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Bi-level Optimization of Blended Composite Panels

Dianzi Liu, Vassili V. Toropov,
Osvaldo M. Querin and David C. Barton

Presentation outline

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- Introduction
- Composite layup rules
- Optimization strategies studied:
  - Smeared stiffness-based method
  - Lamination parameter-based method
- Ply blending scheme
- Wing box example
- Discussion

Issues in optimization of laminated composite panels

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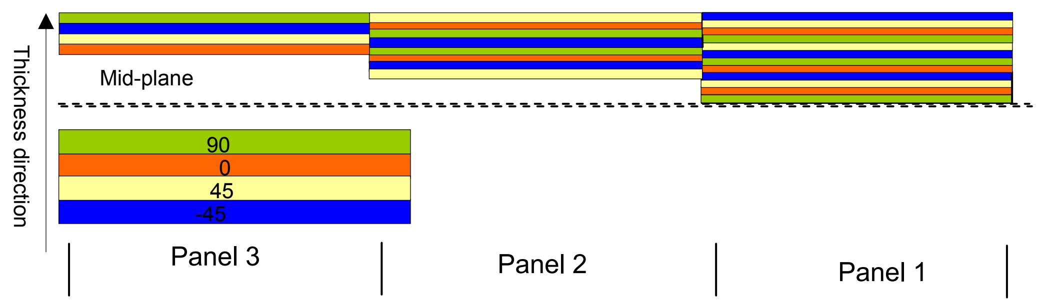
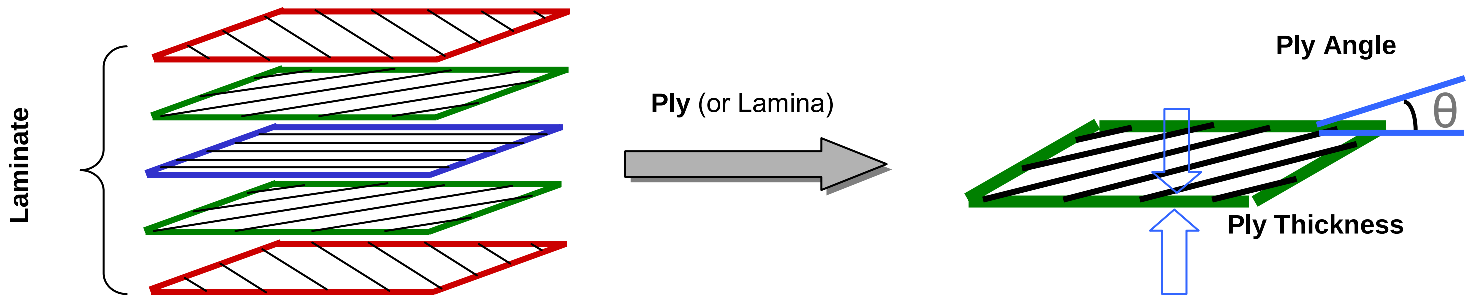
- A modern aircraft structure (e.g. wing box) consists of many composite panels.
- Weight, structural performance and manufacturability are the primary drivers.
- Stacking sequence of the laminas is vital for obtaining the required mechanical characteristics such as bending and buckling behaviour.
- Blending (also referred to as ply compatibility) of plies in the adjacent panels is a very important consideration in the design of composite structures, it is the focus of this study.

An example:



Without blending requirement

Composite layup rules



- Laminates with ply orientations of 0, 90, 45 and -45 degrees are used.
- The stack is symmetric and balanced, i.e. the number of 45º and -45º plies is the same in every component.
- Due to the damage tolerance requirements for the skin at least one set of ±45º plies is placed on the outside.
- The number of plies (N_{max}) of any one orientation placed sequentially in the stack is limited to four.
- A 90º change of angle between two adjacent plies is to be avoided, if possible.
- For every ply angle its percentage in the total stack ≥ 10%.

