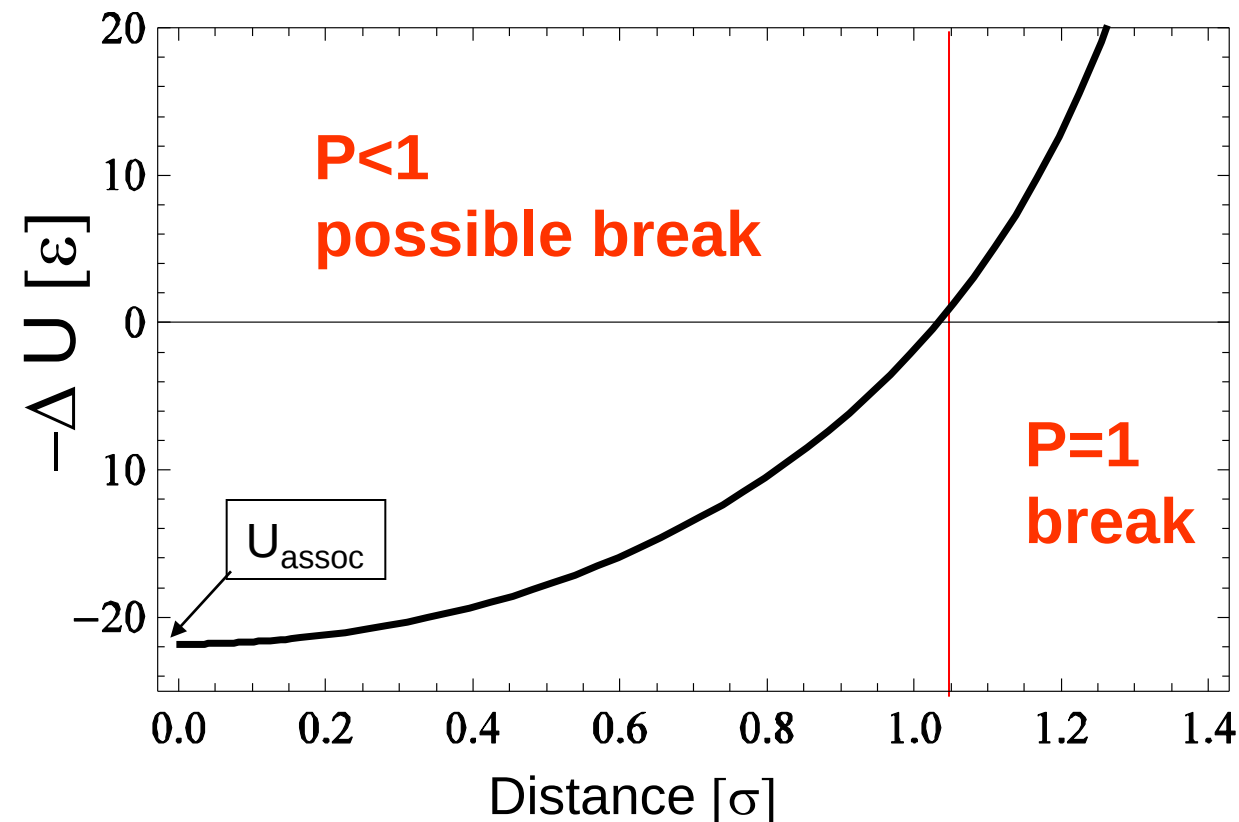


Dynamics of associating polymer (II)

- **Monte Carlo:** attempt to break junction

$$P \sim \exp\left(\frac{-\Delta U}{k_b T}\right)$$

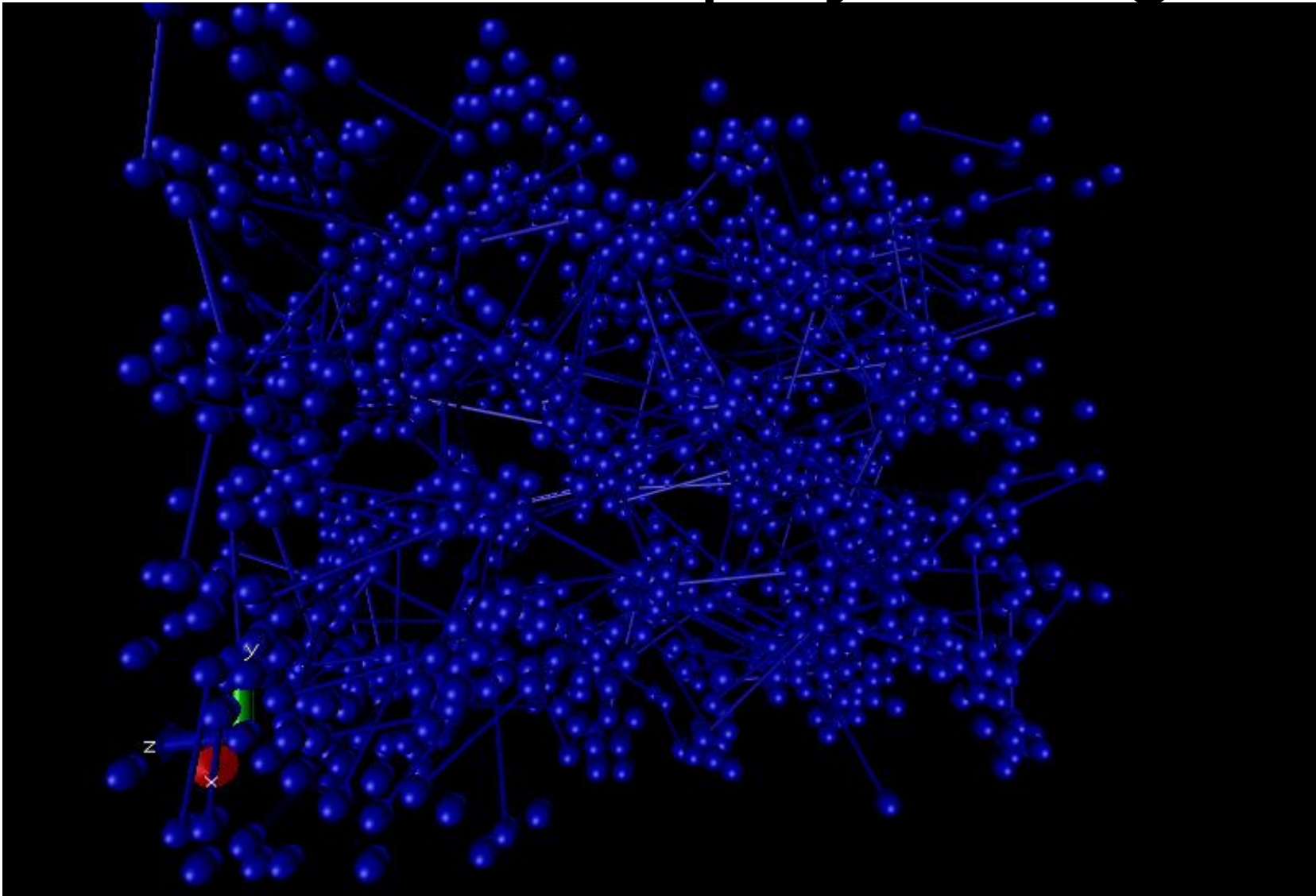
$$\Delta U = U_{possible} - U_{old} = -(U_{assoc} + U_{FENE})$$



