Parametric Resonance in Coupled MEMS Gyroscopes

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Colloquium at Computational Science Research Center, San Diego State University

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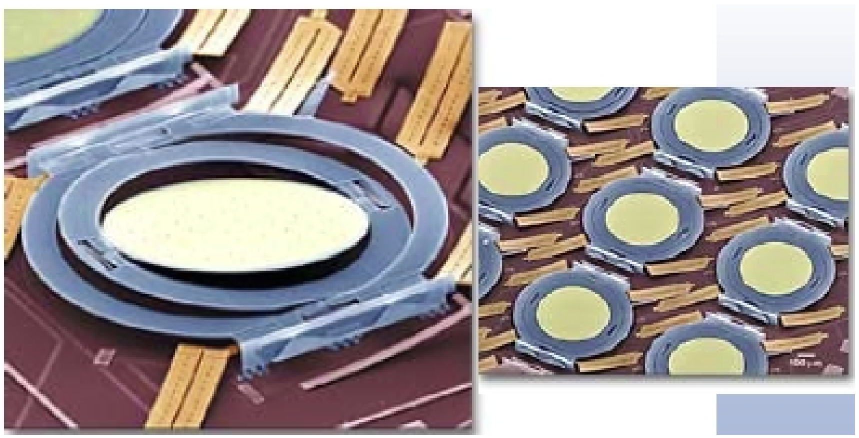
Global Center of Excellence (GCOE), Kyoto University

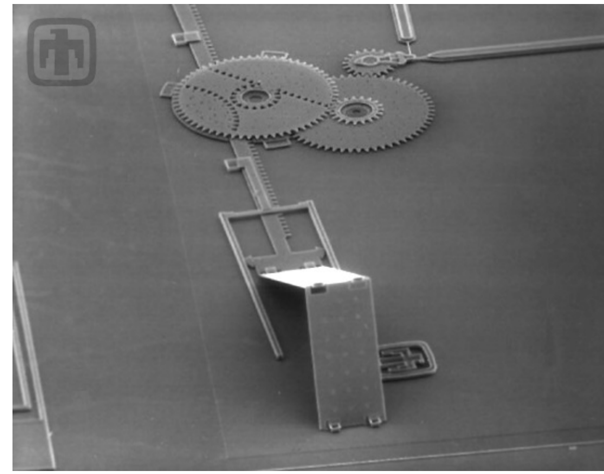
Parametric Resonance in Coupled MEMS Gyroscopes

- Introduction to MEMS and Gyroscope
- Model
- Simulation Results for a Single Gyro
- Simulation Results for the Coupled system (Huy Vu)
- Design

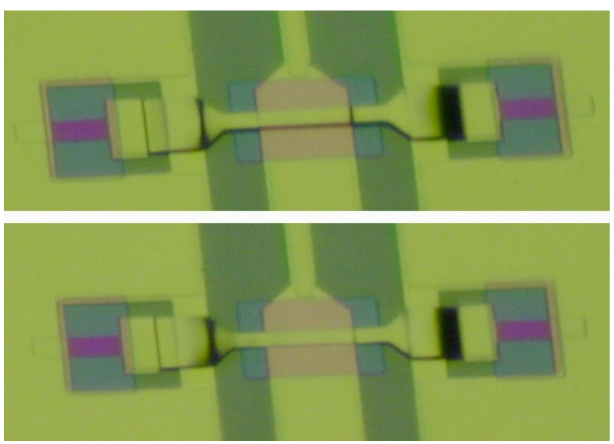
MEMS: Micro Electro Mechanical Systems

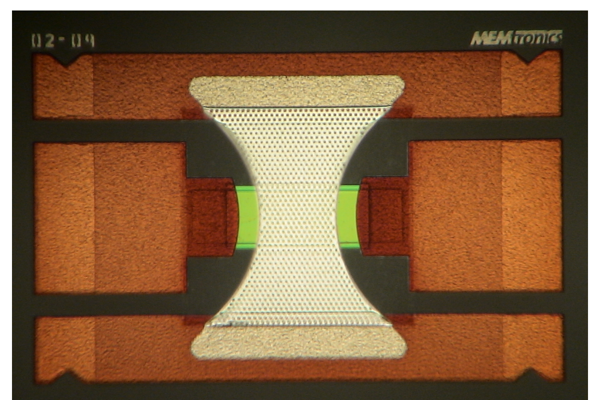
• **Microoptics: Micromirrors (Television)**





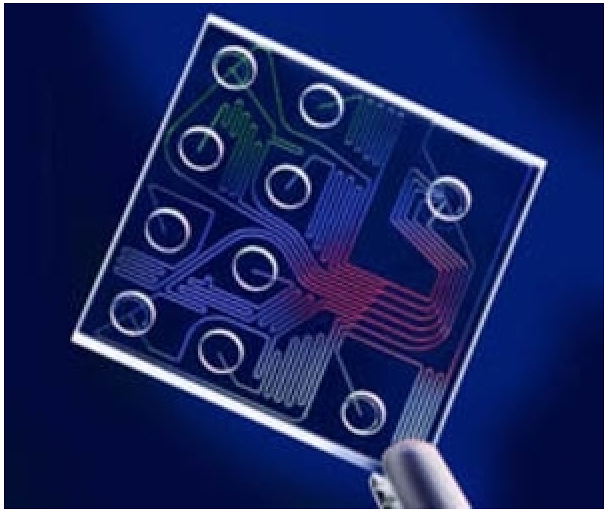
• **RF MEMS: Radio Frequency Switch**



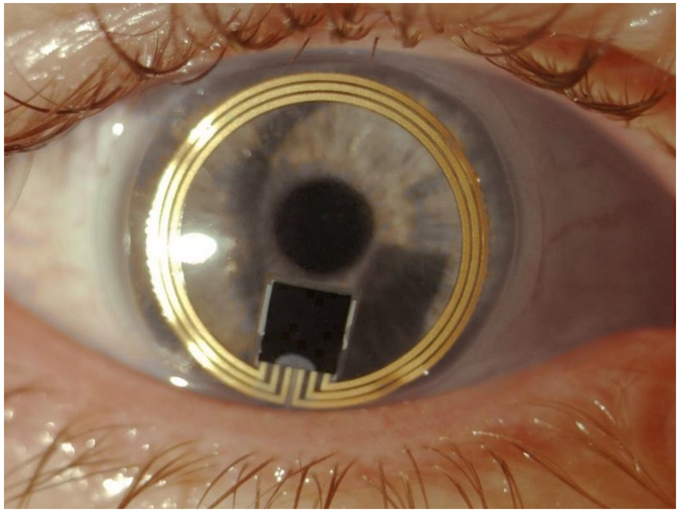


MEMS: Micro Electro Mechanical Systems

• **Microfluidics: Lab-on-a-chip, BioMEMS**

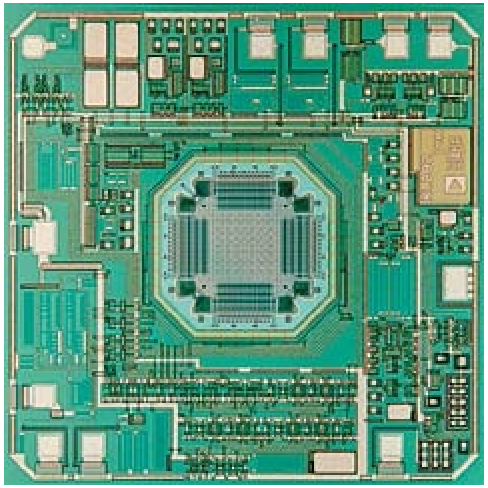


Lens for Glaucoma research (Sensimed AG)

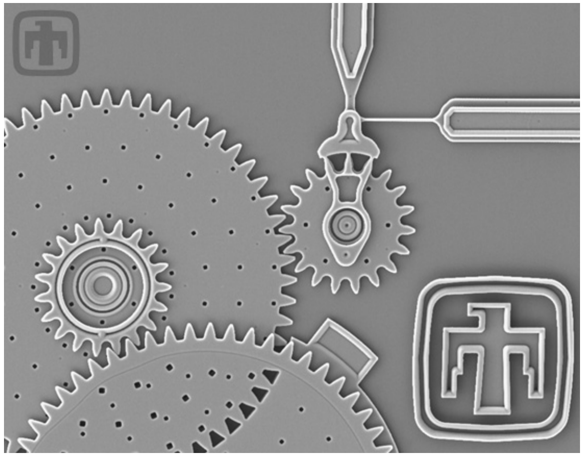


• **Micromechanics:**

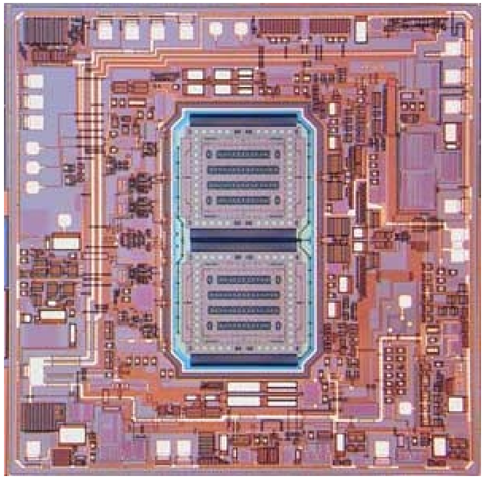
Accelerometer (Analog Devices)



Geers (Sandia)



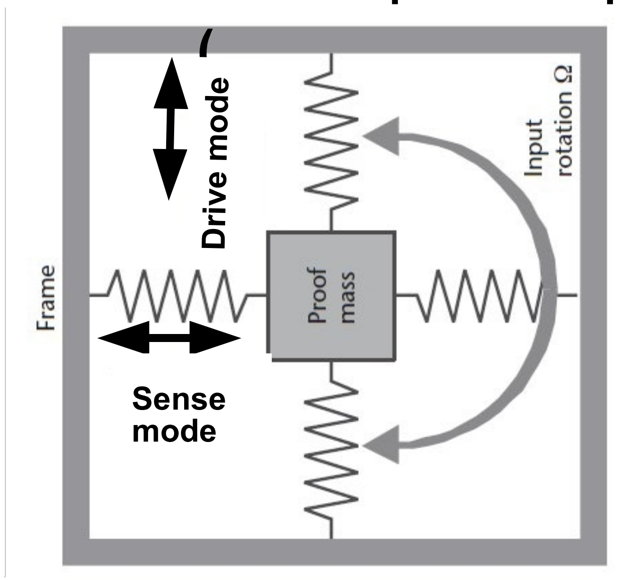
Gyroscope (Analog Devices)



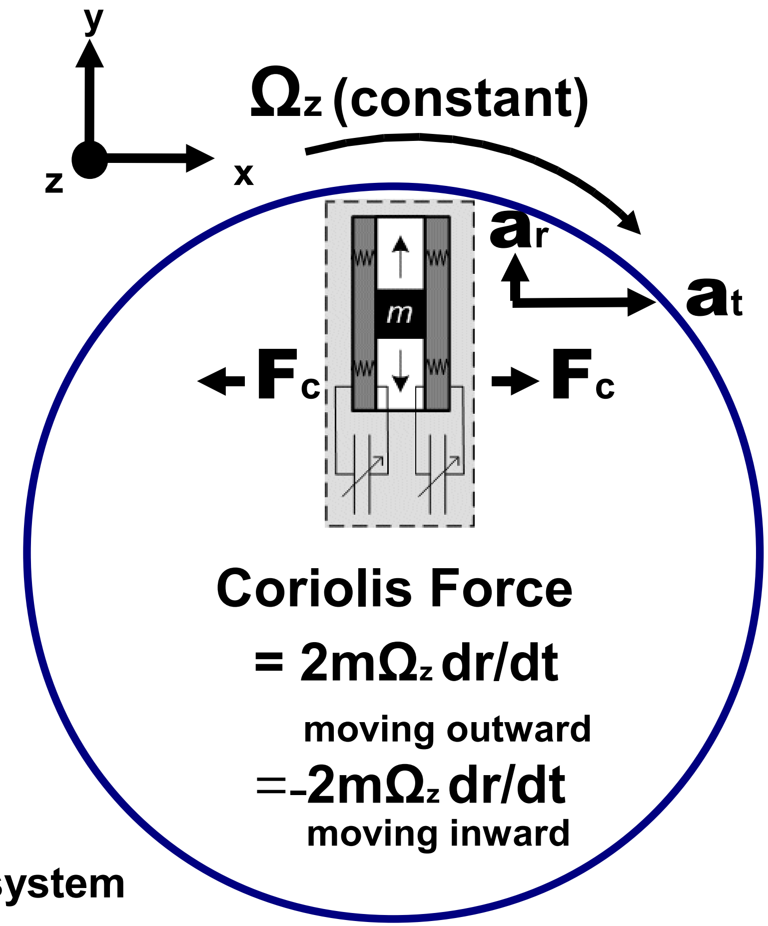
Introduction to MEMS Gyroscope

MEMS Gyroscope principles [2]

Proof mass and operational principle



Gyro on rotating platform



- MEMS Gyroscopes are used in,
  - digital cameras for image stabilization
  - laptops for hard disk protection and gaming system
  - cars for electronic stability control (ESC) and ships as inertial navigation sensor

MEMS gyroscopes measure angular velocity or the rate of rotation

Introduction to MEMS Gyroscope

Functional summary of a MEMS Gyroscope by capacitive sensing

1) The proof mass is driven by a periodic sinusoidal drive signal,

2) On a rotating platform, Coriolis effect causes motion in the sense axis,

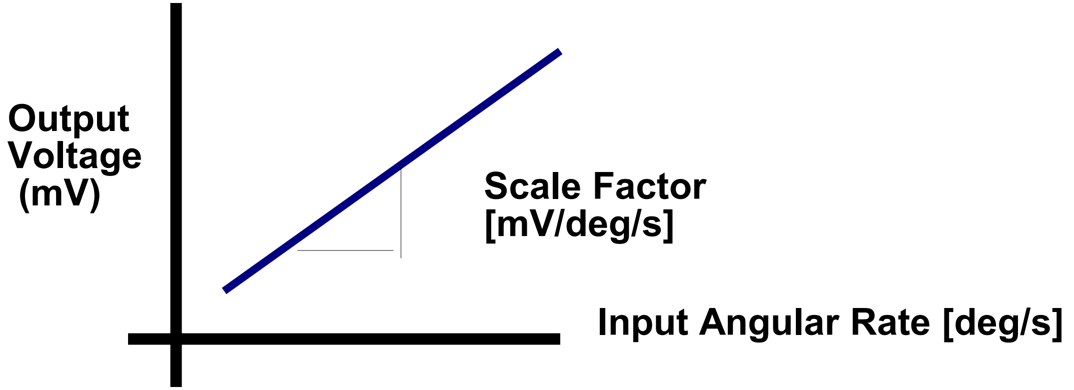
3) The capacitance between the proof mass and sense plate(s) changes,

4) Sense electronics calculate the Coriolis acceleration from the change in capacitance between the proof mass and the sense plate,

5) Sense electronics calculate a rate of rotation of the MEMS gyroscope from the Coriolis acceleration and the drive velocity

Digital output: Sense Electronics generate a pulse stream whose frequency is proportional to the acceleration

Analog Output: Sense Electronics generate a voltage output proportional to the acceleration



Characterization: http://www.youtube.com/watch?v=wuzudYgkcJ8

MEMS Gyroscope: Performance Parameters

Table 1. Performance requirements for different classes of Gyroscopes

Parameter	Rate Grade	Tactical Grade	Inertial Grade
Angle Random Walk °/√h	0.5	0.5-0.05	0.001
Bias Drift, °/h	10-1000	0.1-10	0.01
Scale Factor Accuracy, %	0.1-1	0.01-0.1	0.001
Full Scale Range (°/sec)	50-1000	500	400
Max Shock in 1msec, g's	10^3	10^3-10^2	10^3
Bandwidth, Hz	100	~100	100

Noise floor in the output of gyroscope → Angle Random Walk

Zero-rate-output (ZRO) of gyroscope → Bias Drift

Angular rate input range → Full Scale Range

Bandwidth (drive-mode) in which the gyroscope can produce a meaningful output (sense-mode) → Bandwidth

Drive and sense electronics have an impact on the performance

Parametric Resonance in Coupled MEMS Gyroscopes

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- **Model**
- Simulation Results for a Single Gyro
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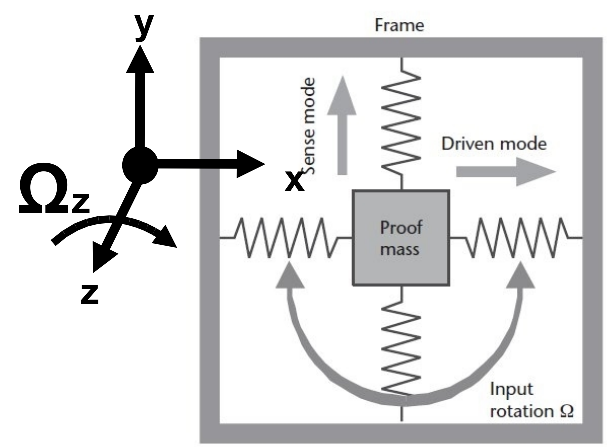
- Design

Model

Total force — Damping force — Restoring force — Excitation force — Coriolis force

$$m\ddot{x}+b_x\dot{x}+k_1x+k_3x^3=F_e(t)+2m\Omega_z\dot{y};$$

$$m\ddot{y}+b_y\dot{y}+k_1y+k_3y^3=-2m\Omega_z\dot{x};$$



m = mass

x = displacement of the mass in x-dir

y = displacement of the mass in y-dir

b_x, b_y = damping constant in x-dir & y-dir respectively

k_1 = linear spring constant

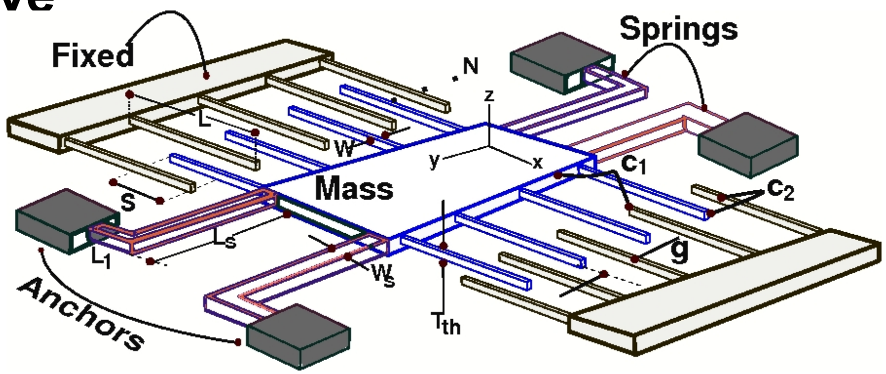
k_3 = nonlinear spring constant

Ω_z = angular rate

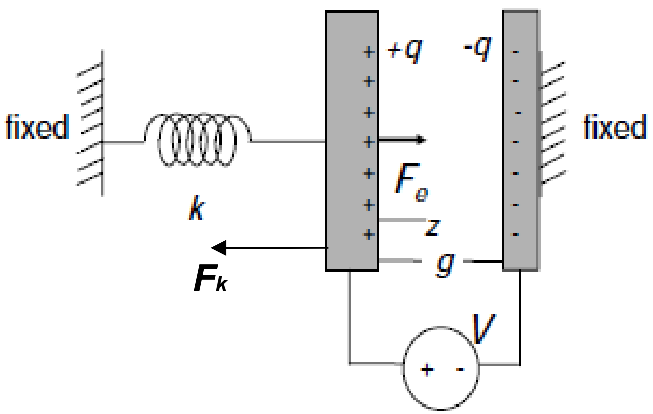
$F_e(t)$ = drive signal = $A_d\cos(\omega_d t)$

Electrostatic Force (inter-digitated comb-drive)