Smeared stiffness-based method (SSBM) concept

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- Smeared stiffness-based method aims at neutralizing the stacking sequence effects on the buckling performance by considering homogeneous stacks with smeared properties. Hence all matrices can be calculated without knowing the stacking sequence:
  - **D** = **A***t^2/12
  - **B** = 0
- Recently introduced in Altair's OptiStruct.
- In this work Ansys was used for the FE analysis.

SSBM: Global level

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▲ Global level optimization problem

- Objective function: weight
- Design variables per panel: $n^{i}_{\pm 45}, n^{i}_{0}, n^{i}_{90}$
- Constraints:
  - Strain: $\varepsilon^{i}_{j} \leq \varepsilon^{allowable}_{j}, i = 1, \cdots, N_{p},$
  - Buckling: $\lambda^{i}_{ce} \geq 1.0, i = 1, \cdots, N_{p},$
  - Ply orientation percentages ≥ 10%

SSBM: Local level

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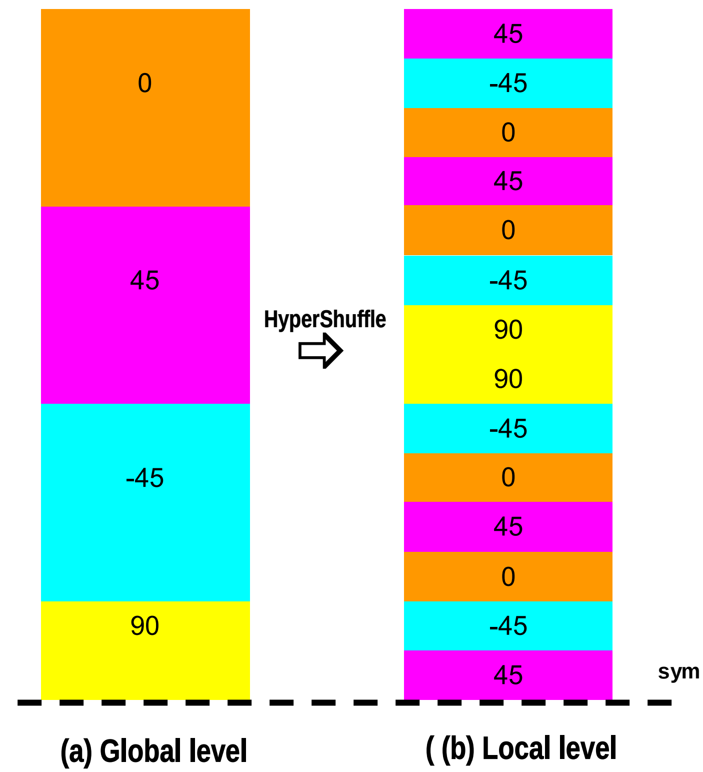
▲ Local level stack arrangement by Altair's HyperShuffle

- The software mimics what a composite designer does.
- The outside layers of the stack consist of layers of ±45° plies.
- The 0° and 90 layers are distributed throughout the stack as homogeneously as possible.
- Layer of 0° or 90° can be placed between the +45° and –45° plies in a ±45° couple.
- At most, two plies of the same orientation (0° or 90°) can be placed next to the mid-plane.
- In this work, the ply shuffling procedure is linked to the ply blending scheme.



Lamination parameters-based method (LPBM)

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- Concept
  - Lamination parameters, together with material parameters, allow to calculate the stiffness matrixes **A** and **D**