Simulation details

- 1000 polymeric chains, 8 beads/chain
- Units: σ (length), ε (energy&temperature), m (mass), $\tau = \sigma(m/\varepsilon)^{1/2}$ (time);
- Box size: $(23.5 \times 20.5 \times 27.4) \sigma^3$ with periodic boundary conditions

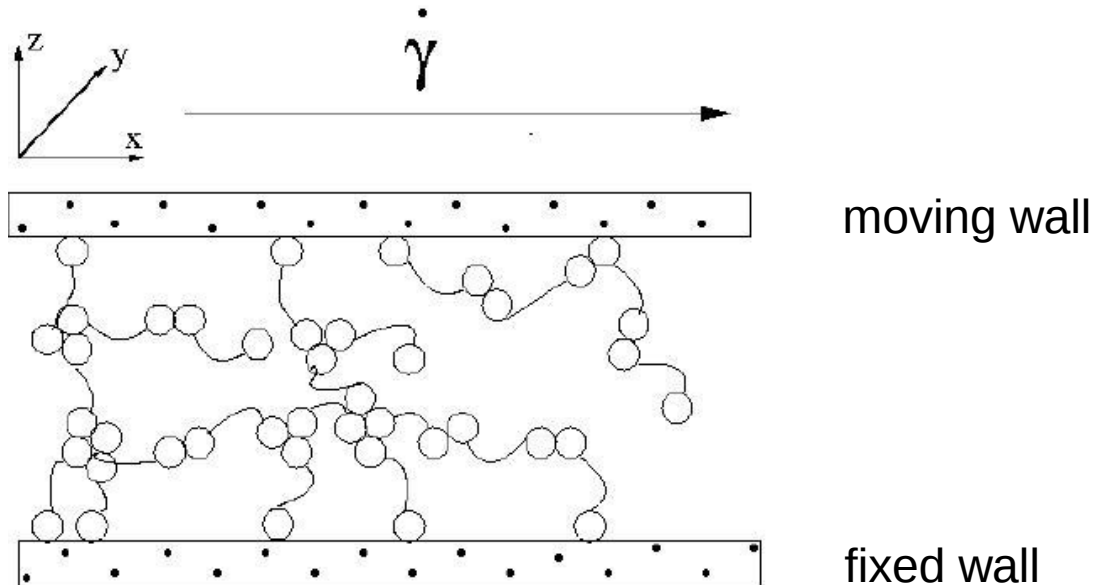
Simulated polymeric gel



$T=1.0$
only
endgroups
shown

Shearing the system

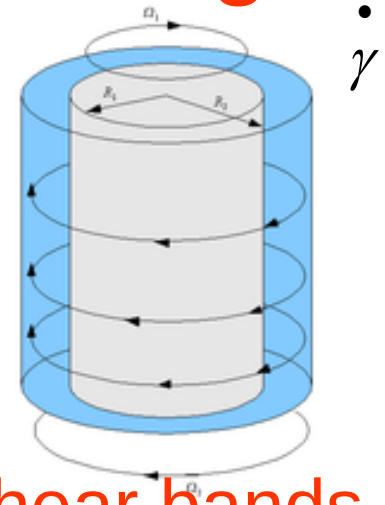
Some chains grafted to wall;
move wall with constant shear rate



Shear banding in polymeric gel

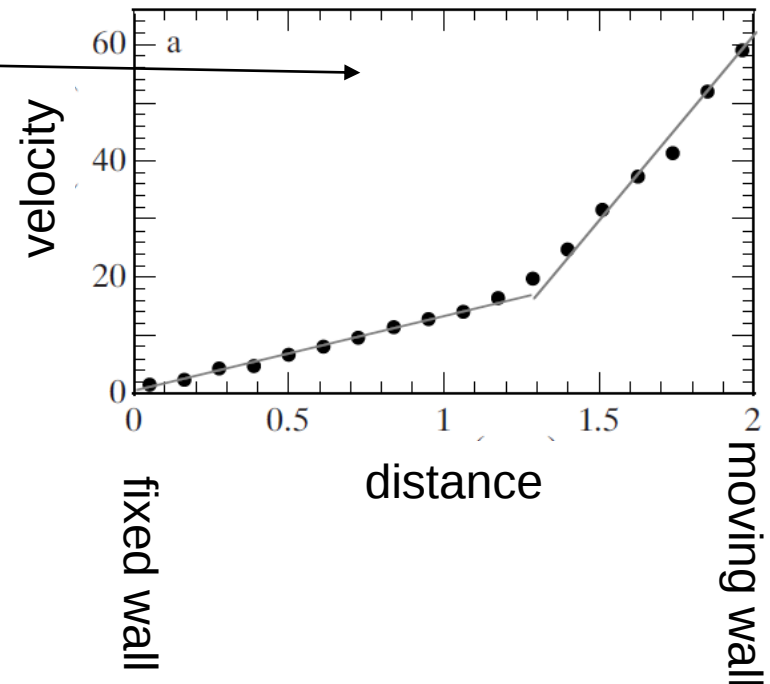
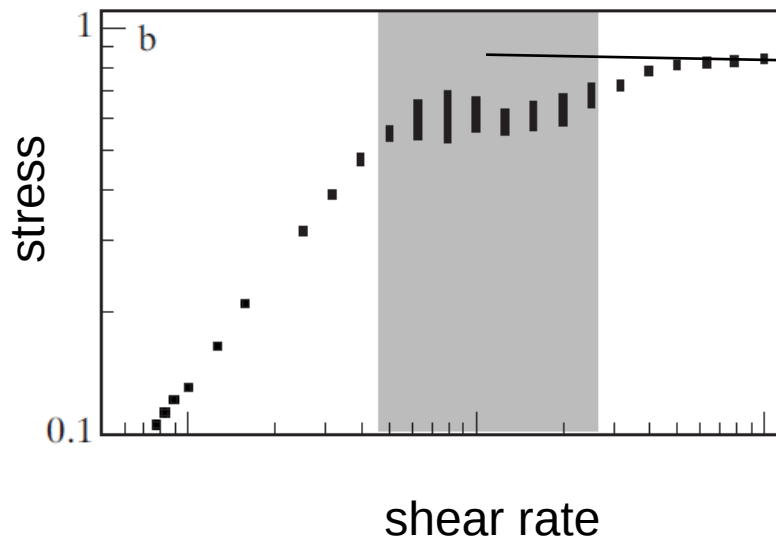
Shear-Banding in Associating Polymers