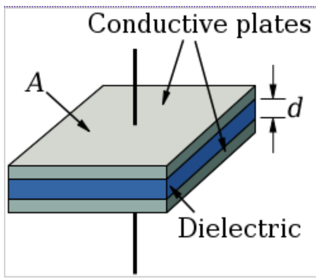
Diagram of Comb-drive



Electrostatic Actuation



Capacitance



Total Force in x-dir

$$F(x,t)=k_x x-\frac{1}{2}\frac{\partial C(x)}{\partial x}V(t)^2$$

Restoring force F_k *Electrostatic force F_e*

$$V(t)=V_{dc}+V_{ac}\cos(\omega t)$$

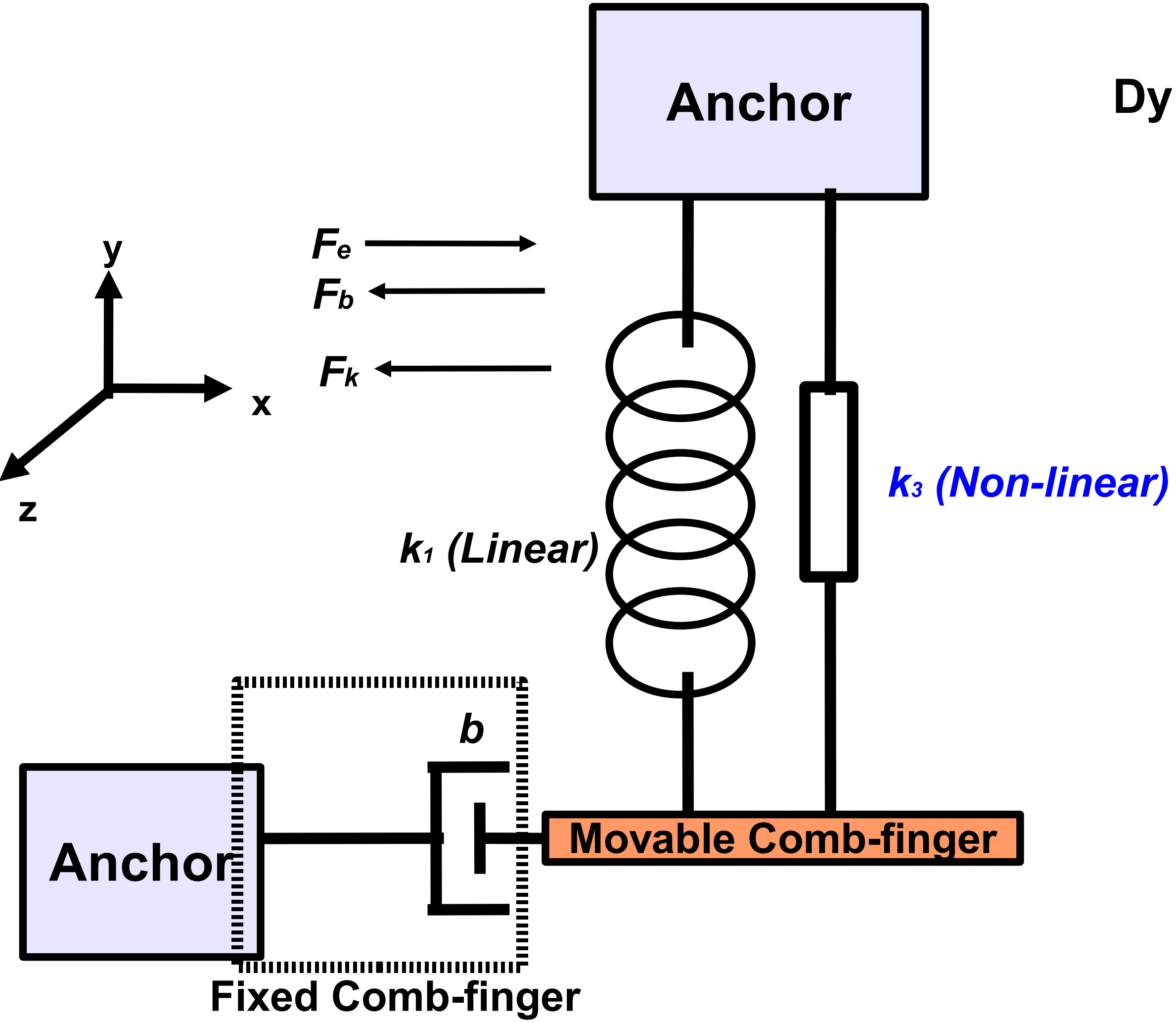
$$C(x)=\frac{2N\epsilon_o(s+x)T_{th}}{g}$$

$$F_e(t)=\frac{N\epsilon_o T_{th}V(t)^2}{g}$$

Note: The excitation force does not depend on x

Forces acting on the Comb-drive

Spring-Mass-Damper System



Dynamics x-dir (linear system)

$$m\ddot{x} + \underbrace{b\,\dot{x}}_{F_b} + \underbrace{k_1 x}_{F_k} = F_e(t)$$

$$F_e(t) = \frac{N \epsilon_o T_{th} V(t)^2}{g}$$

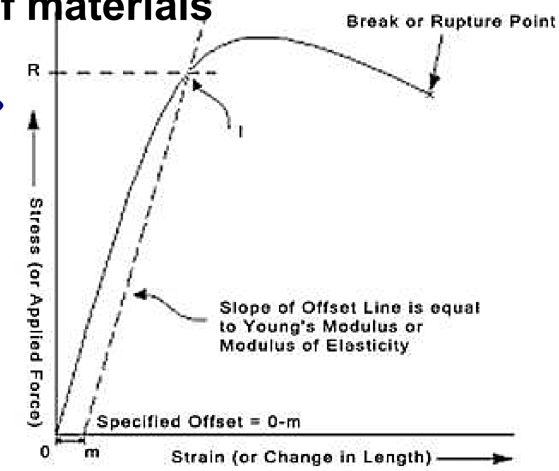
$$k_1 = \frac{3\mathrm{IE}}{L_s^3} = \frac{E T_{th} W s^3}{L_s^3}$$

$$b = \underbrace{\mu_e \frac{A}{z_o}}_{\text{Squeeze-film damping}} + \underbrace{\mu_e \frac{2\mathrm{NLT}_{th}}{g}}_{\text{Slide-film damping}}$$

Sources of Nonlinearity

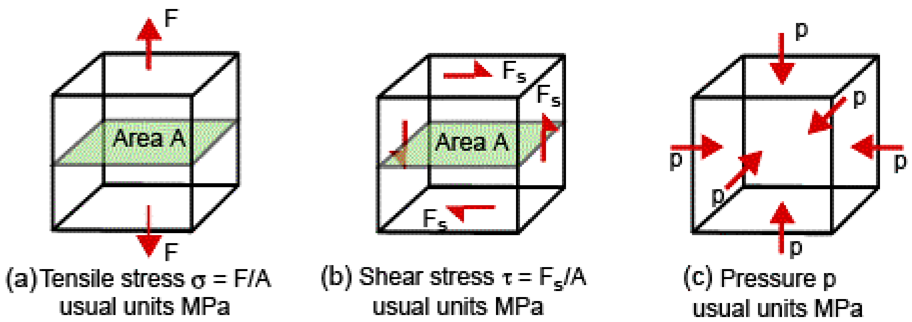
- **Material Nonlinearity: several ways of measuring the stiffness of materials**

1) young's modulus (tensile stress and tensile strain ratio)



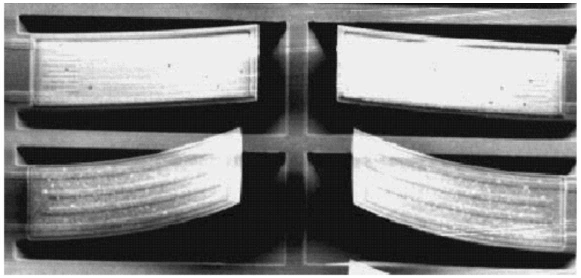
2) shear modulus (shear stress and shear strain)

3) bulk modulus (material resistance to uniform compression)



4) poison's ratio (transverse stress and axial strain ratio)

5) residual stress due to deposition method (causes curling or buckling or a fracture)

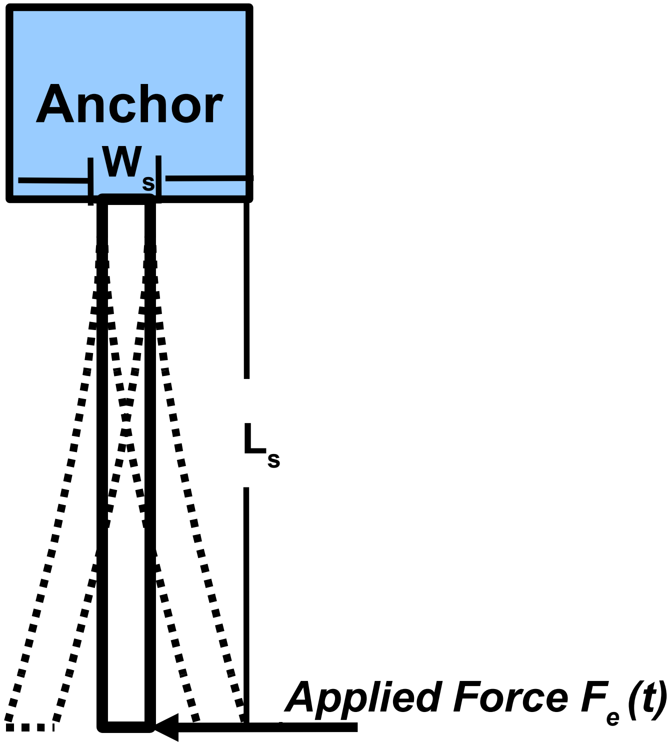


6) Anisotropic material : material properties are <u>not</u> independent of direction, e.g. single crystal silicon (sensitive to device orientation with respect to crystallographic orientations)

- **Geometric Nonliearity (next slide): large deformations can induce nonlinear oscillations**
- **Contact based Nonlinearity: springs come in contact with other parts**

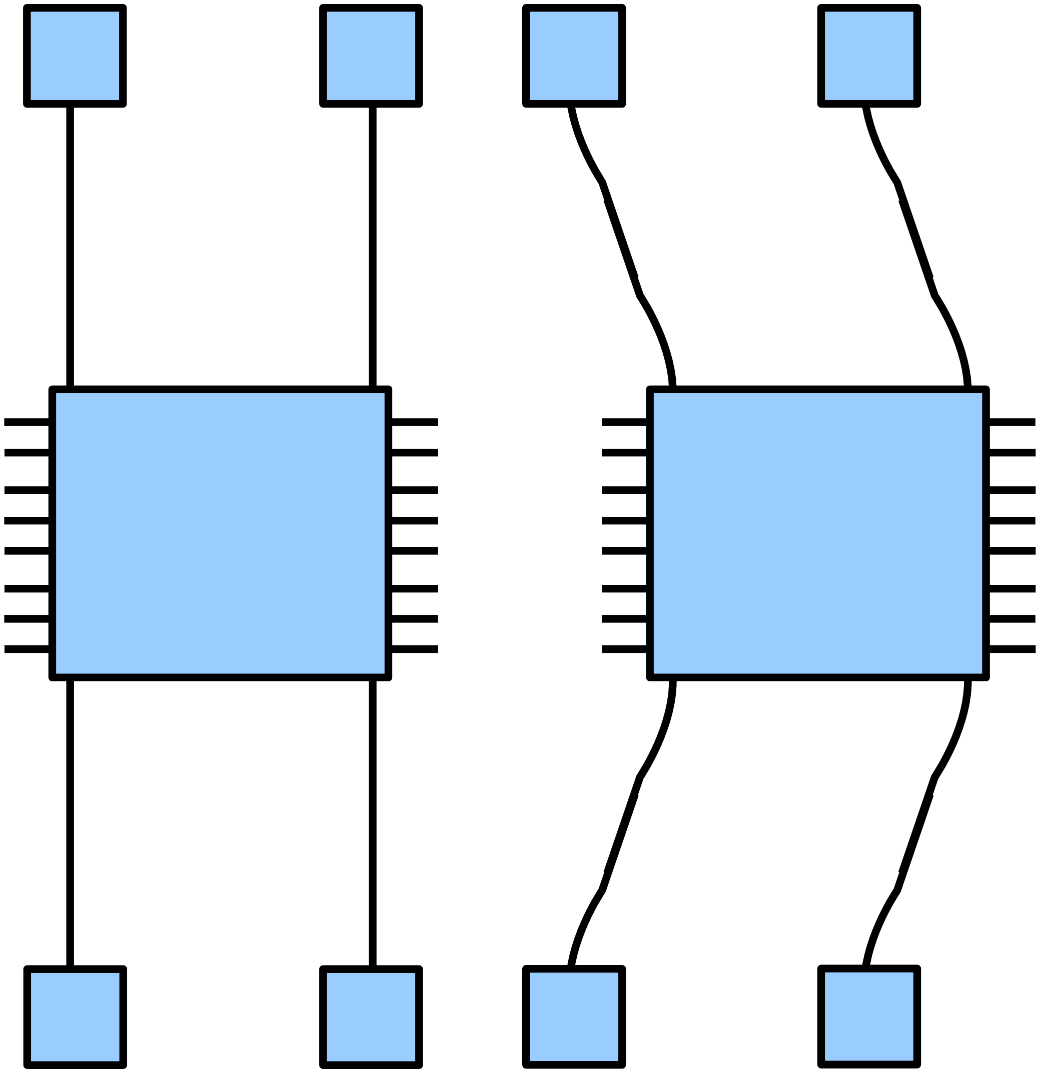
Geometric Nonlinearity

Spring: First mode of displacement



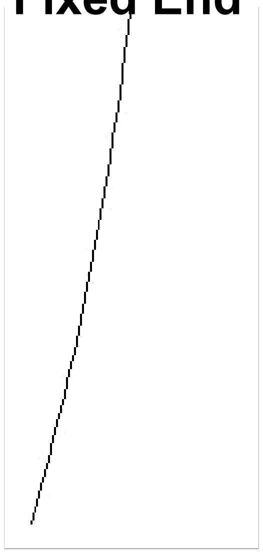
- Given large excitation force, there can be large displacement
- Then a second and/or third mode of displacement can appear in doubly supported beam due to material stress and strain

Spring: Higher modes of displacement



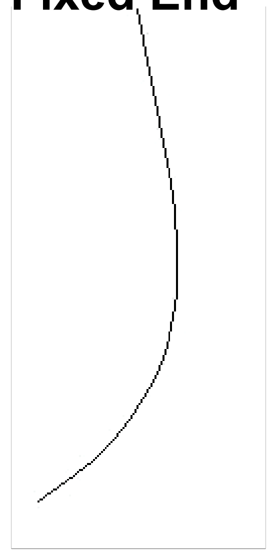
Spring modes and restoring force

Fixed End



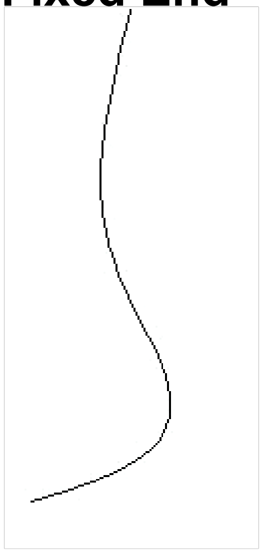
Mode 1

Fixed End

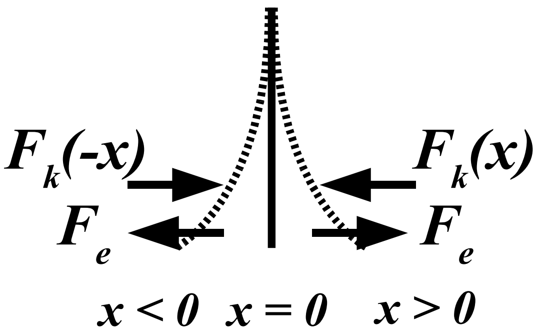


Mode 2

Fixed End



Mode 3



Restoring force

$$F_k(x) = k_1 x + k_2 x^2 + k_3 x^3 + k_4 x^4 ..$$

$$F_k(-x) = -k_1 x + k_2 x^2 - k_3 x^3 + k_4 x^4 ..$$

If symmetrical spring then $F_k(x)$ **is odd**

$$F_k(-x) = -F_k(x)$$

Hence $k_2 = k_4 = 0$ **and** $\boxed{F_k(x) = k_1 x + k_3 x^3}$

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Simulation Results for a Single Gyro

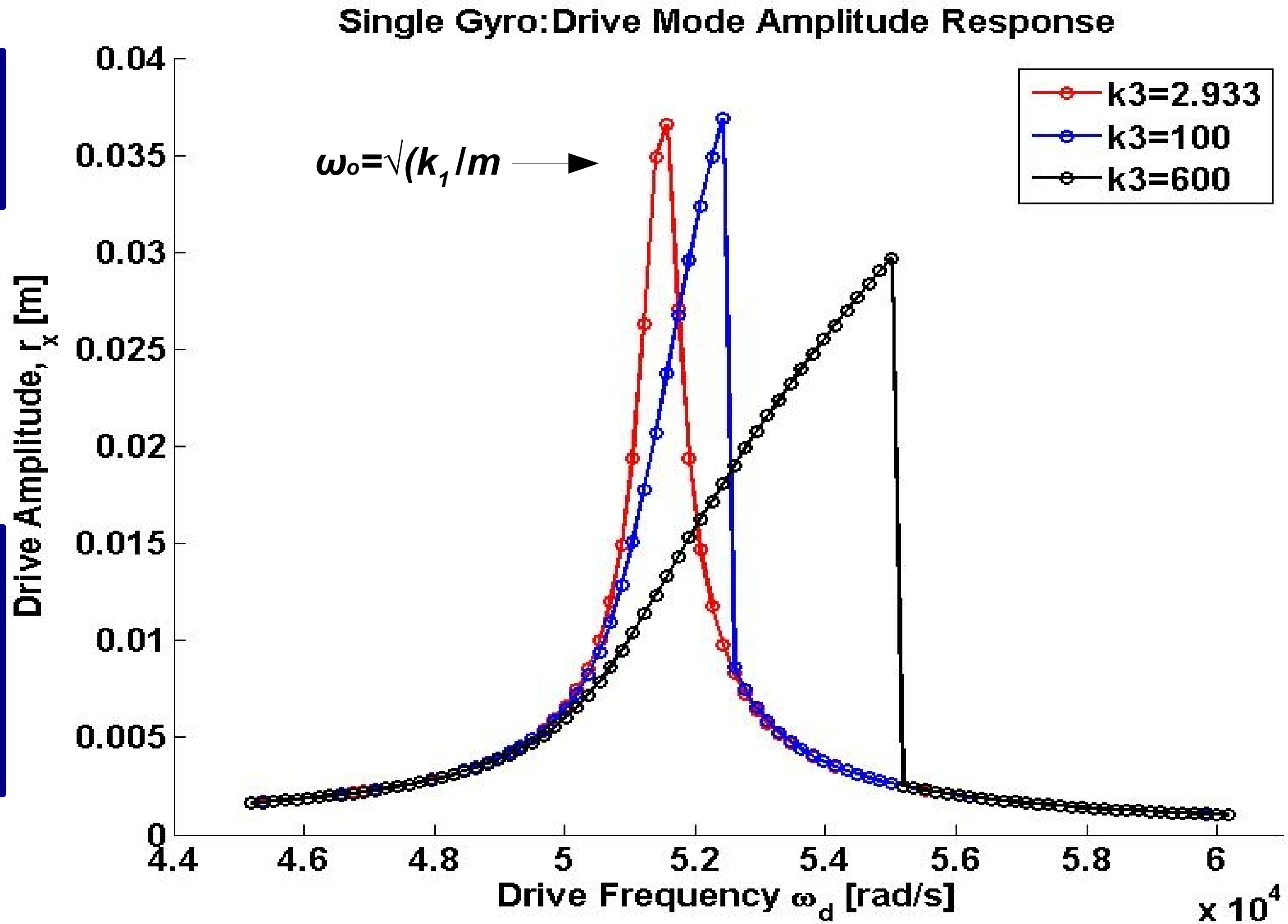
Response of the System in the Drive Mode: k_3 sweep

$$m\ddot{x}+b_x\dot{x}+k_1x+k_3x^3=F_e(t);\, where\, F_e(t)=A_d\cos(\omega_d t)$$

$A_d = 0.001$, $b_x = 5.1472e\text{-}7$, $k_1 = 2.6494$, $m = 1e\text{-}9$ [4]

Saddle-node Bifurcation in a single gyro

Bifurcation Point can be changed by changing nonlinear spring constant



Simulation Results for a Single Gyro

Response of the System in the Drive Mode: A_d sweep

$$m\ddot{x}+b_x\dot{x}+k_1x+k_3x^3=F_e(t);\, where\ F_e(t)=A_d\cos(\omega_d t)$$

A_d= [0.001,0.0015,0.002]