$$
\begin{bmatrix} A_{11} \\ A_{22} \\ A_{12} \\ A_{16} \\ A_{26} \\ A_{66} \end{bmatrix} = h \begin{bmatrix} 1 & \xi_1^A & \xi_2^A & 0 & 0 \\ 1 & -\xi_1^A & \xi_2^A & 0 & 0 \\ 0 & 0 & -\xi_2^A & 1 & 0 \\ 0 & 0 & -\xi_1^A & 0 & 1 \\ 0 & \xi_3^A/2 & \xi_4^A & 0 & 0 \\ 0 & \xi_3^A/2 & -\xi_4^A & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}, \quad \begin{bmatrix} D_{11} \\ D_{22} \\ D_{12} \\ D_{16} \\ D_{26} \\ D_{66} \end{bmatrix} = \left(\frac{h^3}{12} \right) \begin{bmatrix} 1 & \xi_1^D & \xi_2^D & 0 & 0 \\ 1 & -\xi_1^D & \xi_2^D & 0 & 0 \\ 0 & 0 & -\xi_2^D & 1 & 0 \\ 0 & 0 & -\xi_1^D & 0 & 1 \\ 0 & \xi_3^D/2 & \xi_4^D & 0 & 0 \\ 0 & \xi_3^D/2 & -\xi_4^D & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}.
$$

- Lamination parameters:

$$
V_{[1,2,3,4],i}^{A} = \left(\frac{1}{h_i} \right) \int_{-h_i/2}^{h_i/2} [\cos 2\theta, \sin 2\theta, \cos 4\theta, \sin 4\theta] dz
$$

$$
V_{[1,2,3,4],i}^{D} = \left(\frac{12}{h_i^3} \right) \int_{-h_i/2}^{h_i/2} [\cos 2\theta, \sin 2\theta, \cos 4\theta, \sin 4\theta] z^2 dz
$$

LPBM: Global level

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▲ Global level optimization problem

- Objective function: weight
- Design variables per panel: $n^{i}_{\pm 45}, n^{i}_{0}, n^{i}_{90}, V^{D}_{1}, V^{D}_{2}, V^{D}_{3}$
- Constraints:
  - Strain: $\varepsilon^{i}_{j} \leq \varepsilon^{allowable}_{j}, i = 1, \cdots, N_{p},$
  - Buckling: $\lambda^{i}_{ce} \geq 1.0, i = 1, \cdots, N_{p},$
  - Feasibility of lamination parameters
  - Ply orientation percentages ≥ 10%

LPBM: Local level optimization

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▲ Local level optimization procedure

- Stacking sequence is obtained by a permutation GA (permGA) that matches the target lamination parameters from the top level $\mathbf{v}_i^D$ by the lamination parameters $\tilde{\mathbf{v}}_i^D$ computed in the local level optimization subject to the composite layup rule constraints.
- Calculation of lamination parameters is linked to the blending scheme, it is done separately for groups of layers that are shared between panels.
- PermGA is used repeatedly for all sub-stacks that are considered in the the ply blending scheme.