- PEO in Taylor-Couette system



Plateau in stress-shear curve

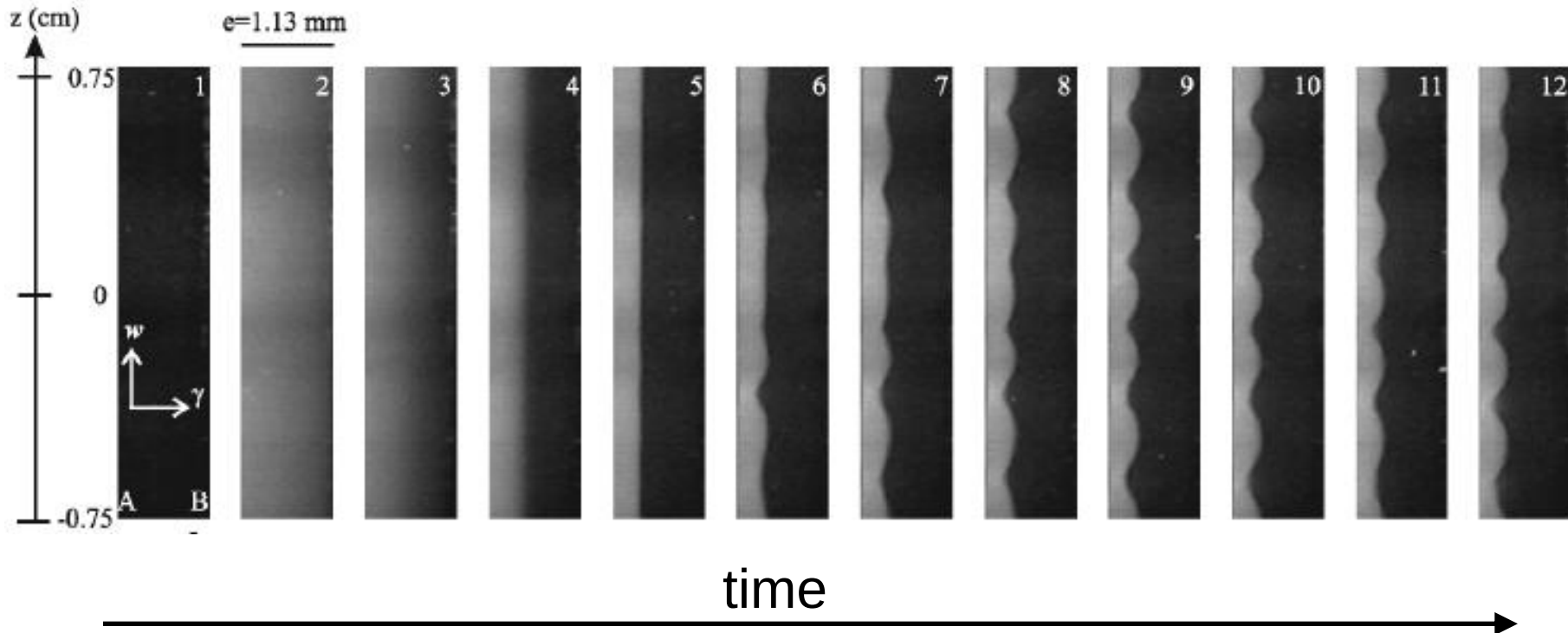
two shear bands



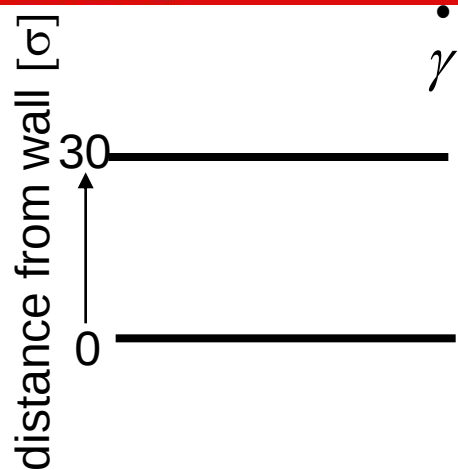
[J.Sprakel et al.,
Phys Rev. E 79, 056306 (2009)]