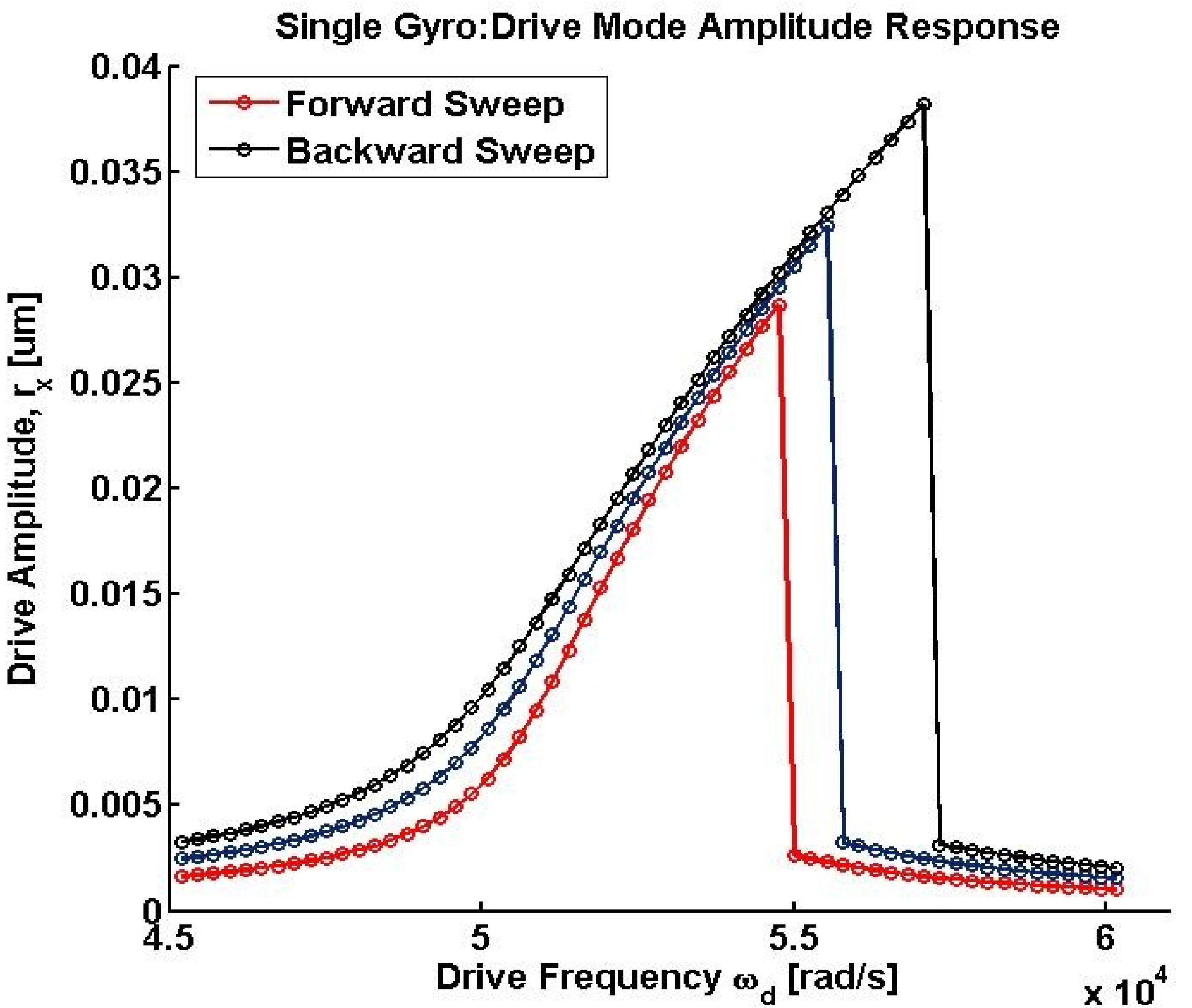
b_x = 5.1472e-7,

k_1 = 2.6494,

k_3 = 600,

m = 1e-9 [4]

Bifurcation Point can be changed by changing amplitude of the input force



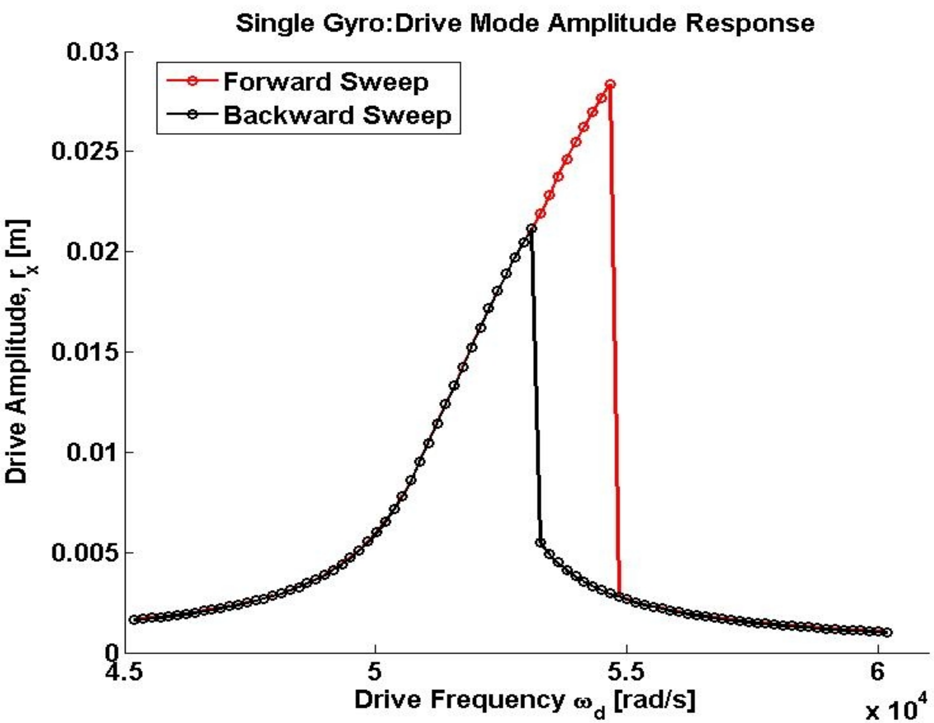
Simulation Results for a Single Gyro

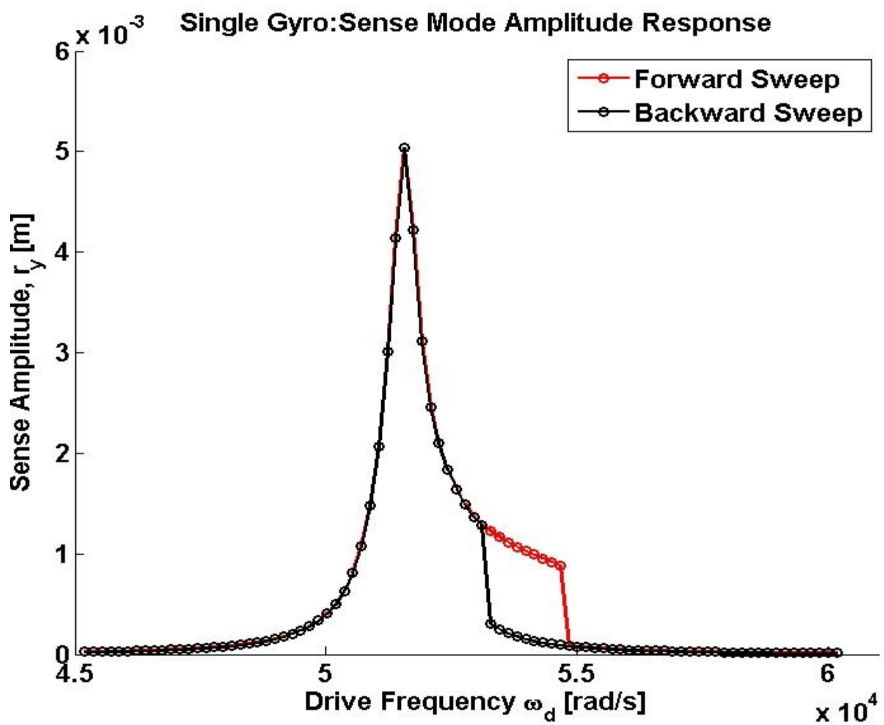
Response of the System in the Drive Mode and Sense Mode

$$m\ddot{x}+b_x\dot{x}+k_1x+k_3x^3=F_e(t)+2m\Omega_z\dot{y}\,;\,where\ F_e(t)=A_d\cos(\omega_d t)$$

$$m\ddot{y}+b_y\dot{y}+k_1y+k_3y^3=-2m\Omega_z\dot{x}$$

A_d = 0.001, b_x = 5.1472e-7, k_1 = 2.6494, k_3 = 600, m = 1e-9, Ω_z = 100 rad/s [4]





Simulation Results for a Single Gyro

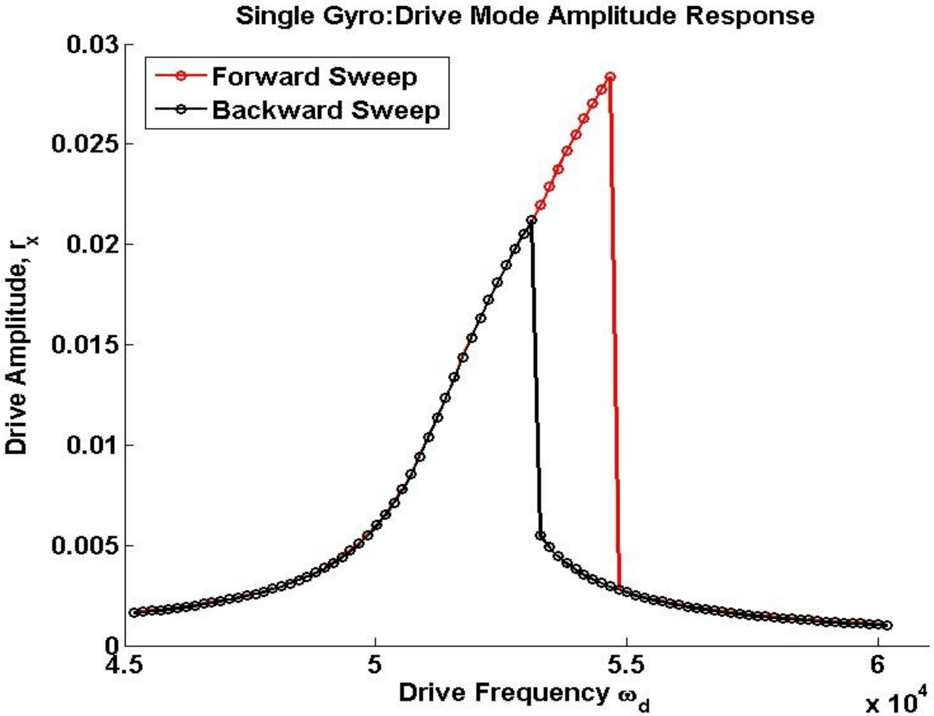
Response of the System in the Drive Mode and Sense Mode

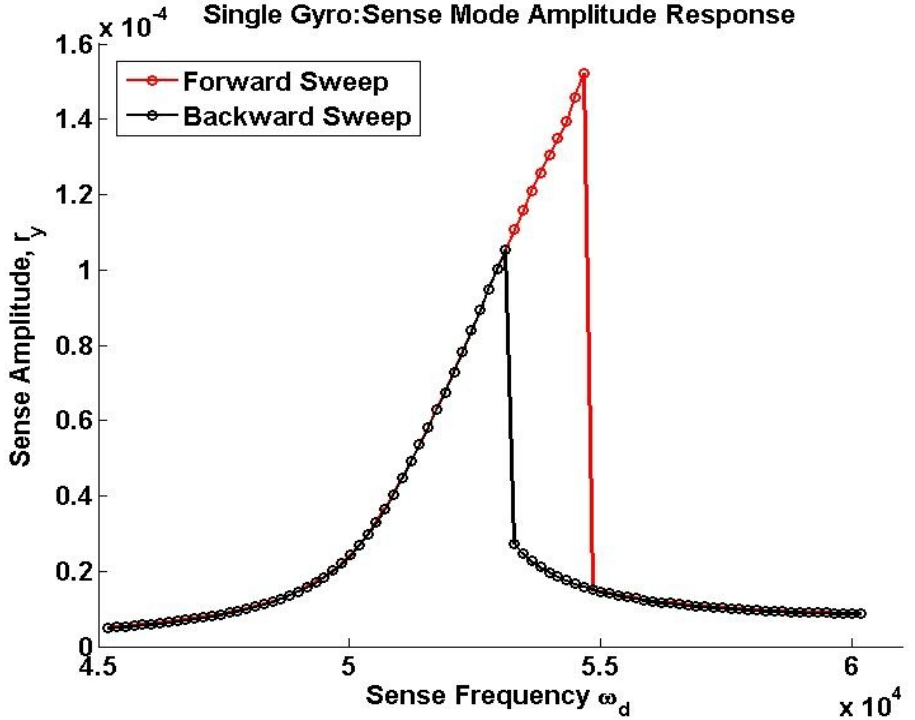
$$m\ddot{x}+b_x\dot{x}+k_1x+k_3x^3=F_e(t)+2m\Omega_z\dot{y}\,;\ where\ F_e(t)=A_d\cos(\omega_d t)$$

$$m\ddot{y}+b_y\dot{y}+k_{1y}y+k_{3y}y^3=-2m\Omega_z\dot{x}$$

A_d= 0.001, b_x = 5.1472e-7, k_1 = 2.6494, k_2 = 600, m = 1e-9, Ω_z = 100 rad/s [4]

$\boxed{k_{1y} = 5,\ k_{3y} = 600}$





Simulation Results for a Single Gyro

Response of the System in the Drive Mode and Sense Mode

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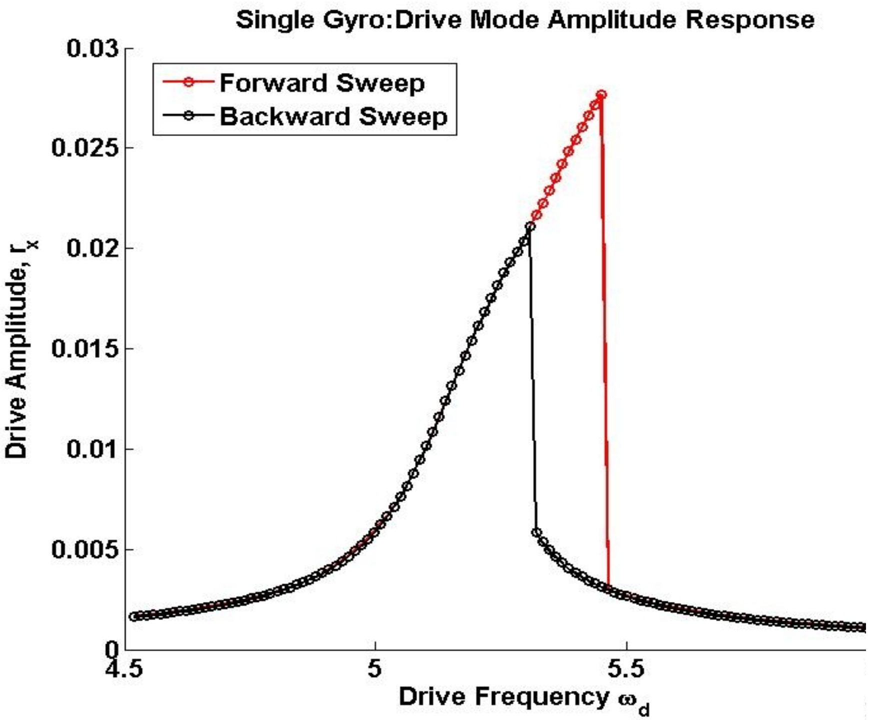
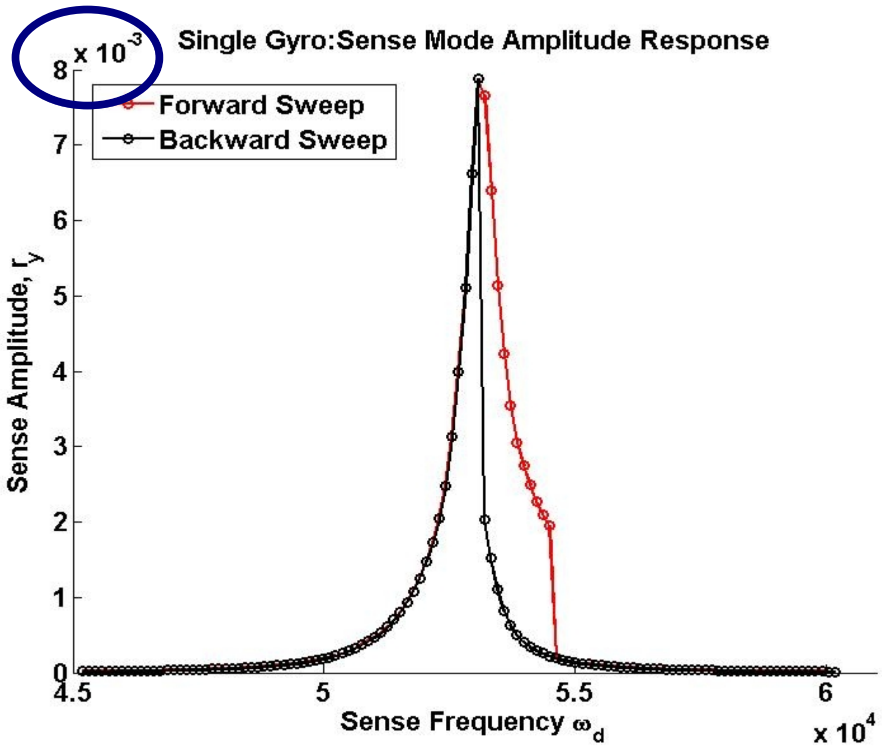
$$m\ddot{y}+b_y\dot{y}+k_{1y}y+k_{3y}y^3=-2m\Omega_z\dot{x}$$

A_d= 0.001, b_x = 5.1472e-7, k_1 = 2.6494, k_3 = 600, m = 1e-9, Ω_z = 100 rad/s [4]

$\boxed{k_{1y} = 2.82,\ k_{3y} = 50}$





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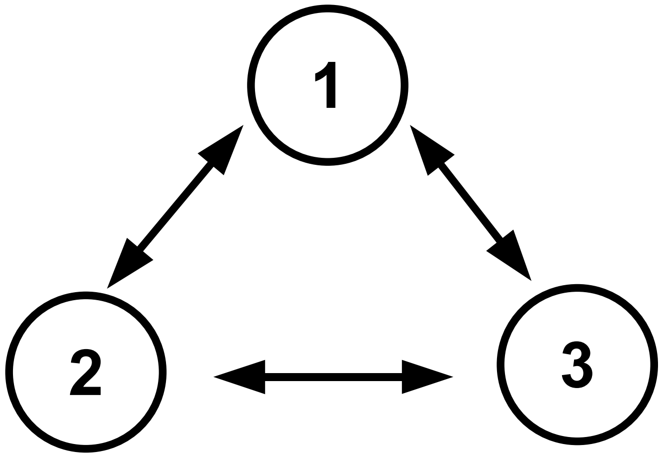
Coupled System

Bi-directionally coupled ring of gyros



$$m\ddot{x}_j + b\dot{x}_j + k_1 x_j + k_3 x_j^3 = F_e(t) + \lambda(x_{j+1} - 2x_j + x_{j-1}) + 2m\Omega_z \dot{y}_j;$$
$$m\ddot{y}_j + b\dot{y}_j + k_1 y_j + k_3 y_j^3 = -2m\Omega_z \dot{x}_j;\; j = 1,2,3$$

Assumptions: mass, spring constants, damping coefficients do not vary, diffusive coupling function: $x_{j+1} - x_j$

λ = coupling parameter (gain or bias current of an amplifier)

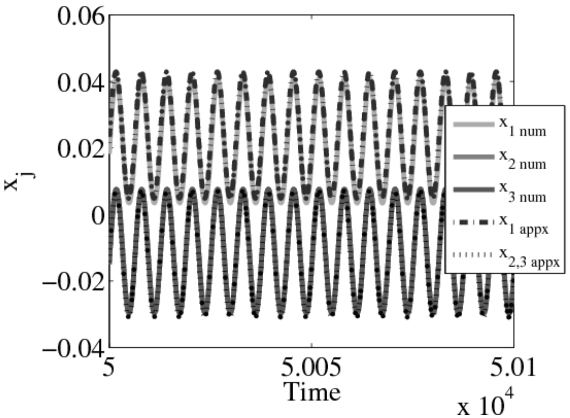
x_j , y_j = displacement of jth element in the n-coupled system (here n =3)

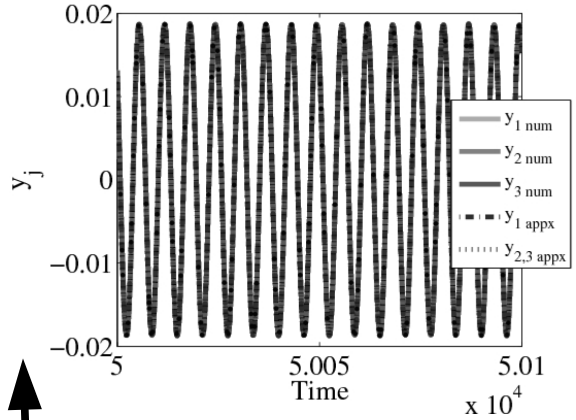
$F_e(t)$ = drive signal = $A_d \cos(\omega_d t)$