LPBM: Flow chart at local level

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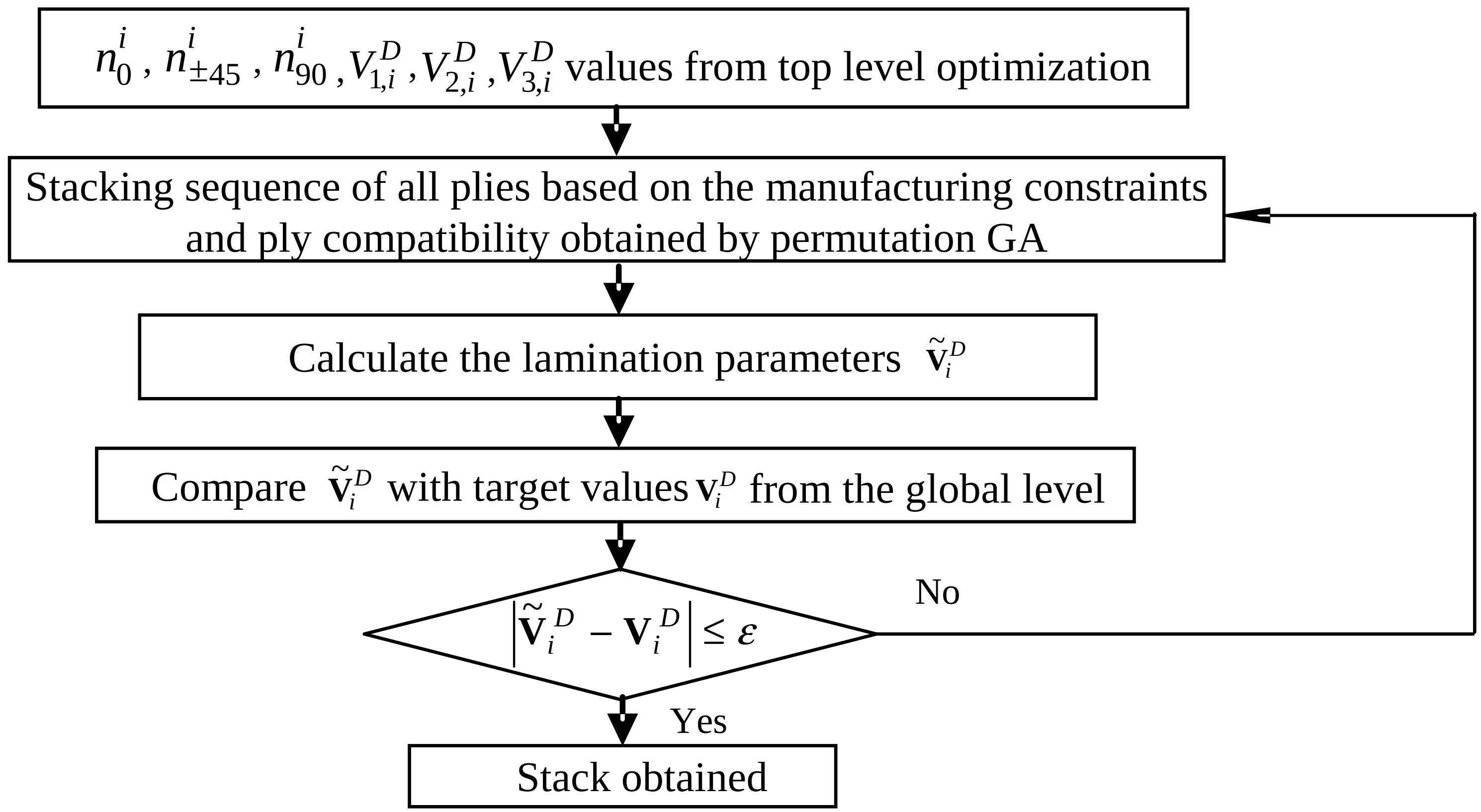