Shear-banding in viscoelastic fluids

- Interface instabilities in worm-like micelles



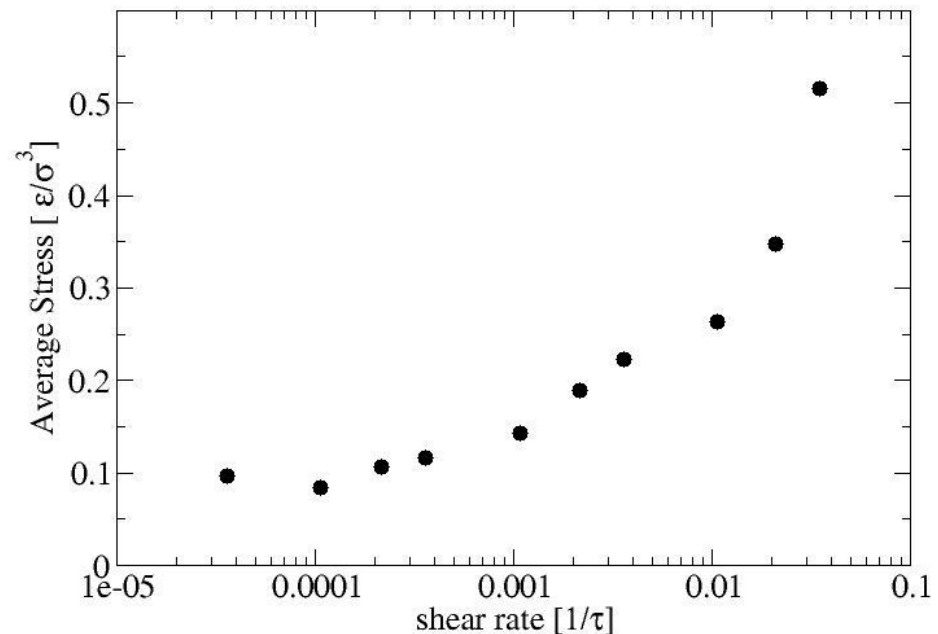
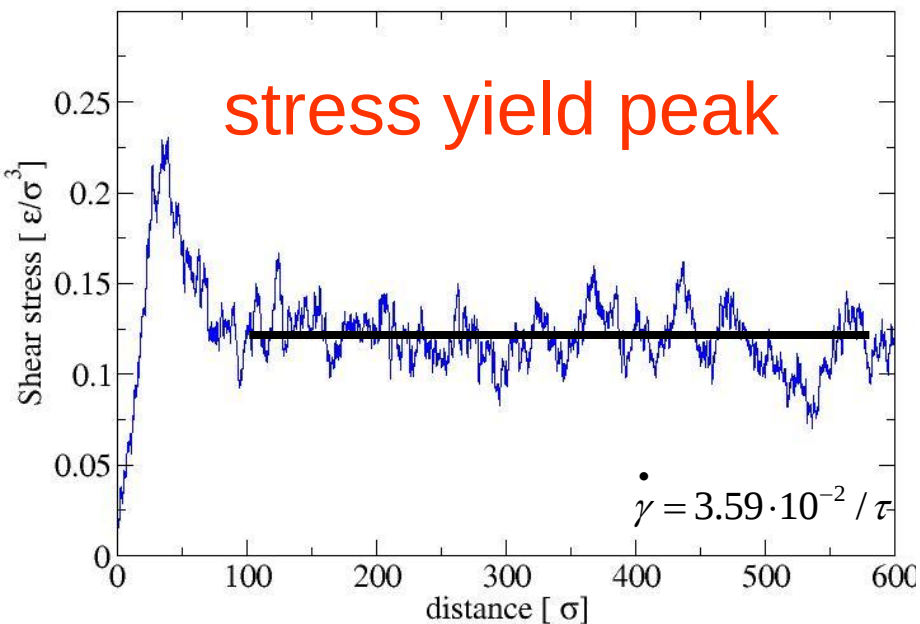
[Lerouge et al., PRL 96,088301 (2006).]



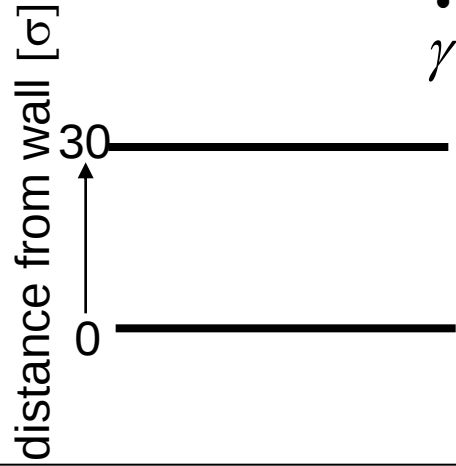
Stress under constant shear

All results $T=0.35 \ \varepsilon$ ($<$ micelle transition $T=0.5 \ \varepsilon$)

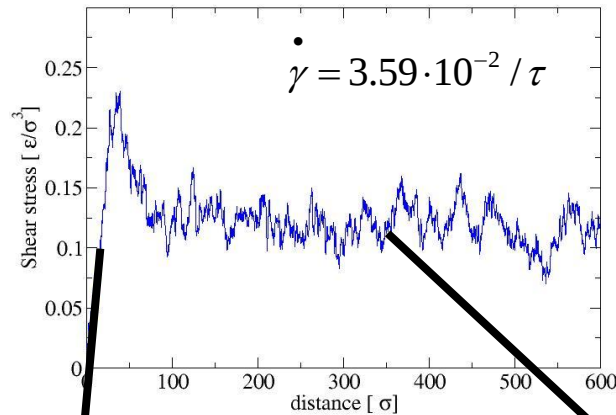
$T=0.35$



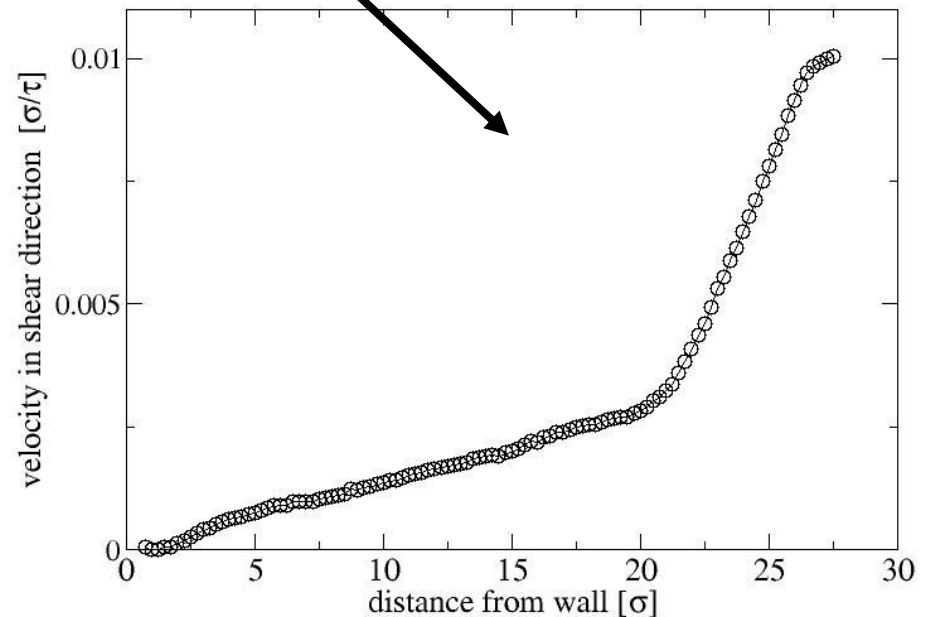
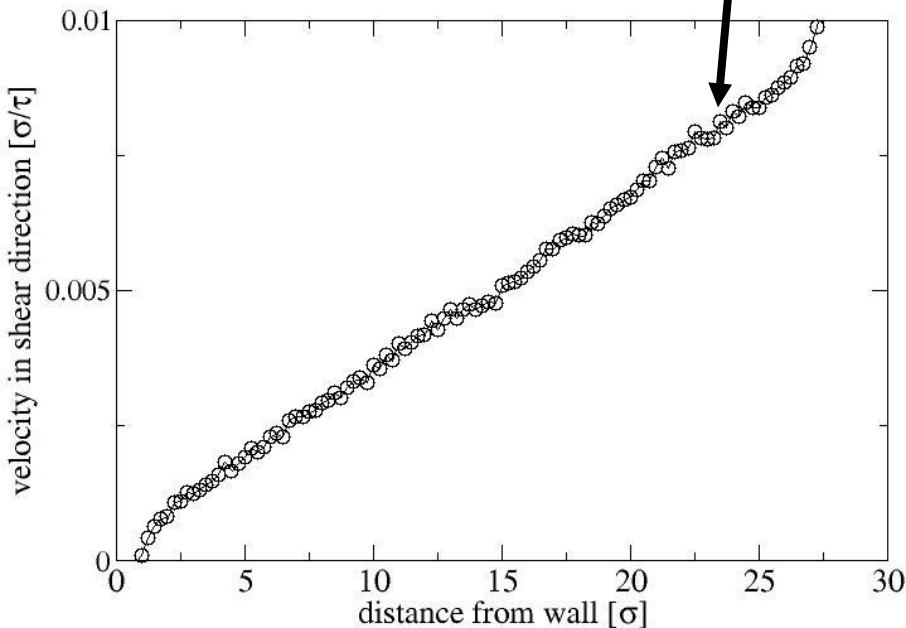
Velocity profiles