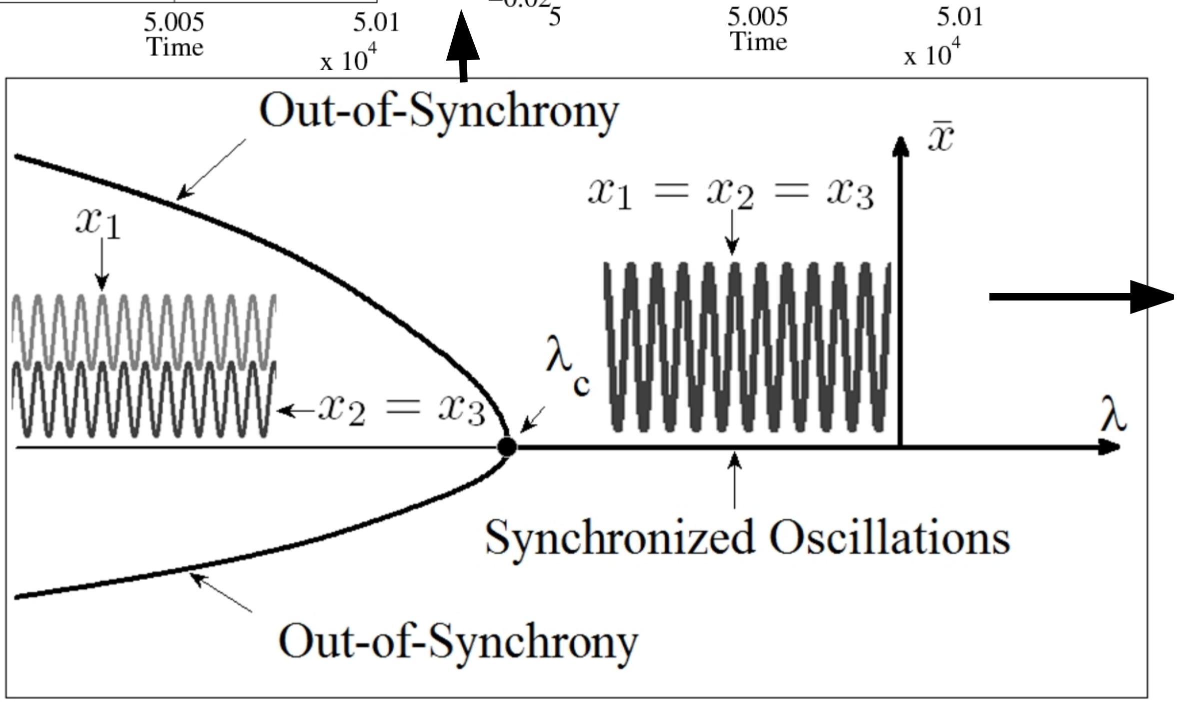
Simulation Results for the Coupled System (Huy Vu)

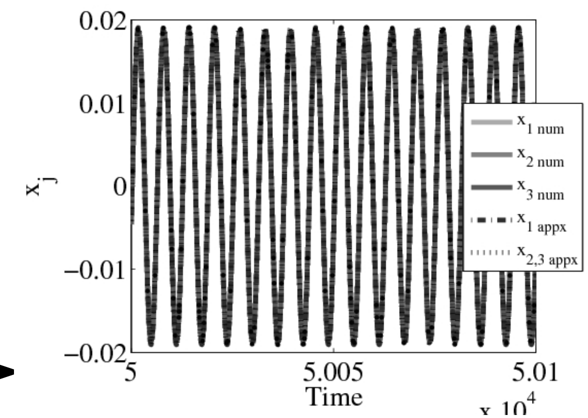
Using perturbation method on dimension-less form...

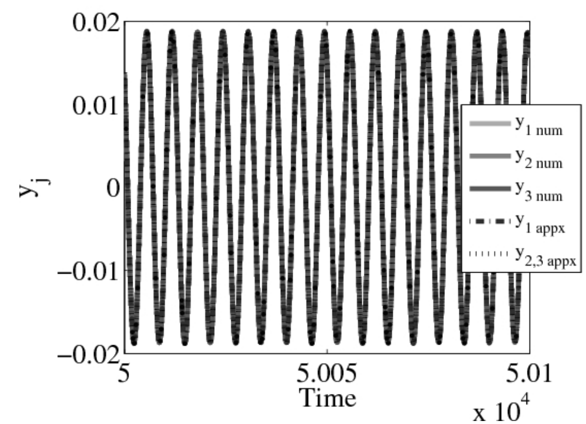
- **Supercritical Pitchfork Bifurcation in an individual Gyro**
- **Bistable potential well**





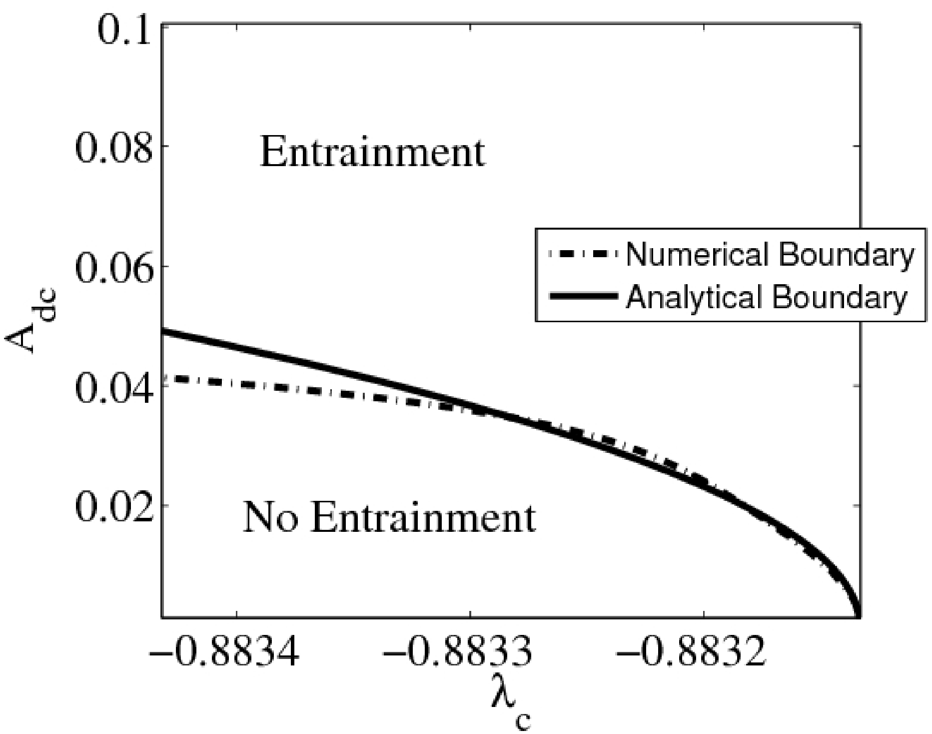




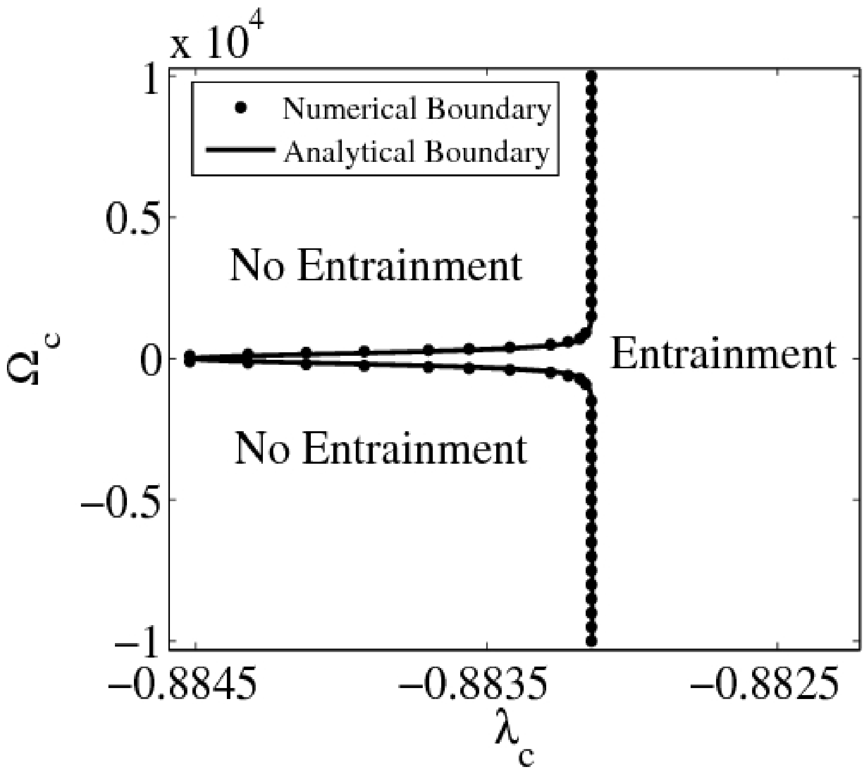


Simulation Results for the Coupled System (Huy Vu)

Two parameter Bifurcation Diagram
Around the critical value of λc



Ω = 3000 rad/s

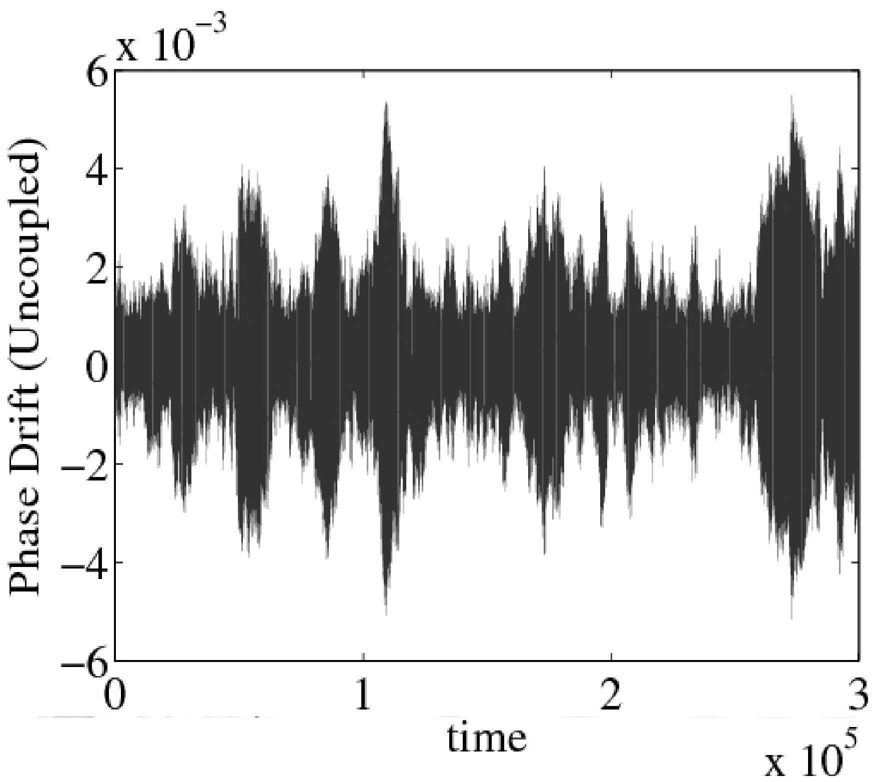


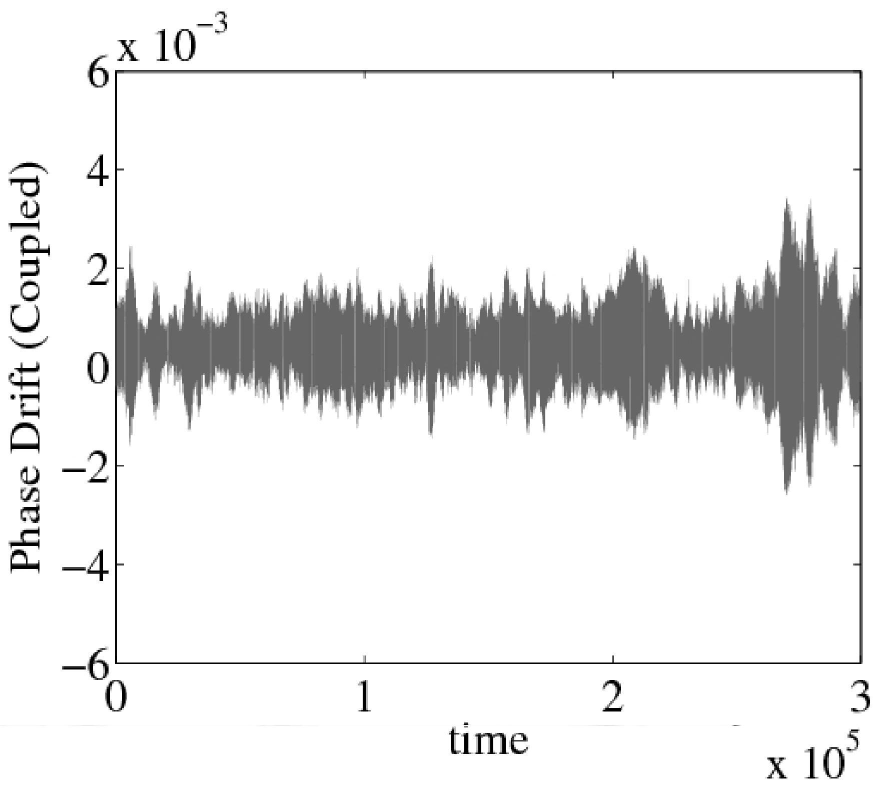
Ad = 0.001

Simulation Results for the Coupled System (Huy Vu)

Phase drift is reduced for an individual gyro in a coupled system

- **random variation in mass is 10%**
- **wideband Gaussian noise is added to the equations**
- **difference between uncorrupted and corrupted signal (y-dir) for many gyros is taken**

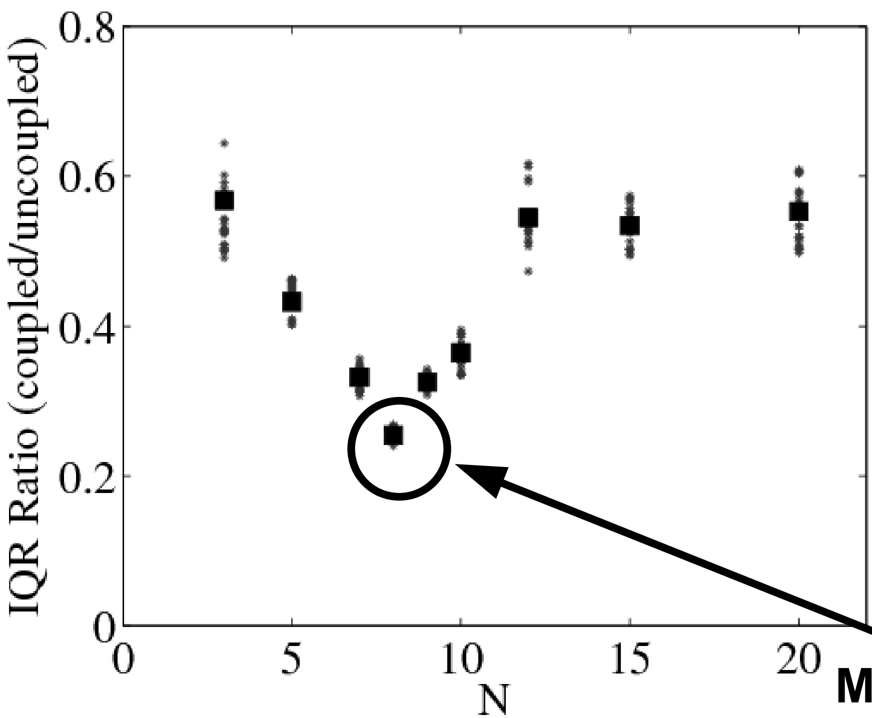


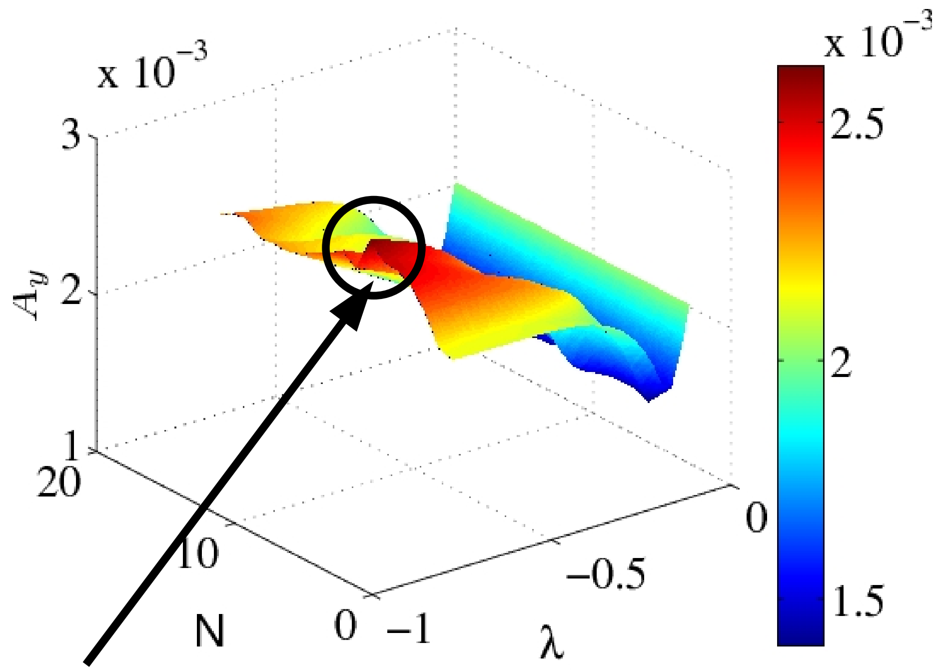


Simulation Results for the Coupled System (Huy Vu)

Minimum phase drift in coupled system

N = total number of gyroscopes in a ring
- **-large number of sets of gyros**
- **-values with 50% variation from the mean value of individual phase drift is averaged across a large time period**
- **-Ratio of phase drift in coupled and uncoupled is computed**
- **-These ratios in a coupled system are averaged**





Minimum phase drift

Parametric Resonance in Coupled MEMS Gyroscopes

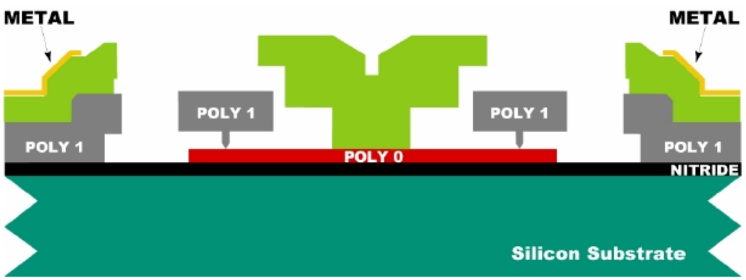
- Introduction to MEMS and Gyroscope
- Model
- Simulation Results for a Single Gyro
- Simulation Results for the Coupled system (Huy Vu)
- Design

Design

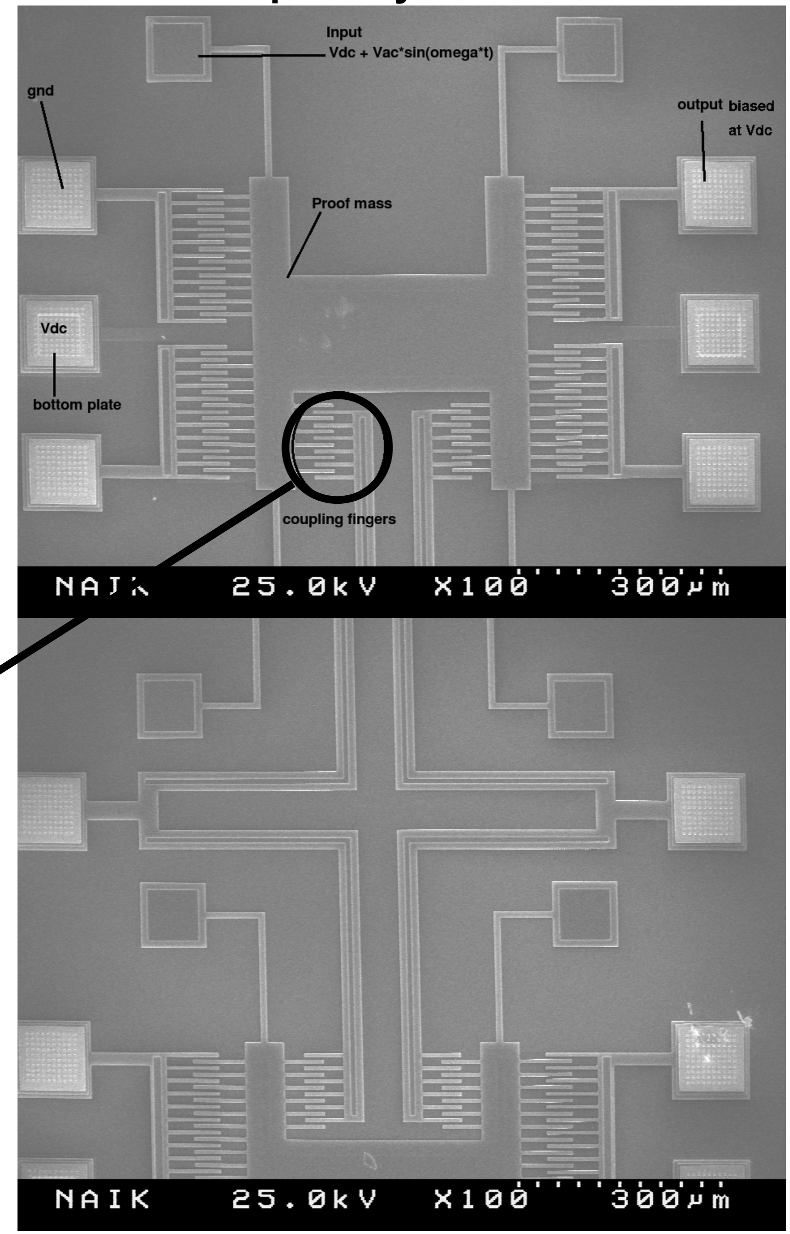
SDSU Feb 19,2010

Layout of Coupled Comb-drives

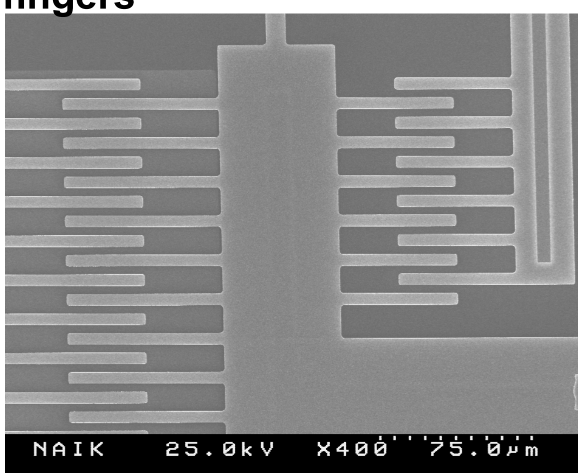
PolyMUMPs Run 86 (Jan 09)



SEM 01: Coupled Gyro Test Structure



SEM 02: Detailed View of Coupling fingers



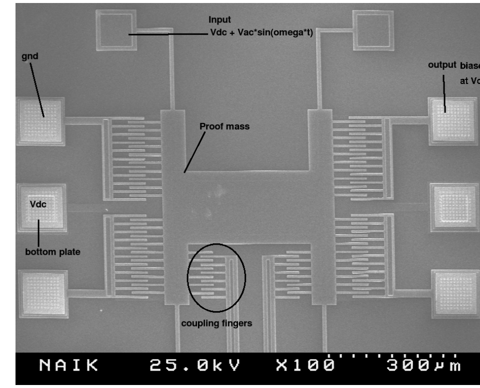
Experimental Results

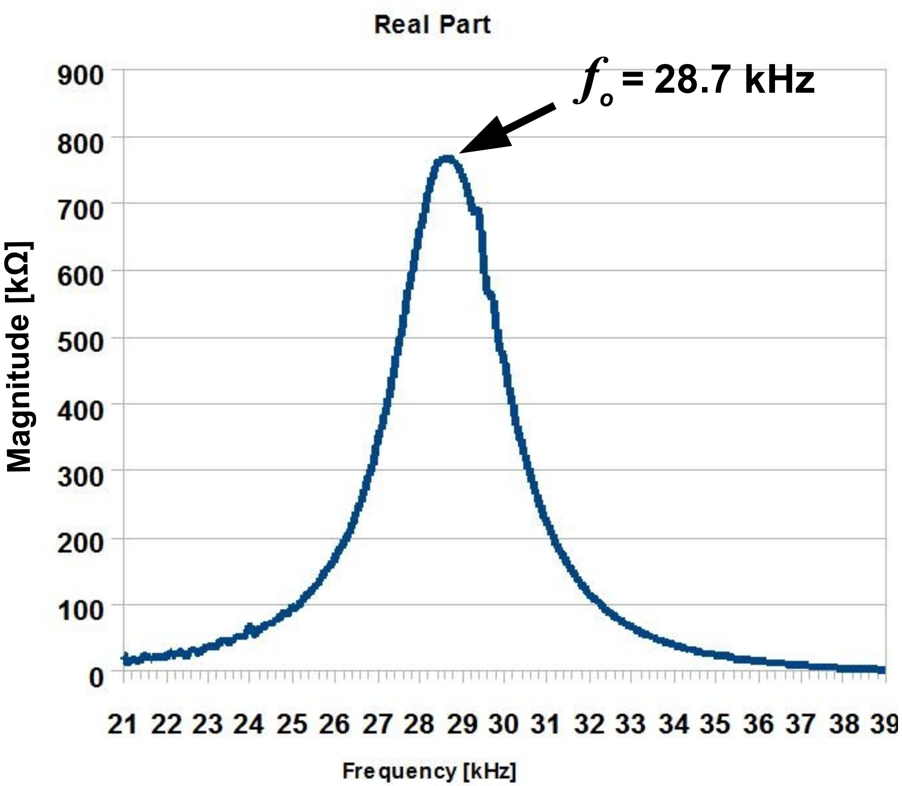
$|V_{ac}|$ = 1 V, V_{dc} = 6 V

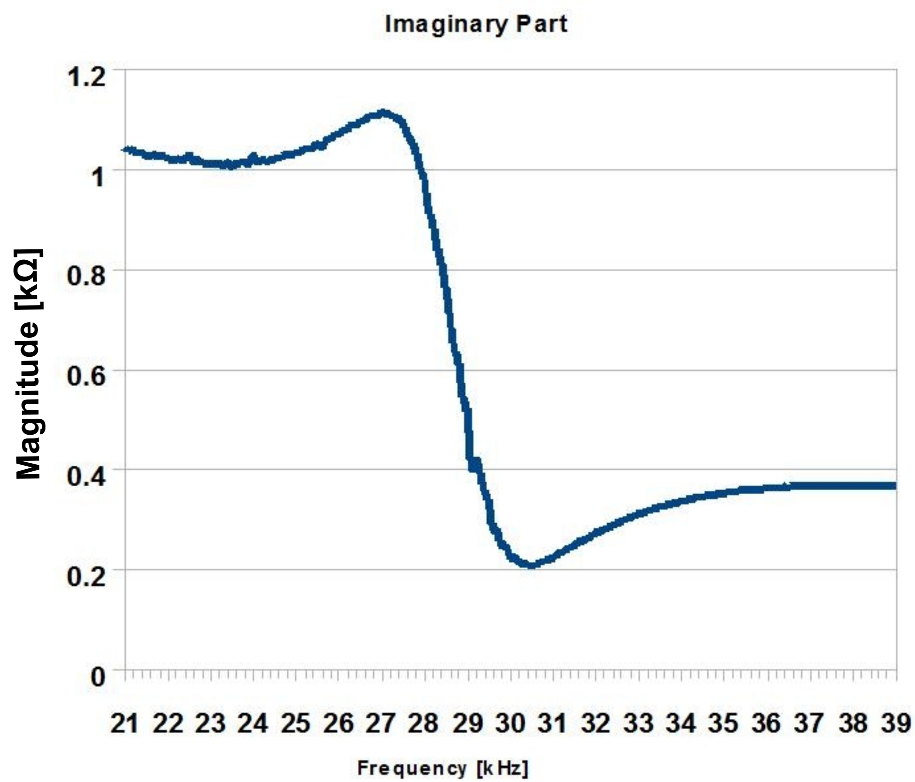
f_{ac} sweep

k_1 = 22.18 N/m
b = 8.5703 x 10^{-6} Ns/m

Coupled Gyro Test Structure
Top Element, x-dir







Issues

- How to induce large vibrations in drive mode

- How to enhance read-out in sense mode

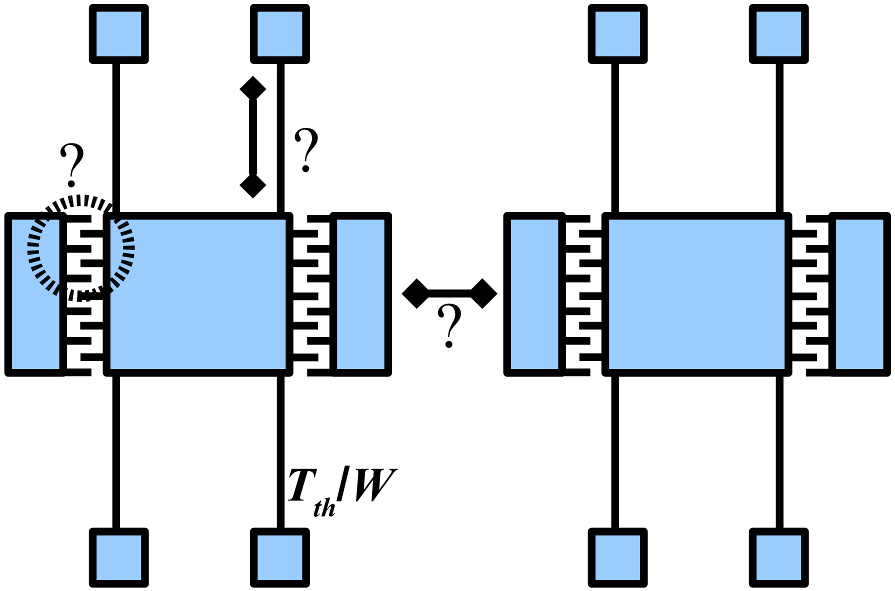
- Controlling stiffness k_1 and k_3

- Anisoelasticity and quadrature error
  - x and y mode cross-coupling due to fabrication variation and imperfection

- Damping
  - viscous anisodamping (surfboard effect)
  - anchor loss
  - parasitic effects (through substrate, die-attach, package)
  - electronics

- Coupling

- How to decrease out of plane movement i.e. high aspect ratio (T_{th}/W)



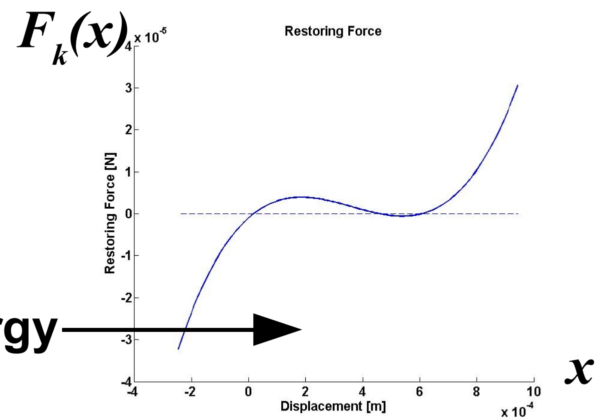
Tuning Nonlinearity

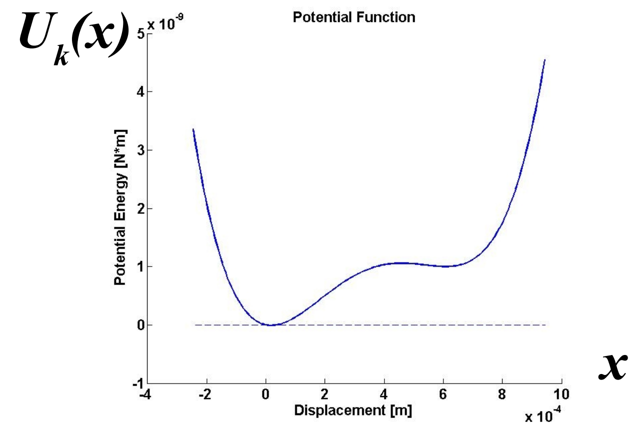
- Restoring force

$$F_k(x) = k_1 x + k_3 x^3$$

$$U_k(x) = \frac{1}{2} k_x x^2 + \frac{1}{4} k_3 x^4$$

- Desired force and energy





- Tuning k_1 (linear stiffness) can affect k_3 (nonlinear stiffness) and vise-a-versa
- Tune nonlinearity by parametric excitation or parametric coupling

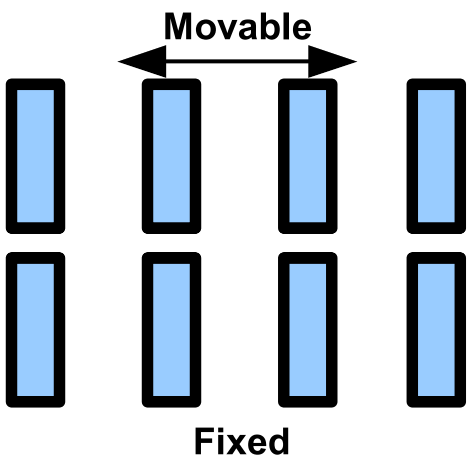


Fig. 01 Comb-fingers in stable or attractive state

$$F_{ke}(x) = (a_1 x + a_3 x^3) V_a^2$$
$$where, V_a = DC + A\cos(\omega_d t)$$

Total potential function can have bi-stable characteristics

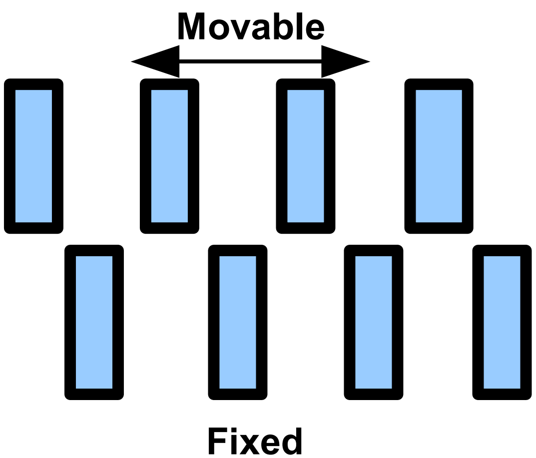
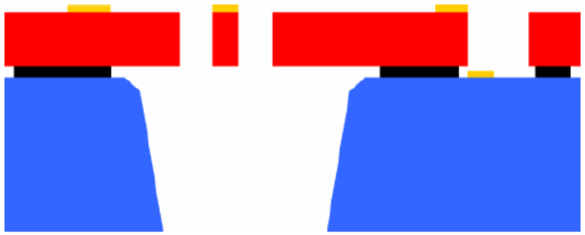
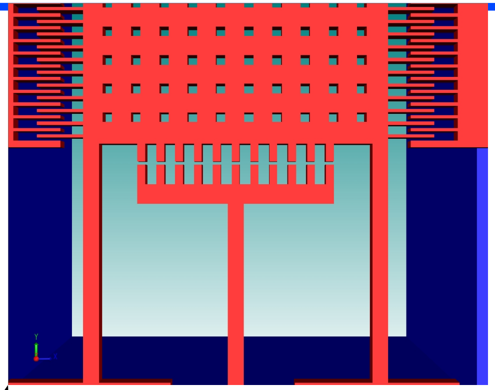


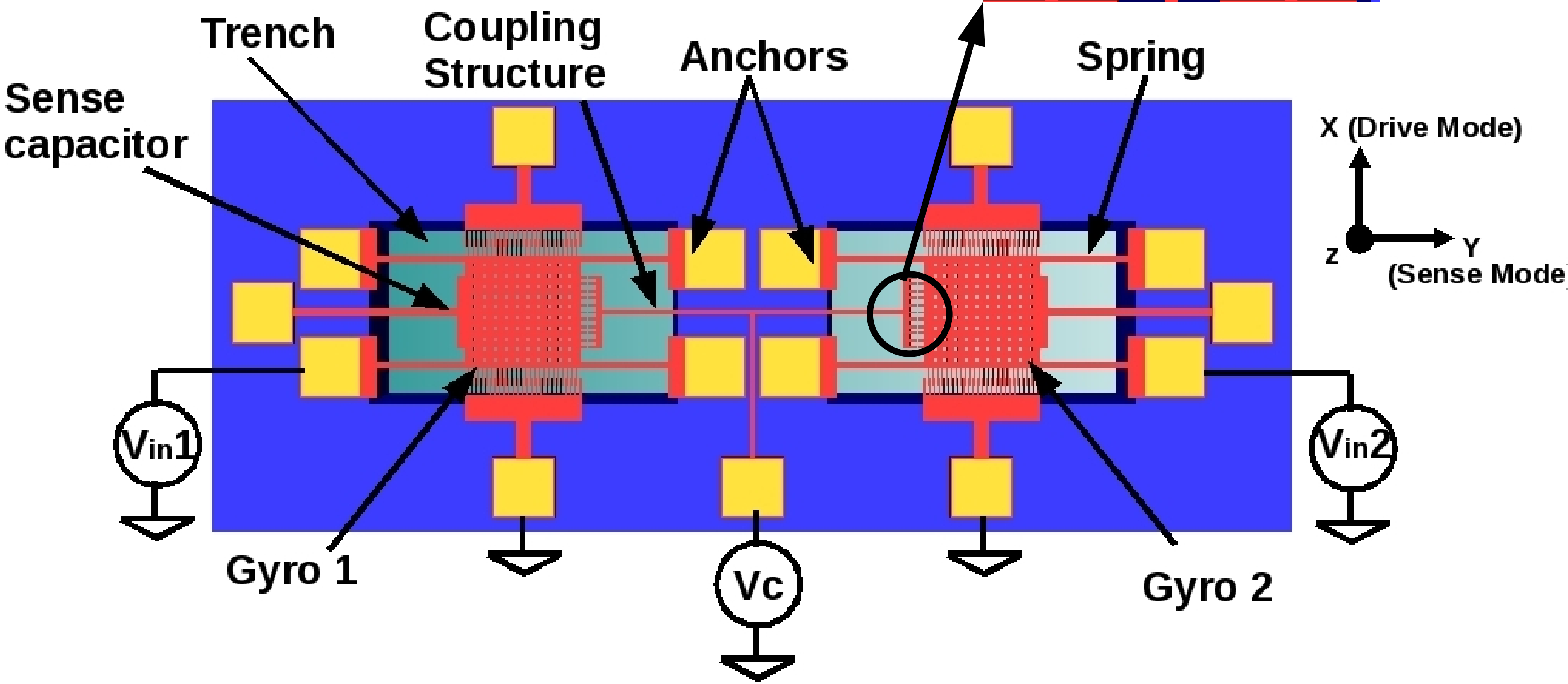
Fig. 02 Comb-fingers in unstable or repulsive state

Design

SOIMUMPs Run 28 (July 09)