Before yield peak:
homogeneous



After yield peak:
2 shear bands



Velocity profile over time

- Fluctuations of interface

moving
wall

distance from wall [σ]

fixed
wall

2

4

6

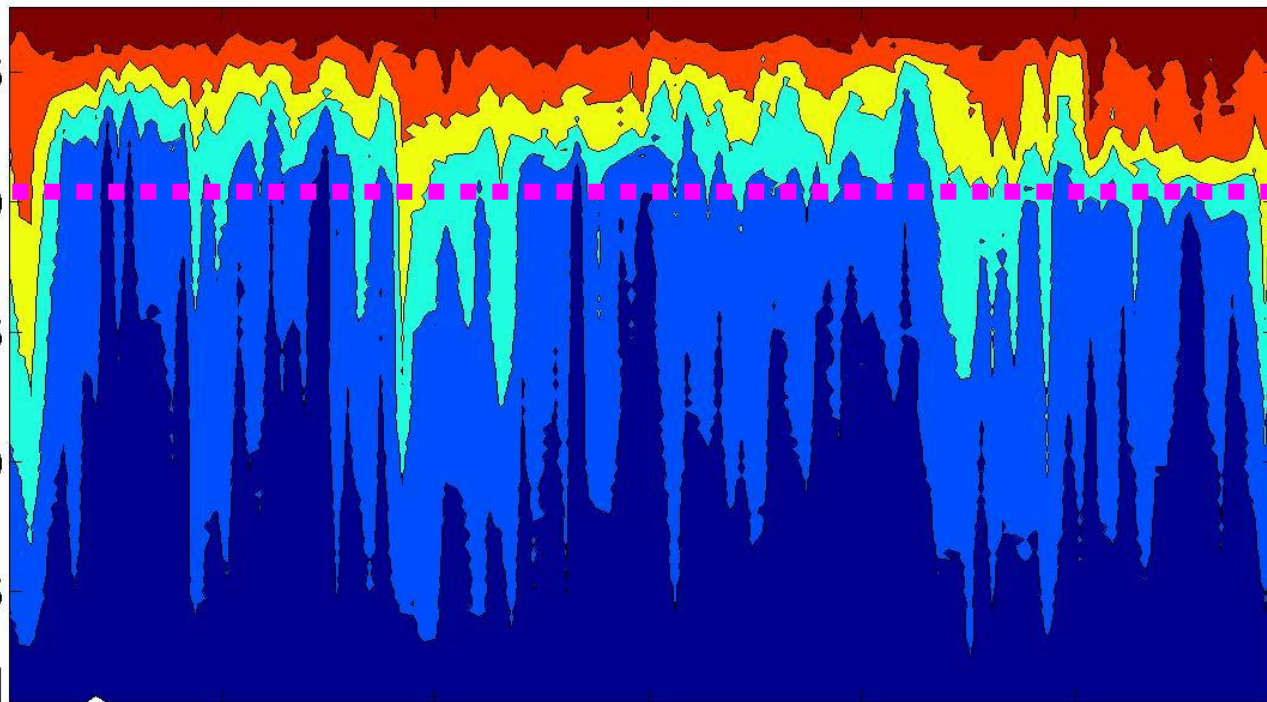
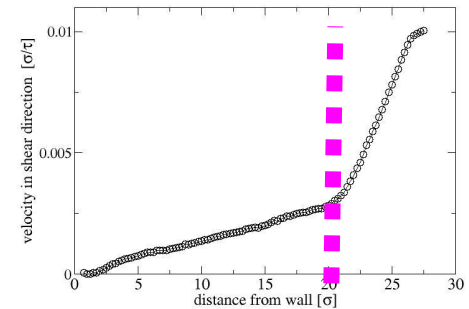
8

10

time [τ]