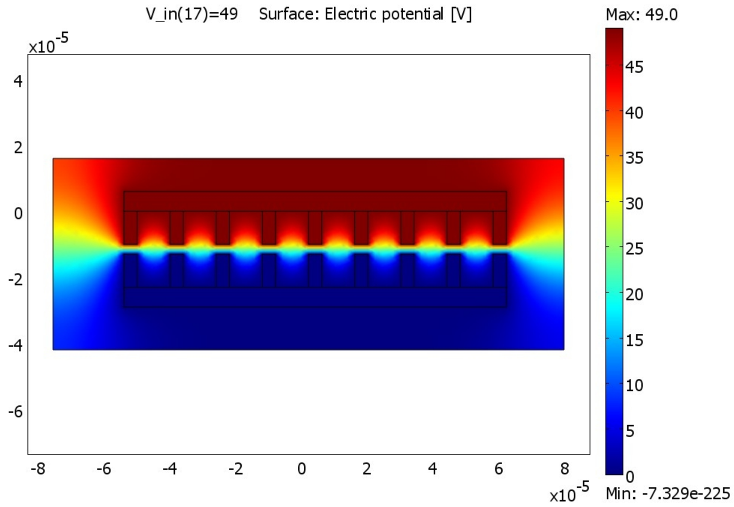


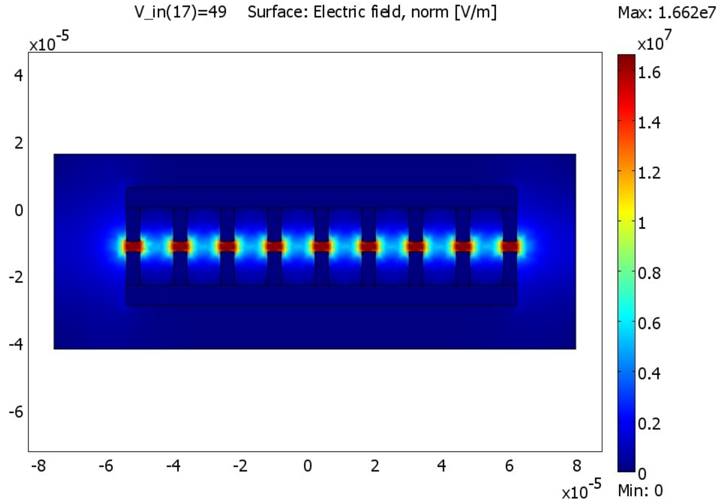


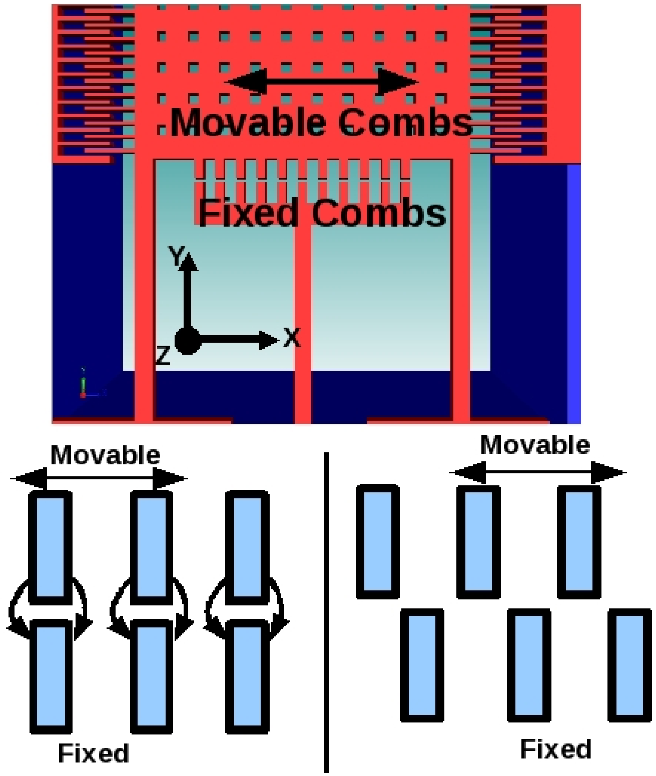


Substrate PolySilicon Top Metal

Design


V_in(17)=49 Surface: Electric potential [V]
Max: 49.0
x10^-5
Min: -7.329e-225

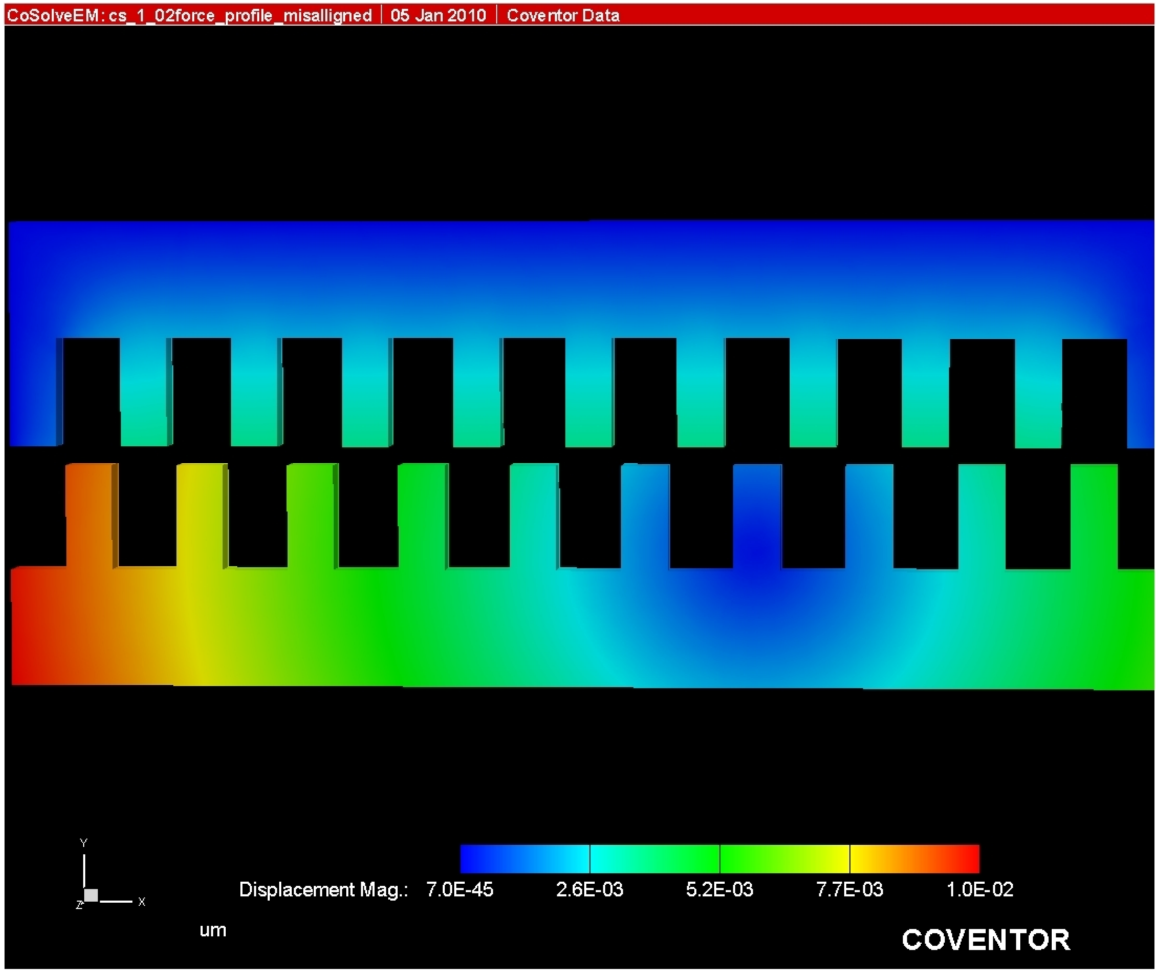

V_in(17)=49 Surface: Electric field, norm [V/m]
Max: 1.662e7
x10^7
x10^-5
Min: 0


Movable Combs
Fixed Combs
Y
X
Z
Movable
Fixed
Movable
Fixed

Design

Misaligned Fingers (negative displacement)

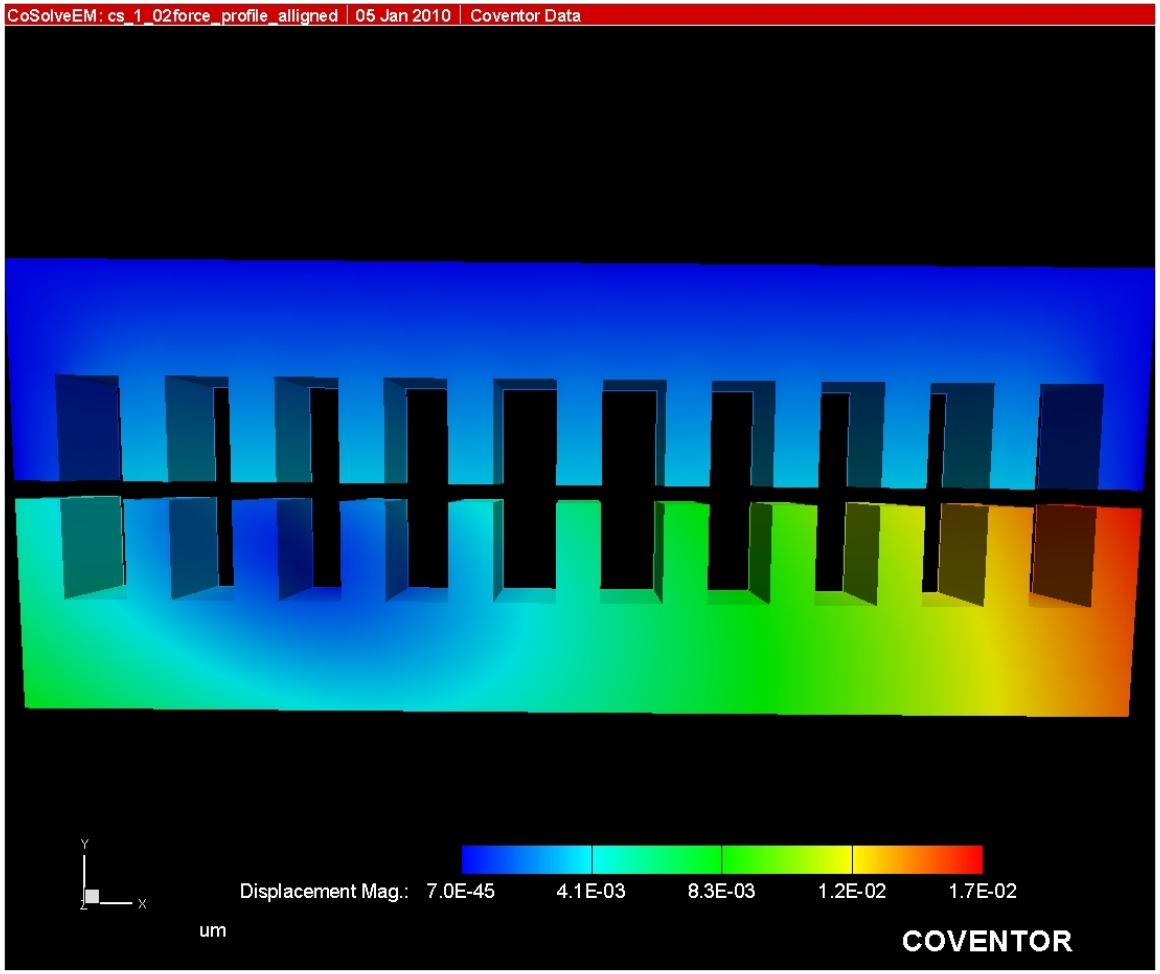
Vdc = 40 V



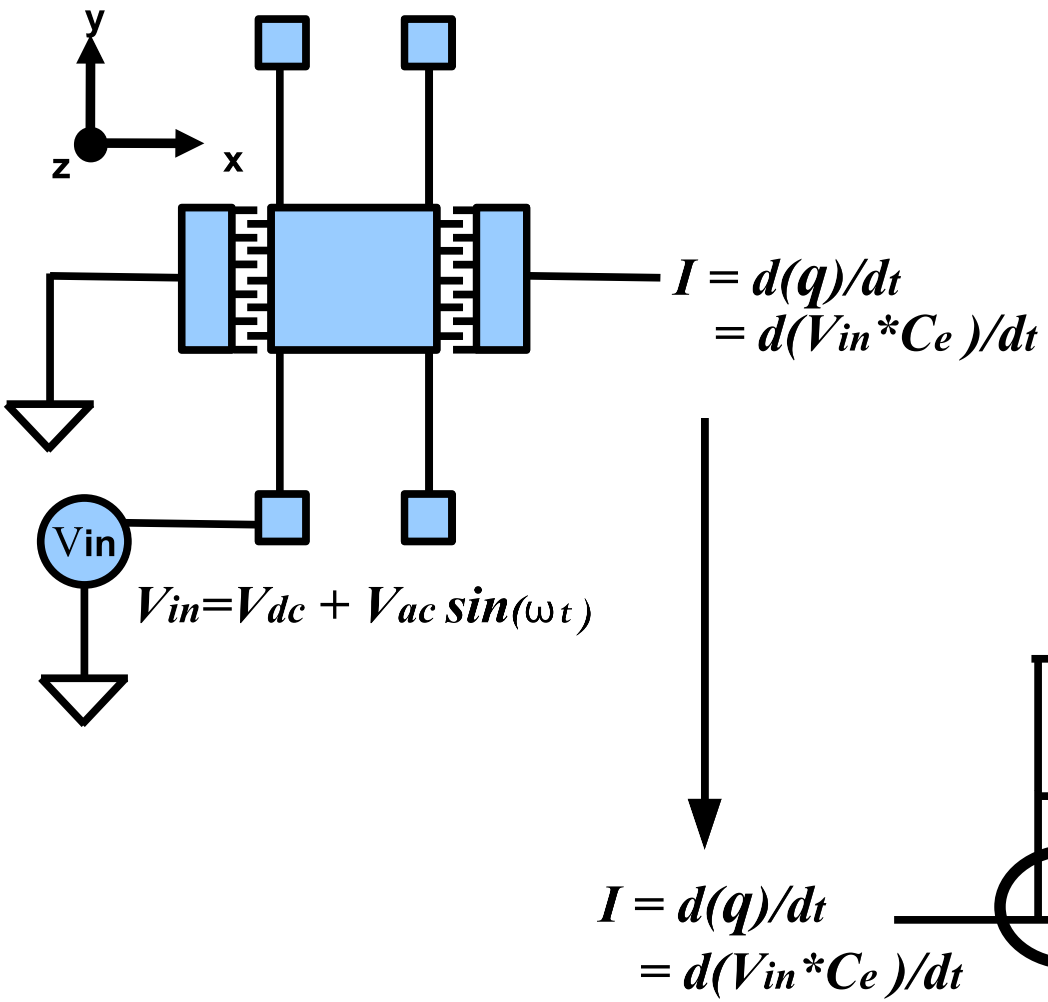
Design

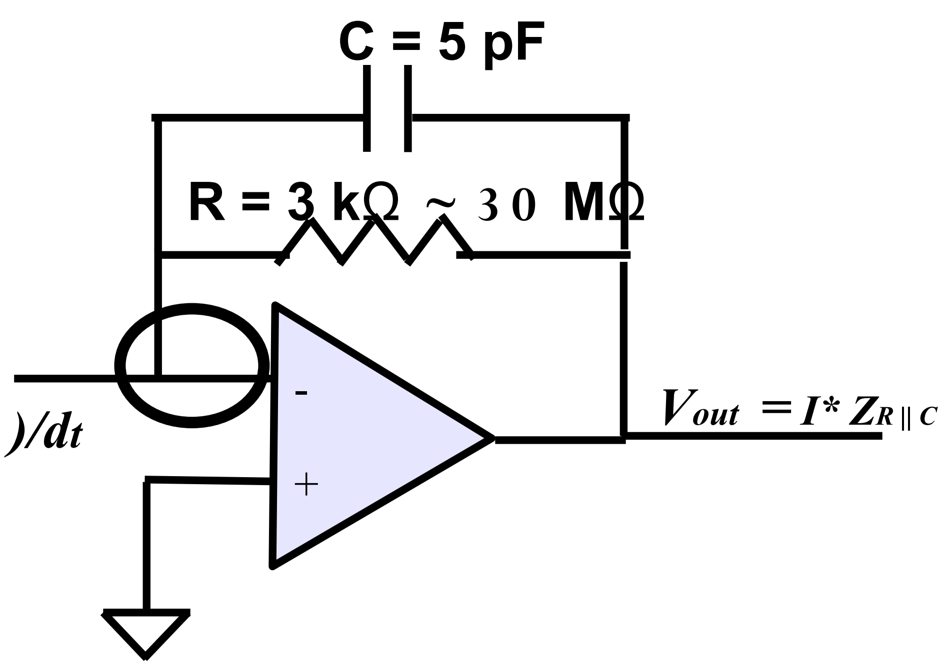
Aligned Fingers (positive displacement)

Vdc = 40 V



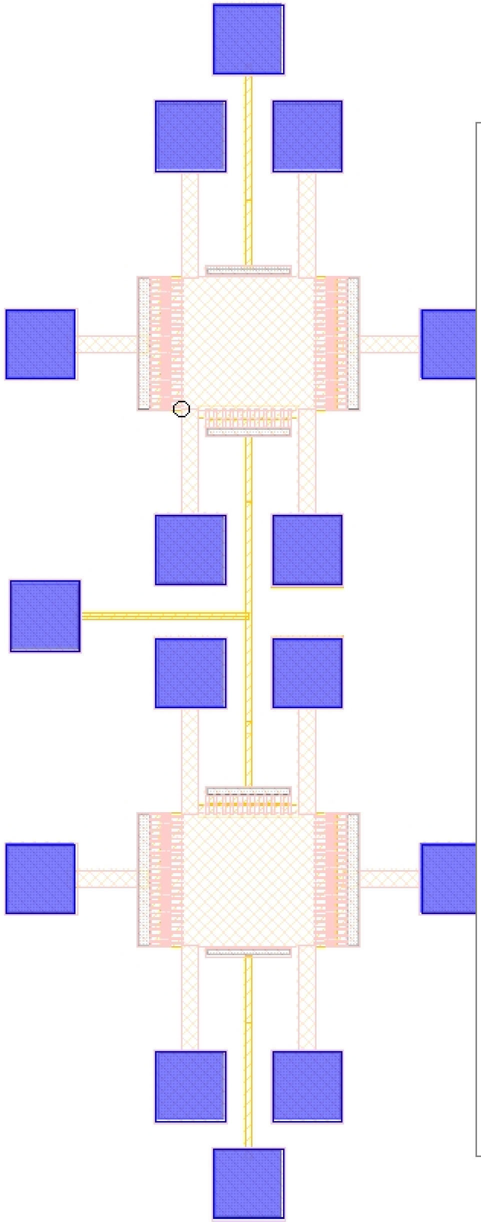
Characterization

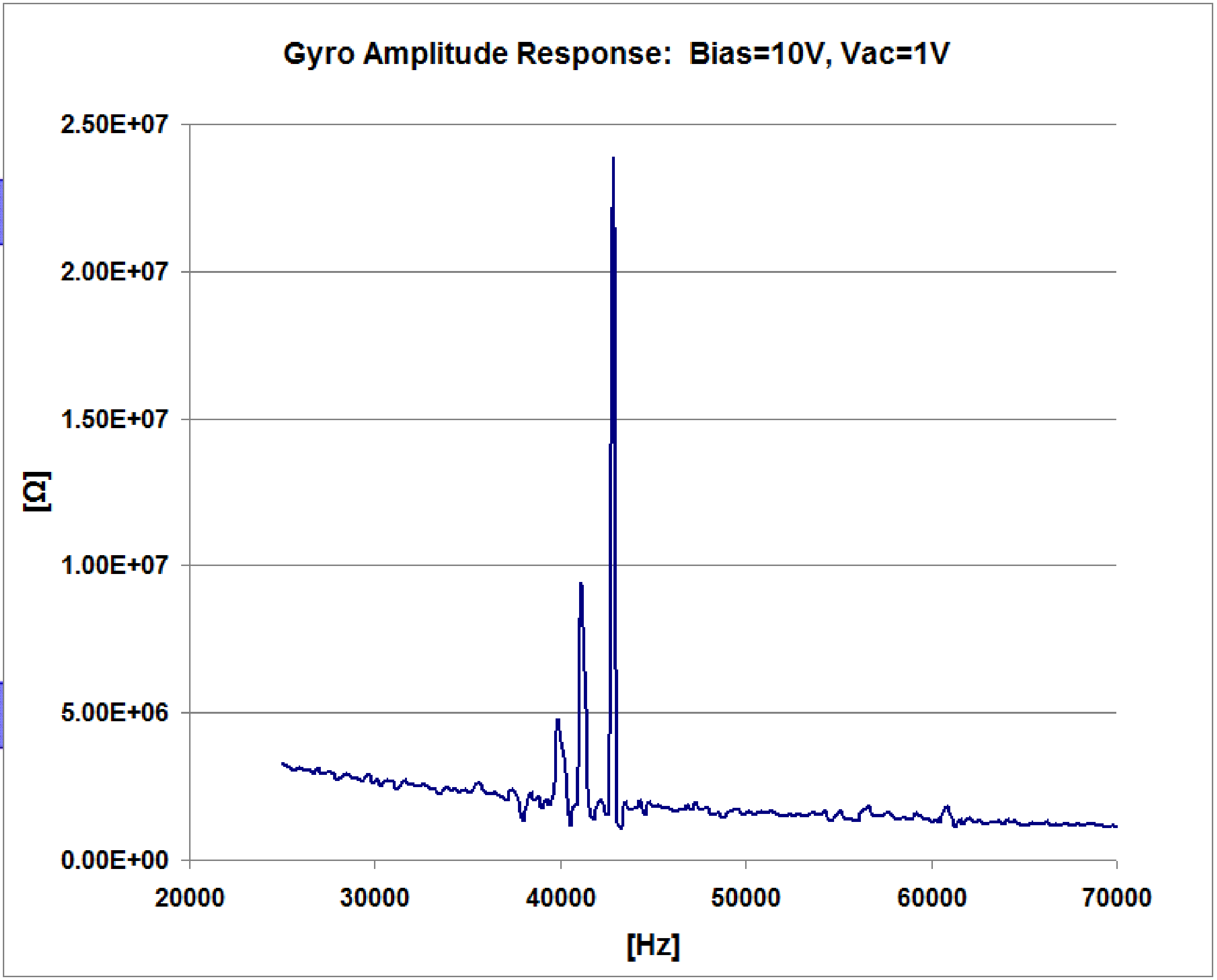




- Virtual ground created by the op amp
- Drop in impedance draws current out

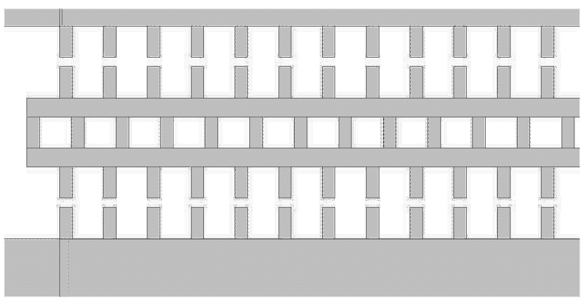
Characterization x-dir

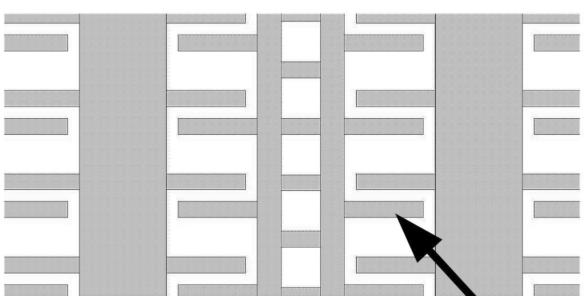


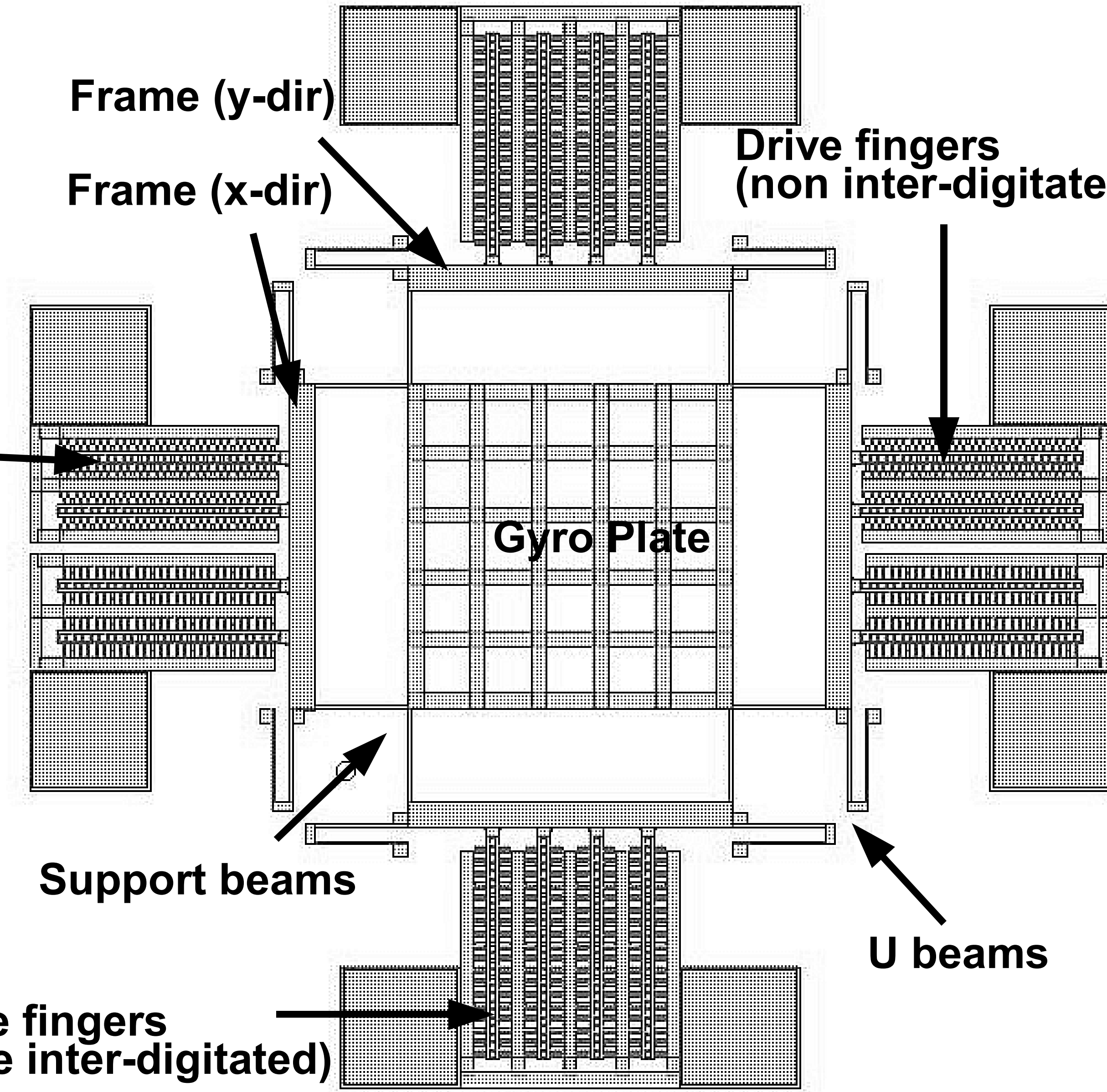


New Design

- Differential
- Parametric Excitation
- Symmetry about x and y

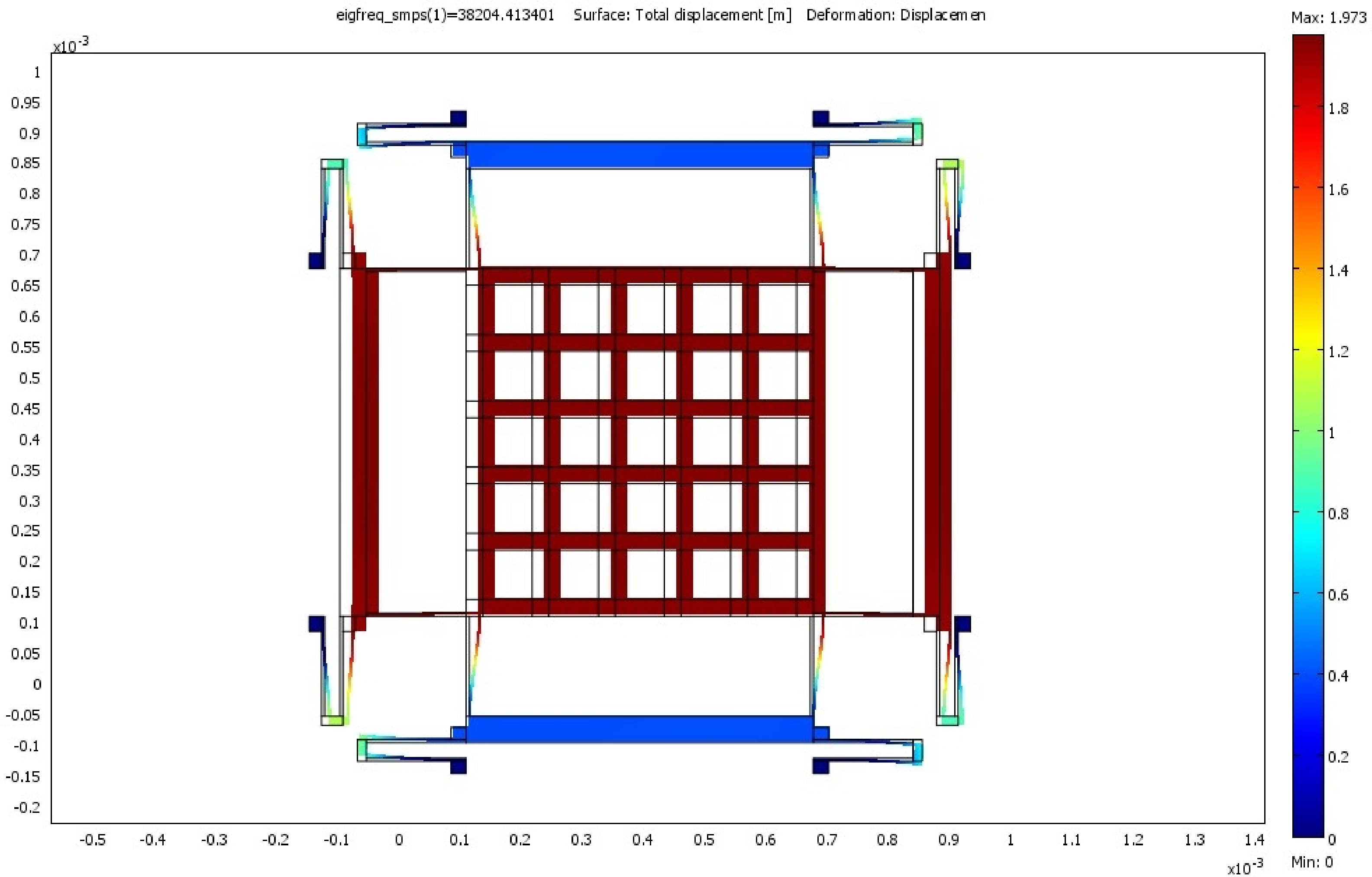






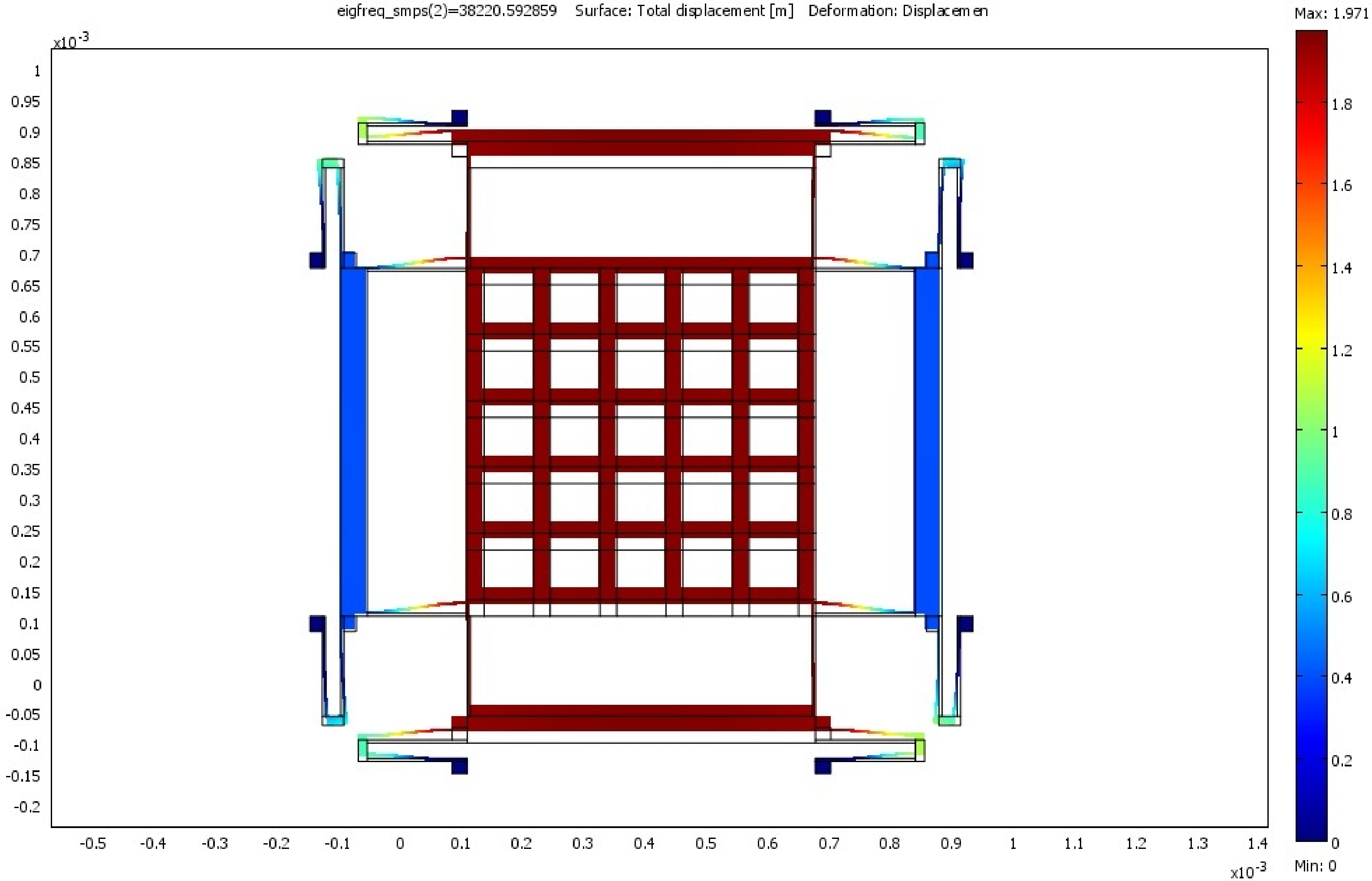
New Design

First Eigenmode (x-dir): ~ 38204 Hz



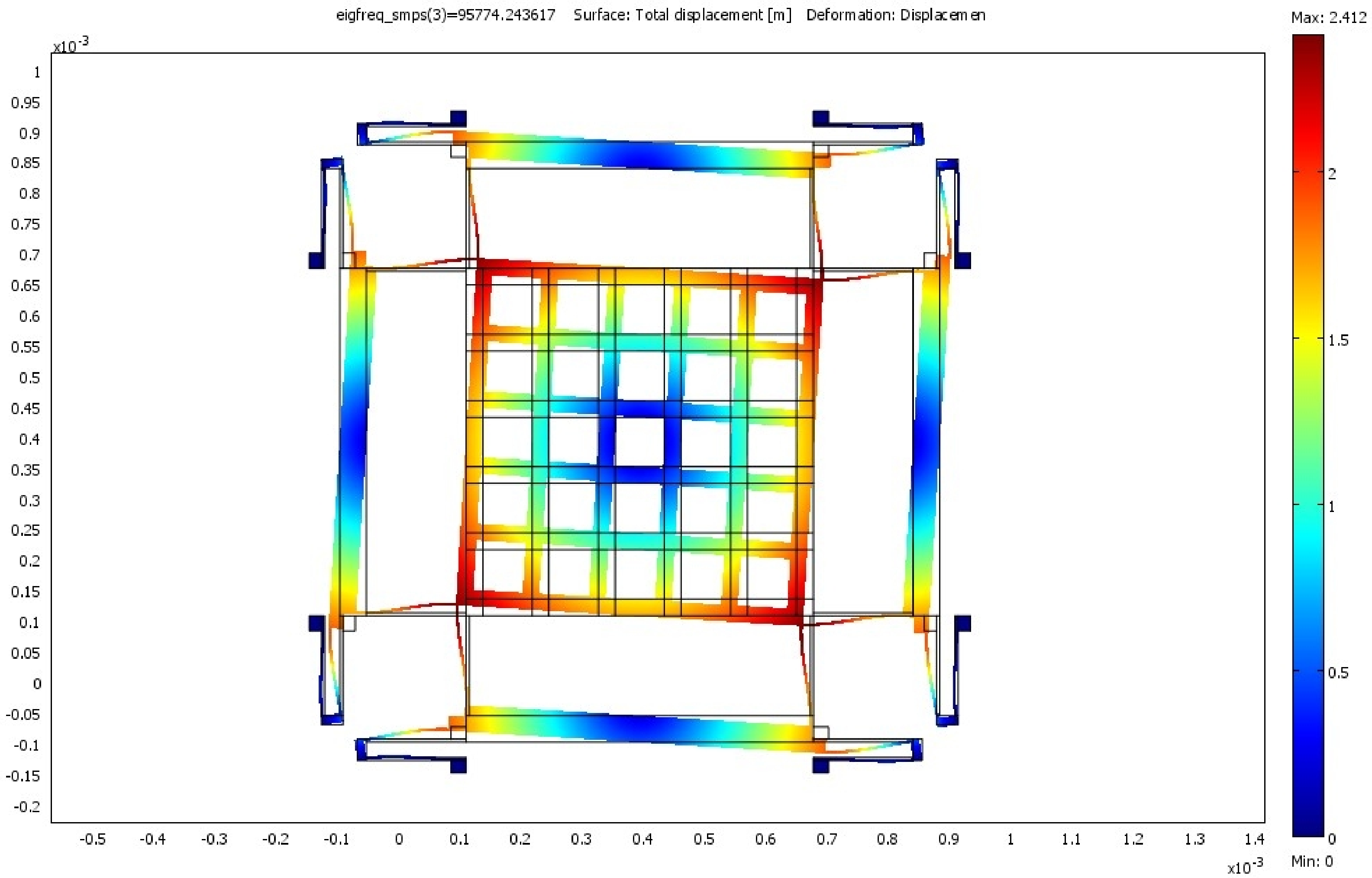
New Design

Second Eigenmode (y-dir): ~ 38220 Hz



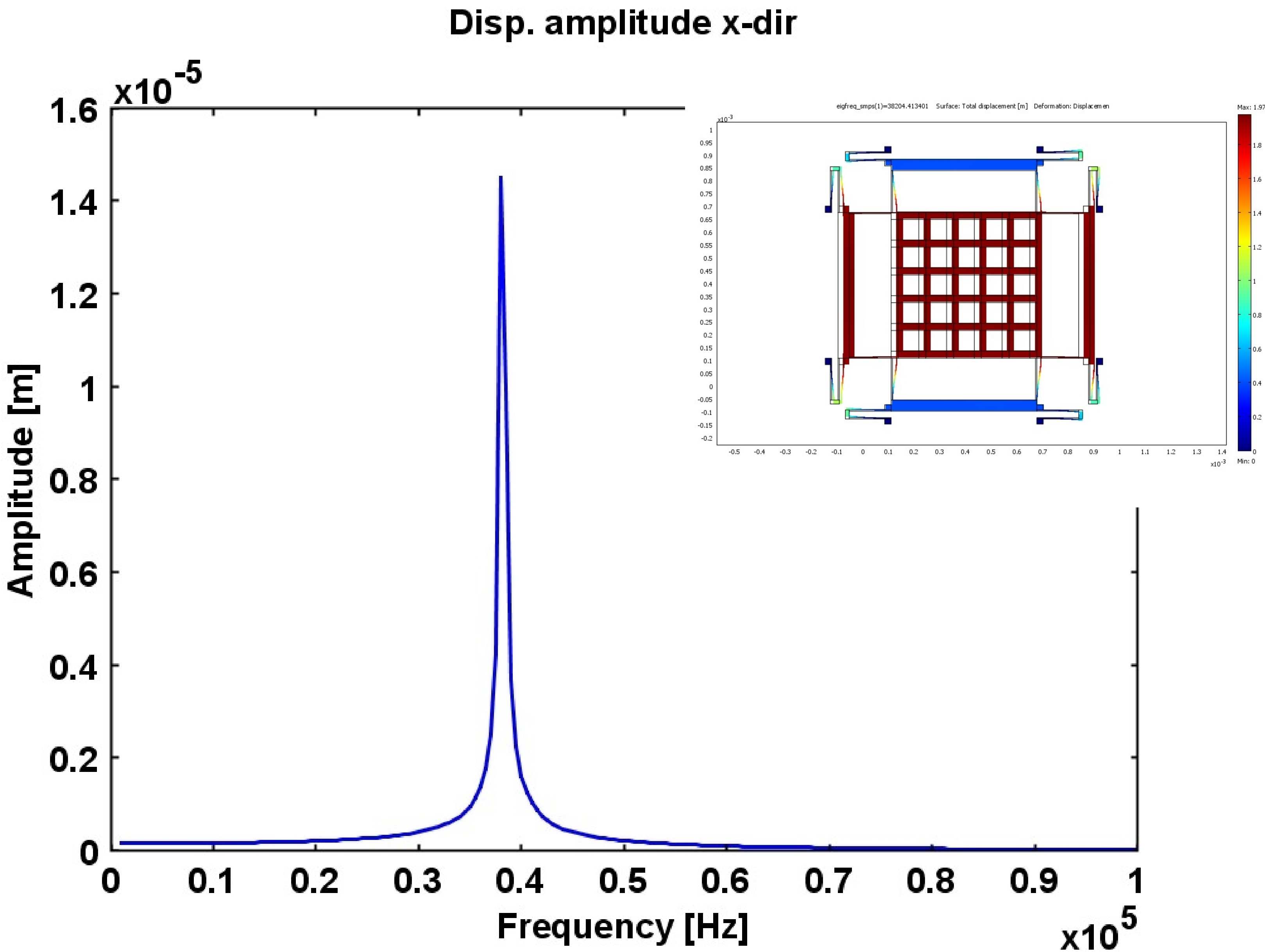
New Design

Third Eigenmode (torsional): ~ 95774 Hz



New Design

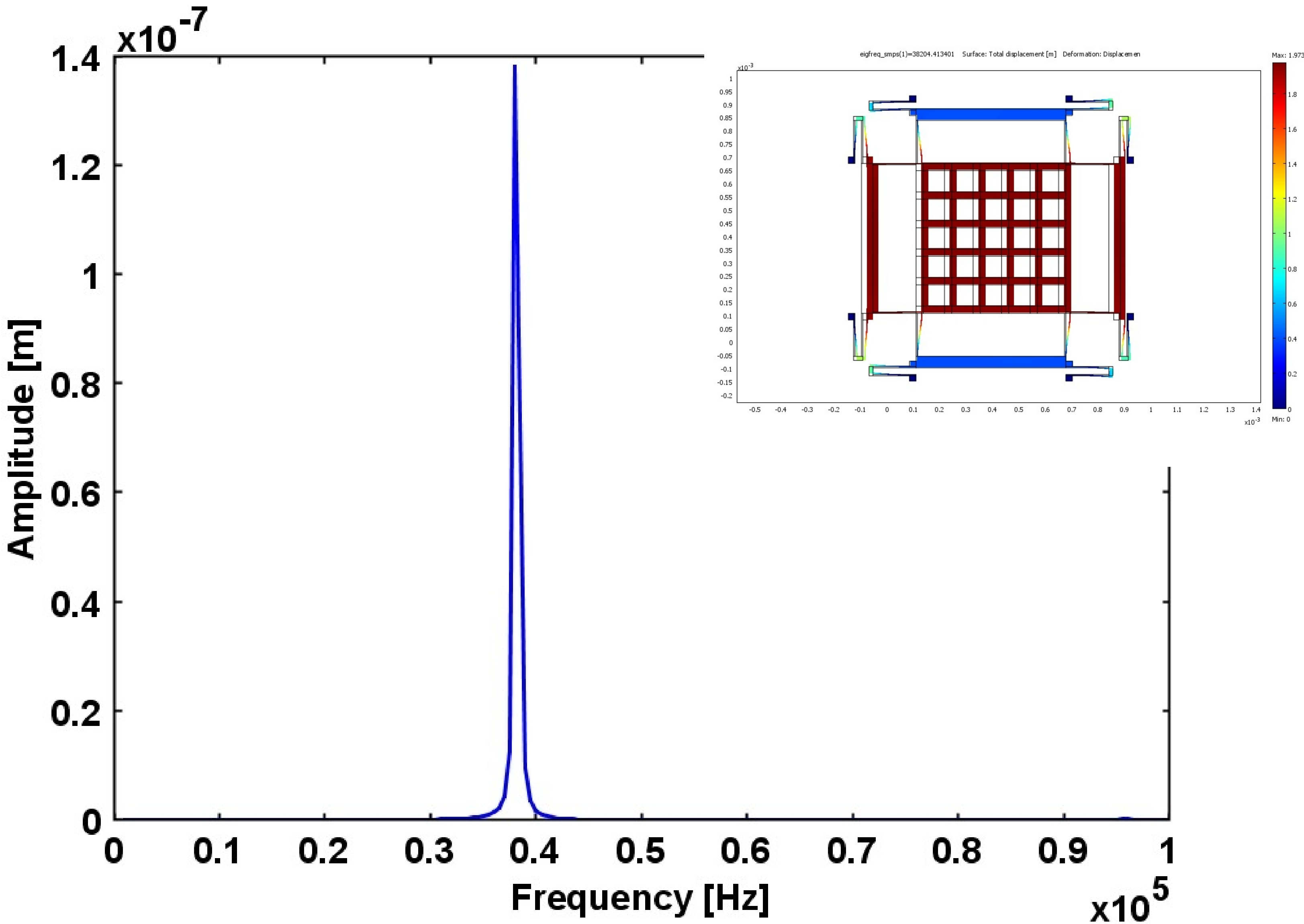
Frequency Response with Force amplitude = 56.7 uN (x-dir)



New Design

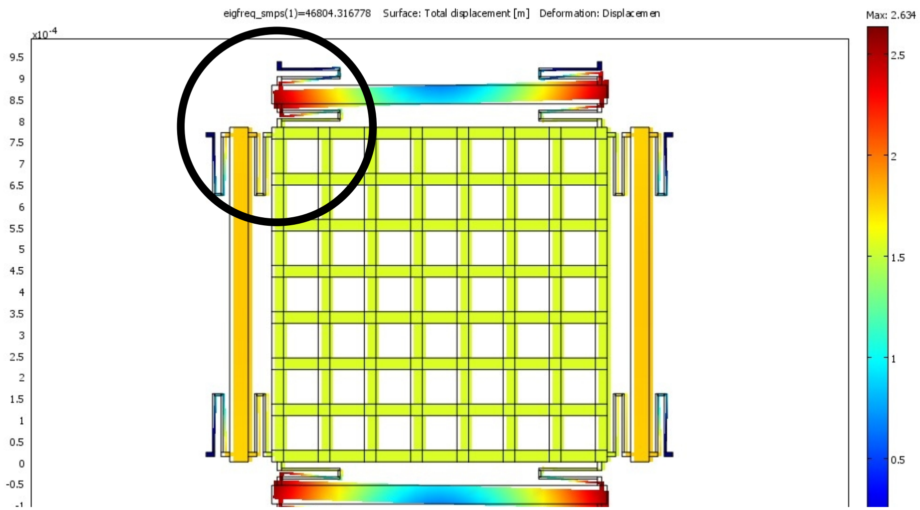
Frequency Response with Force amplitude = 56.7 uN (x-dir)

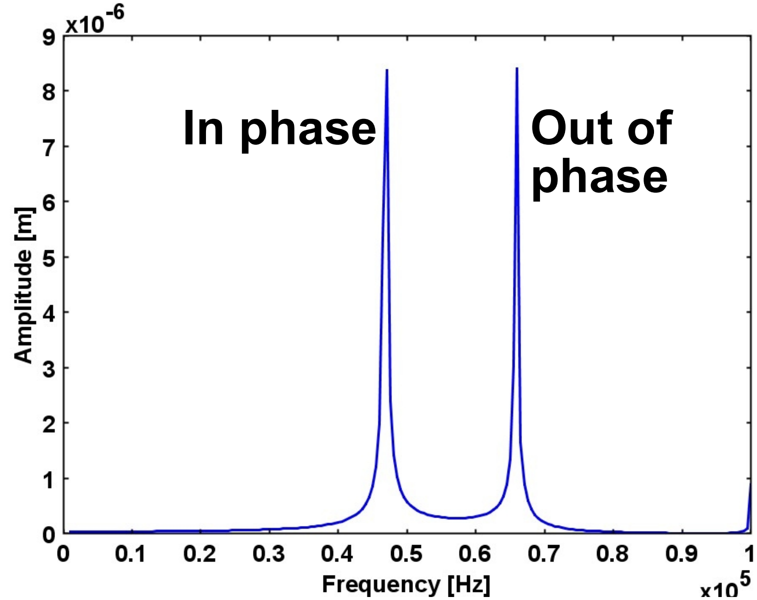
Disp. amplitude y-dir



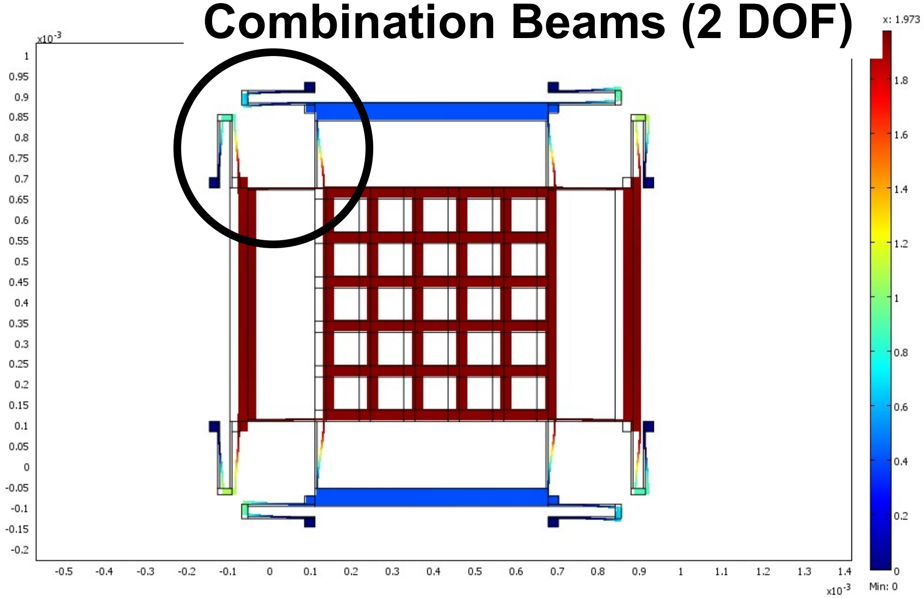
Design iteration: frequency response

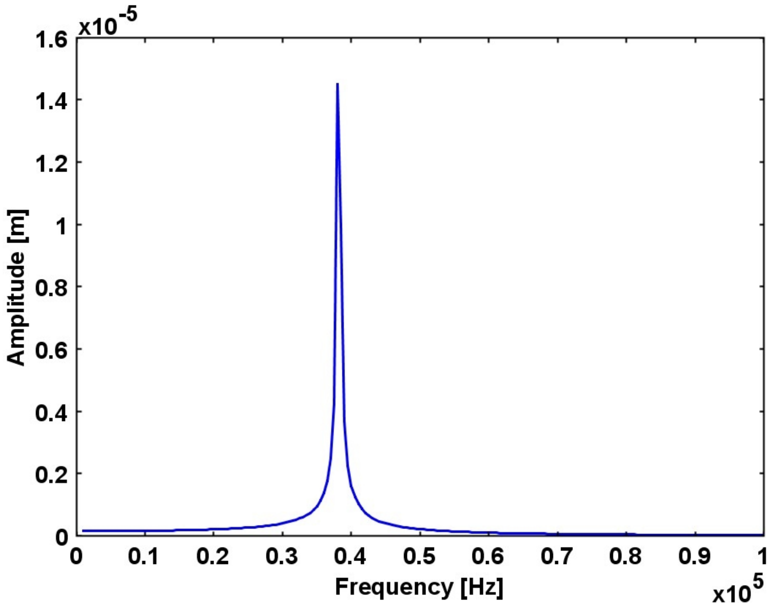
U Beams (4 DOF)





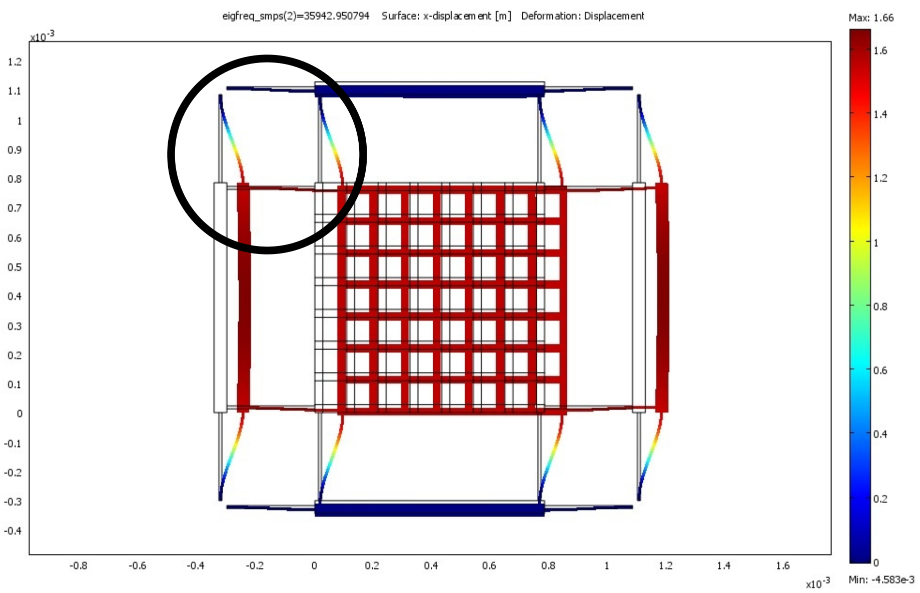
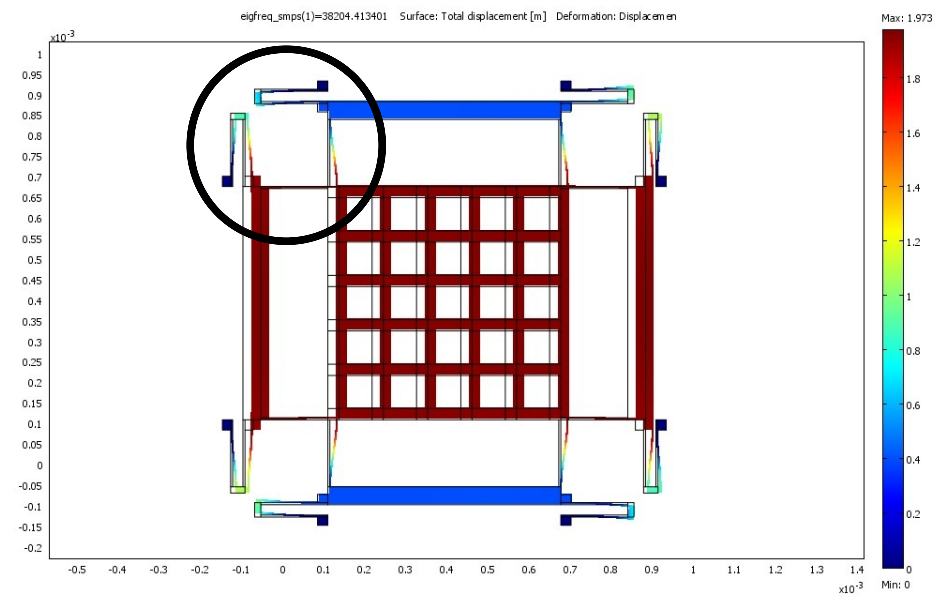
Combination Beams (2 DOF)





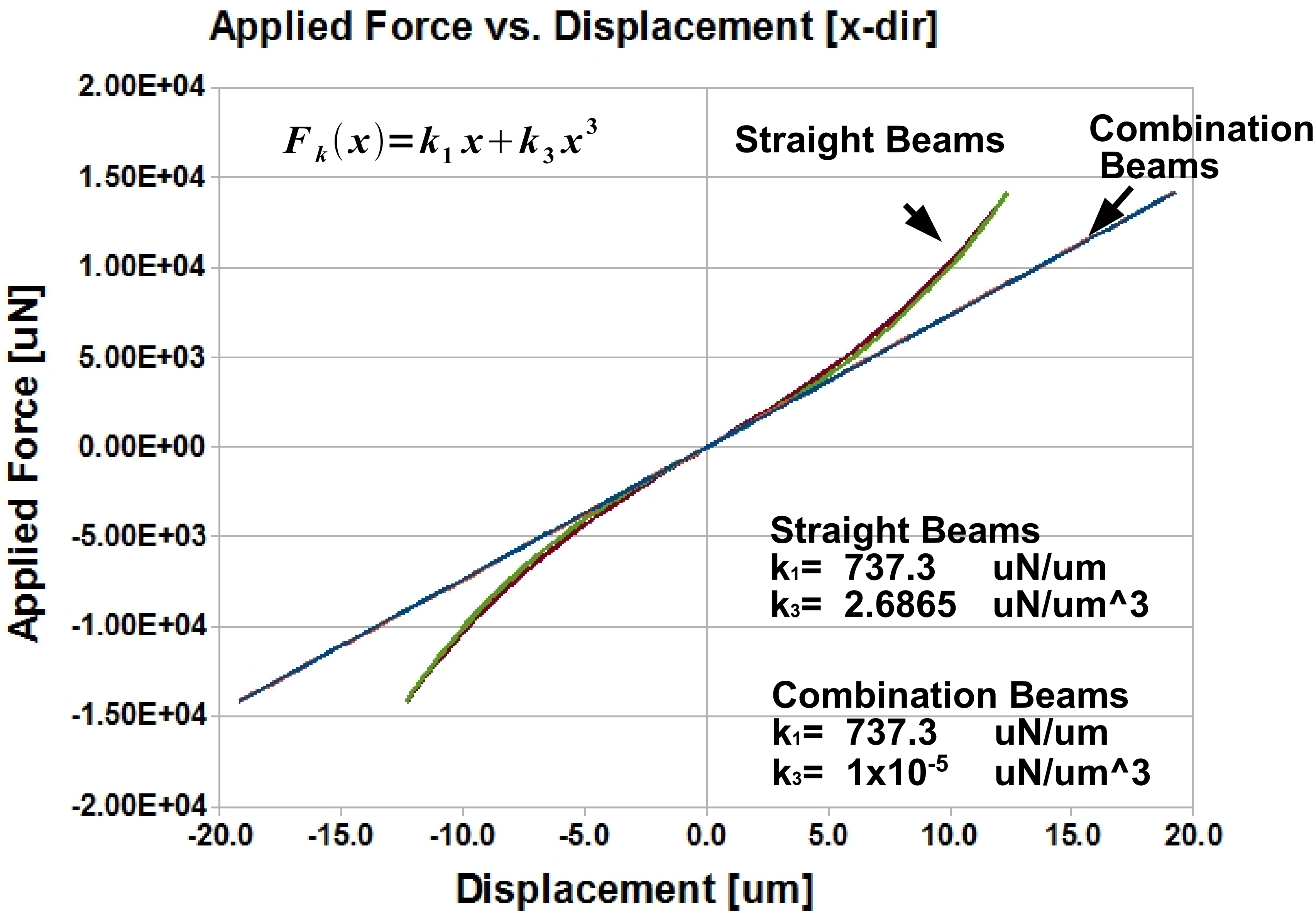
New Design

Parameter estimation

Straight Beams	Combination Beams
	

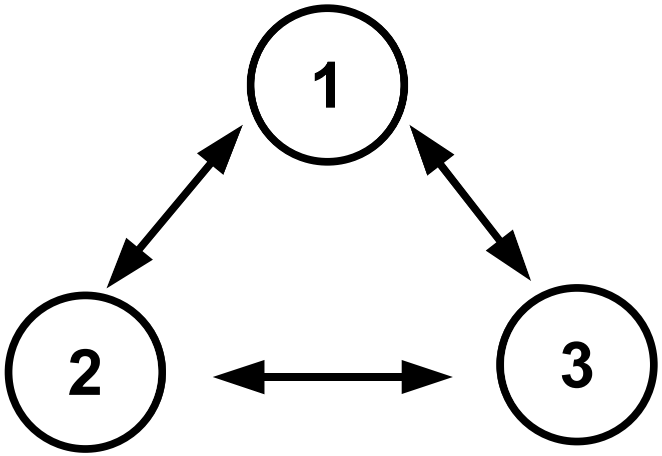
New Design