$\times 10^4$

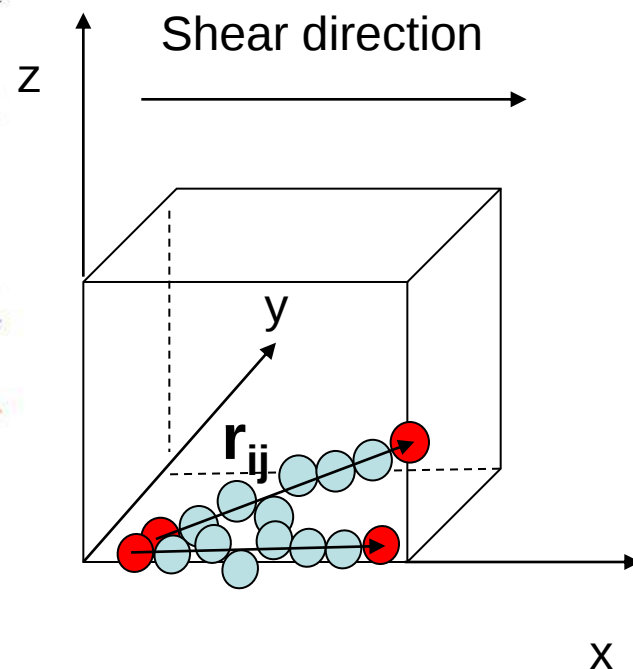
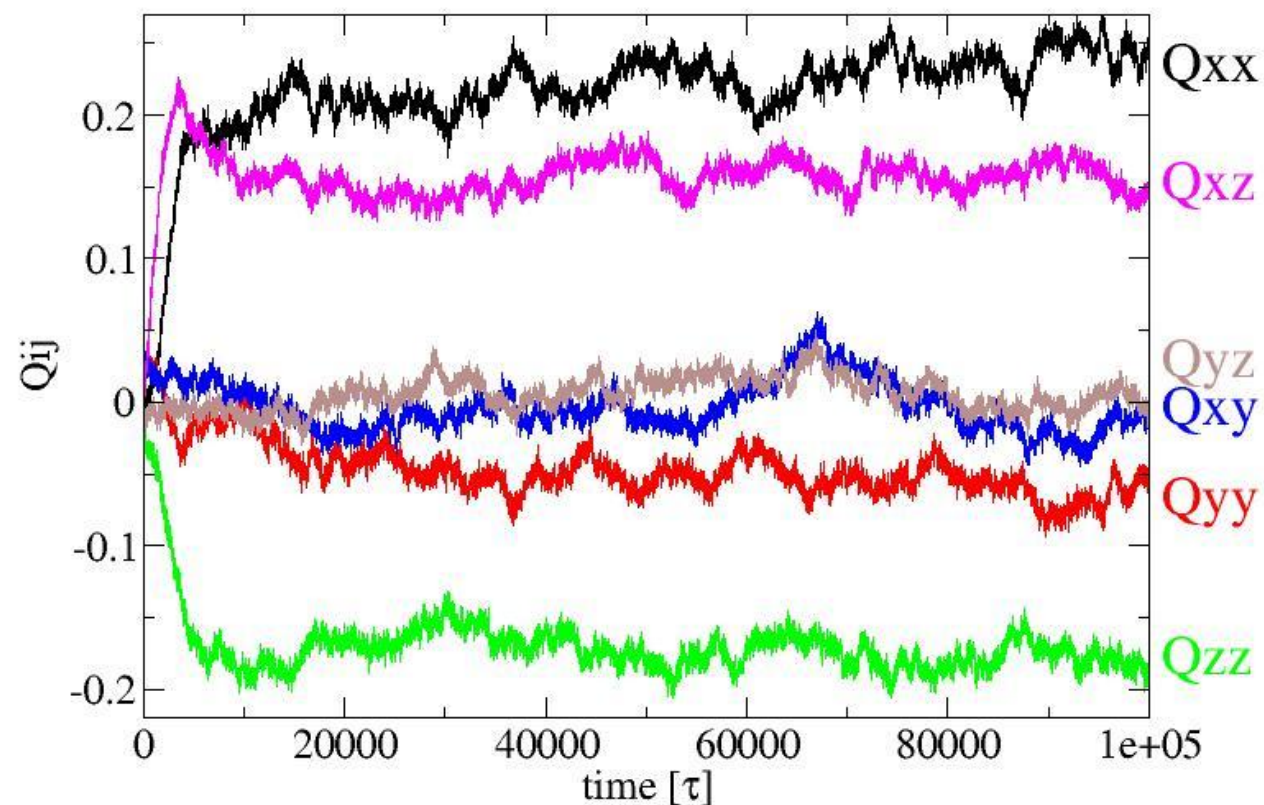
$\times 10^{-3}$
velocity [σ/τ]



Chain Orientation

$$Q_{ij} = \left\langle \frac{3}{2} \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) \right\rangle$$

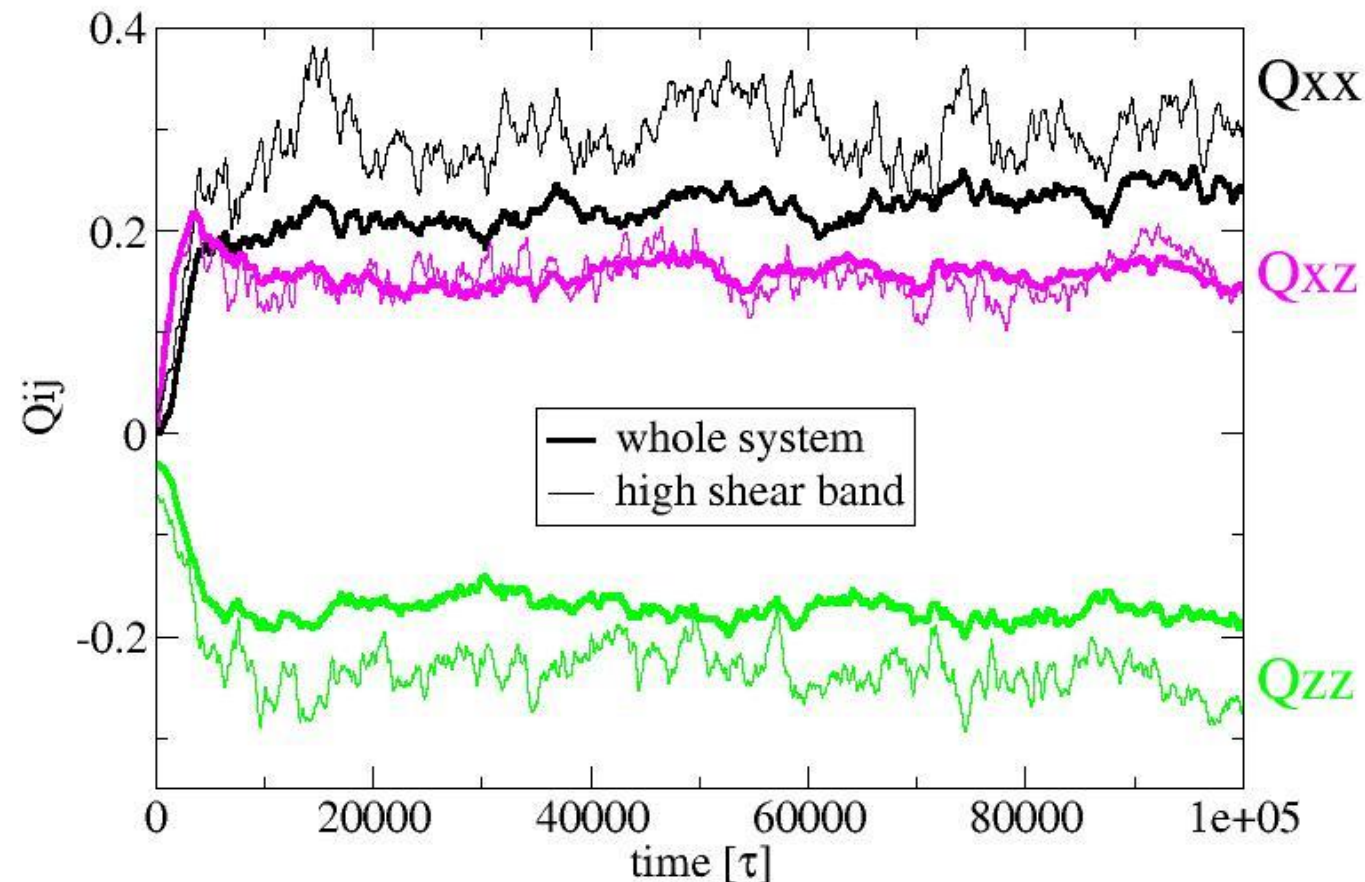
$$Q_{xx}=1 \quad Q_{zz}=-0.5$$



Chain orientation

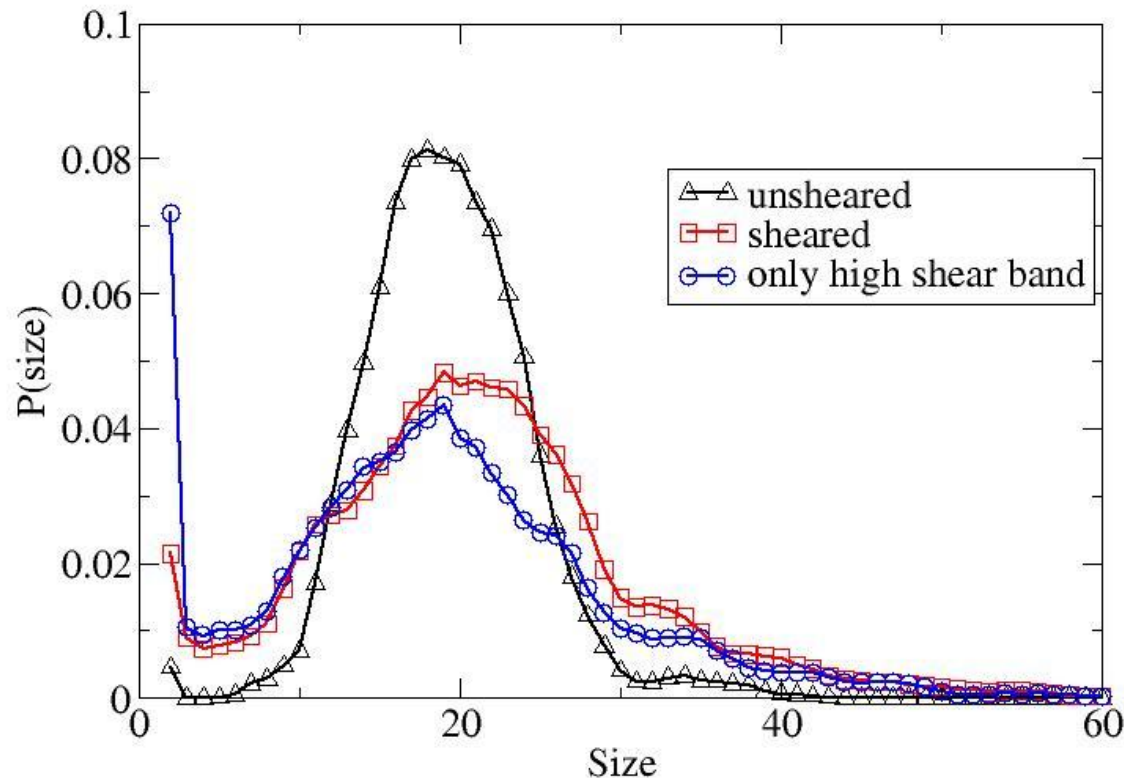
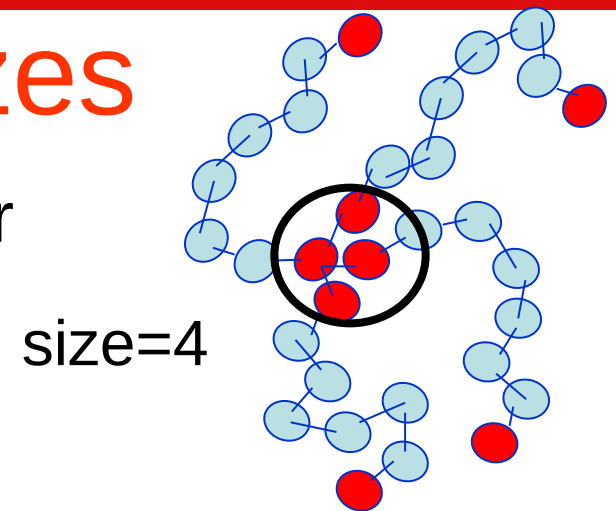
$$Q_{ij} = \left\langle \frac{3}{2} \left(\frac{r_i r_j}{r^2} - \frac{1}{3} \delta_{ij} \right) \right\rangle$$

- Effects more outspoken in high shear band



Aggregate sizes

- Sheared: more smaller and larger aggregates
- High shear band: largest aggregates as likely



Conclusions

- MD/MC simulation reproduces experiments
 - Plateau in shear-stress curve
 - Shear banding observed
 - Temporal fluctuations in velocity profile
- Microscopic differences between sheared/unsheared system
 - Chain orientation
 - Aggregate size distribution
- Small differences between shear bands
- Current work: local stresses, positional order, secondary flow, network structure

Equation of Motion

K. Kremer and G. S. Grest. Dynamics of entangled linear polymer melts: A molecular-dynamics simulation. *Journal of Chemical Physics*, 92:5057, 1990.

$$\ddot{r}_i = \nabla \sum_{j \neq i} U_{ij} - \Gamma \dot{r}_i + W_i(t)$$

$$U_{ij} = U_{ij}^{LJ} + U_{ij}^{FENE} \quad \bullet \text{Interaction energy}$$

$$\Gamma \quad \bullet \text{Friction constant;}$$

•Heat bath coupling – all complicated interactions

$$W \quad \bullet \text{Gaussian white noise}$$

• $\langle W_i^2 \rangle = 6 k_B T \Gamma$ (fluctuation dissipation theorem)