Parameter estimation by curve-fitting simulated data with the model



Coupled System: Parametric Excitation

Bi-directionally coupled ring of gyros



$$\left.\begin{aligned} m\ddot{x}_j + b\dot{x}_j + k_1 x_j + k_3 x_j^3 &= F_{ke}(x,t) + \lambda(x_{j+1} - 2x_j + x_{j-1}) + 2m\Omega_z \dot{y}_j; \\ m\ddot{y}_j + b\dot{y}_j + k_1 y_j + k_3 y_j^3 &= -2m\Omega_z \dot{x}_j;\ j=1,2,3 \end{aligned}\right\} \quad (1)$$

$$F_{ke}(x) = (r_1 x + r_3 x^3) V_a^2$$

where, $V_a = DC + A\cos(\omega_d t)$, r_1, r_3 = electrostatic coefficients

$$\boxed{\begin{aligned} m\ddot{x}_j + b\dot{x}_j + (k_1 - r_1 V_a^2) x_j + (k_3 - r_3 V_a^2) x_j^3 &= \lambda(x_{j+1} - 2x_j + x_{j-1}) + 2m\Omega_z \dot{y}_j; \\ m\ddot{y}_j + b\dot{y}_j + k_1 y_j + k_3 y_j^3 &= -2m\Omega_z \dot{x}_j;\ j=1,2,3 \end{aligned}} \quad (2)$$

Thank You

References

[1] V. Apostolyuk, 'Theory and Design of Micromechanical Vibratory Gyroscopes', MEMS Handbook Vol.1:Design Methods in MEMS/NEMS 173, Springer, New York, 2005

[2] P. Prendergast, B. Kropf, 'How to use programmable analog to measure MEMS gyroscopes',

http://www.embedded.com/columns/technicalinsights/197002302?_requestid=334309

[3] C. Acar, Robust Micromachined Vibratory Gyroscopes, PhD Dissertation (UC Irvine) 37-39, (2004)

[4] H. Vu, V. In, A Palacios, 'Two-time scale Analysis of a Ring of Coupled Vibratory Gyroscopes', Submitted to APS/123-QED, (2009)

Contact Info

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