Predictor-corrector algorithm

1) Predictor: Taylor: estimate at $t+\delta t$

$$\vec{r}^p(t + \delta t) = \vec{r}(t) + \delta t \vec{v}(t) + \frac{1}{2} \delta t^2 \vec{a}(t) + \frac{1}{6} \delta t^3 \vec{b}(t) + \dots$$

$$\vec{v}^p(t + \delta t) = \vec{v}(t) + \delta t \vec{a}(t) + \frac{1}{2} \delta t^2 \vec{b}(t) + \dots$$

$$\vec{a}^p(t + \delta t) = \vec{a}(t) + \delta t \vec{b}(t) + \dots$$

$$\vec{b}^p(t + \delta t) = \vec{b}(t) + \dots$$

2) From $\vec{r}^p$ calculate forces and acceleration $\vec{a}^c(t + \delta t)$ at $t+\delta t$

3) Estimate size of error in prediction step:

$$\Delta \vec{a}(t + \delta t) = \vec{a}^c(t + \delta t) - \vec{a}^p(t + \delta t)$$

4) Corrector step:

$$\vec{r}^c(t + \delta t) = \vec{r}^p(t + \delta t) + c_0 \Delta \vec{a}(t + \delta t)$$

$$\vec{v}^c(t + \delta t) = \vec{v}^p(t + \delta t) + c_1 \Delta \vec{a}(t + \delta t)$$

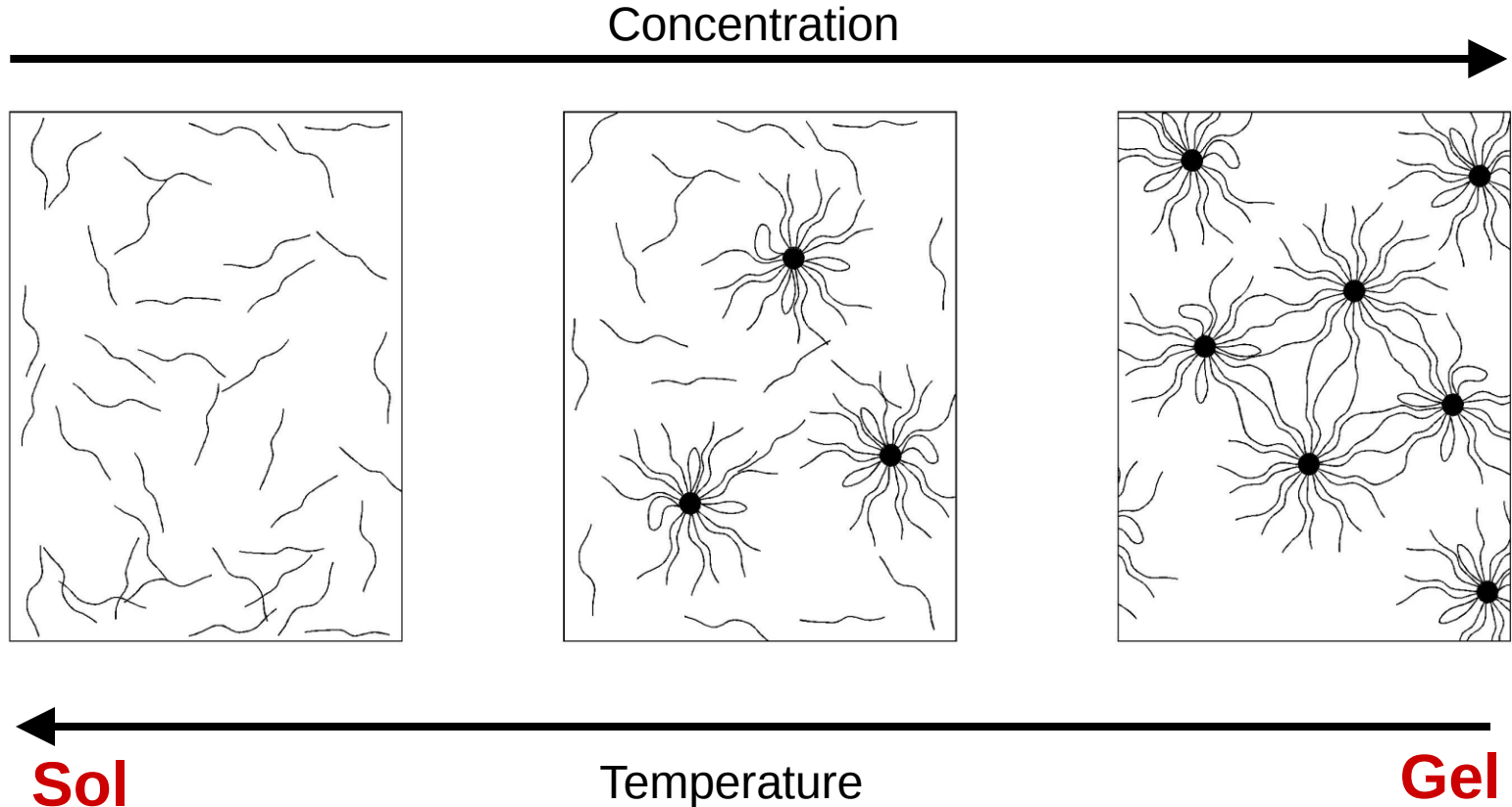
$$\vec{a}^c(t + \delta t) = \vec{a}^p(t + \delta t) + c_2 \Delta \vec{a}(t + \delta t)$$

$$\vec{b}^c(t + \delta t) = \vec{b}^p(t + \delta t) + c_3 \Delta \vec{a}(t + \delta t)$$

$$\Delta t = 0.005 \tau$$

Polymeric gels

Associating: reversible junctions between endgroups



Simulation details

- 1000 polymeric chains, 8 beads/chain
- Units: σ (length), ε (energy&temperature), m (mass), $\tau = \sigma(m/\varepsilon)^{1/2}$ (time);
- Box size: $(23.5 \times 20.5 \times 27.4) \sigma^3$ with periodic boundary conditions
- Concentration = $0.6/\sigma^3$ (in overlap regime)
- Radius of gyration:

$$R_g^2 = \frac{1}{N} \sum_{k=1}^N (r_k - r_{mean})^2 = 2.69\sigma^2$$

- Bond life time $> 1 / \text{shear rate}$