Flow chart of the panel stacking sequence optimization process at local level

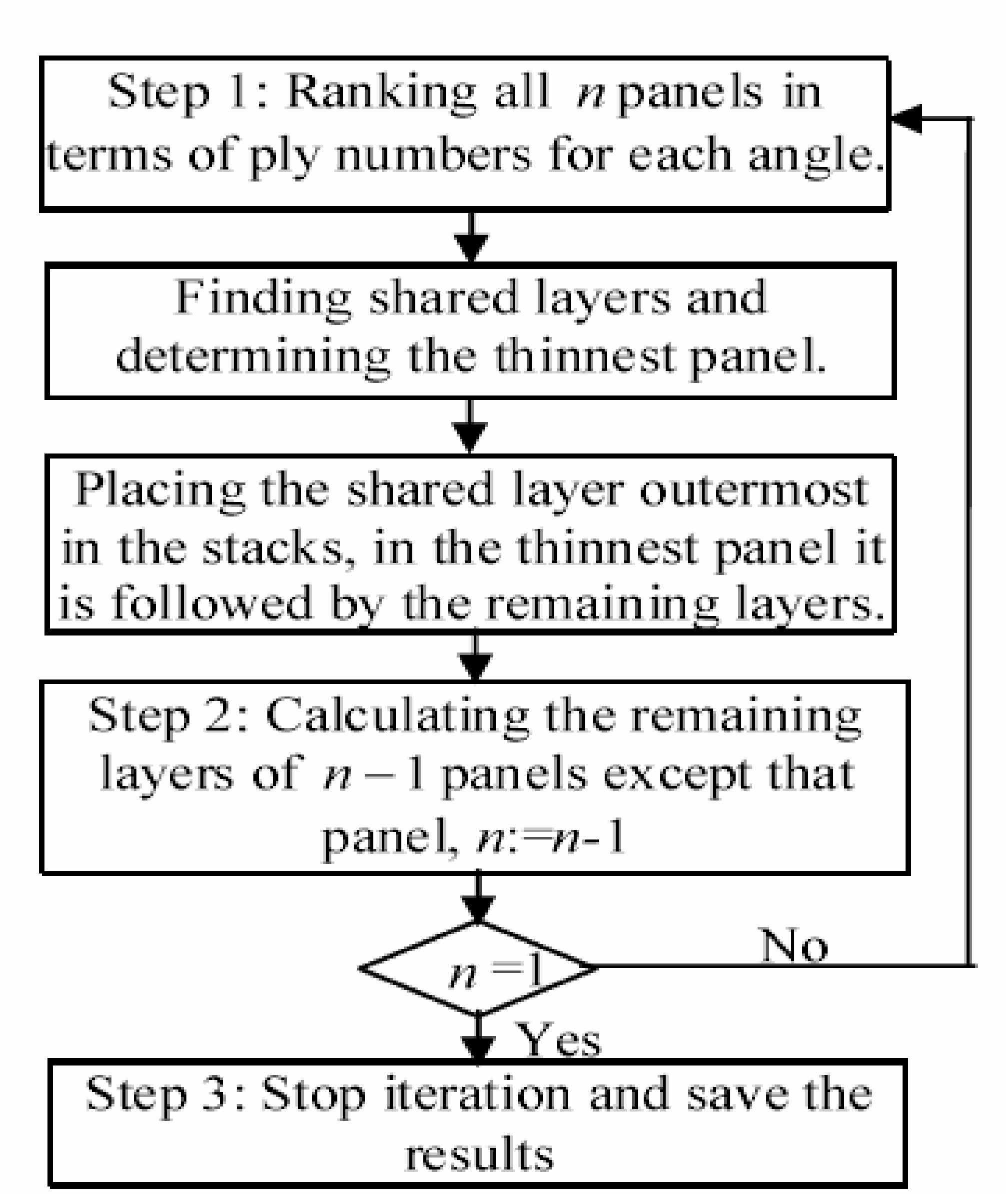
Blending requirement

- Blending (or compatibility) of plies in the adjacent panels is a very important consideration in the design of composite aircraft structures.
- Related research
  - Liu and Haftka (2001) defined the *composition continuity* and the *stacking sequence continuity measures.* Toropov *et al.* (2005) used this approach for optimization of aeronautical composite components.
  - Gürdal's group (since 2002) developed two blending methods, *inward* and *outward* blending, to improve the ply continuity between adjacent panels using a *guide based GA.*
  - Liu and Krog (2008) developed a *card sequence* approach to identifying a laminate stacking sequence in individual wing panels satisfying inter-panel continuity constraints.

Blending scheme

- Shared Layers Blending procedure
  1) Ranking of all panels in terms of the numbers of plies of each angle is performed.
  2) For each ply angle, the minimum number of plies (out of all panels) is selected. This set of three ply numbers (for 0°, 90° and ±45° plies) defines the first set of layers shared among all panels.
  3) The first set of shared layers is placed outermost in the stack and its stacking sequence is arranged using either SSBM or LPBM.
  4) In the thinnest panel, finding the remaining layers (after the first set of shared layers has been dealt with), these will go through the local blending at the final stage.
  5) Next, for the remaining layers of all the panels, except the thinnest panel, the same procedure is applied as at the first stage. This is repeated until the last panel is considered.
  6) Finally, for the adjacent panels common remaining layers are determined and local blending is performed.

Blending scheme: Flow chart



Blending procedure example

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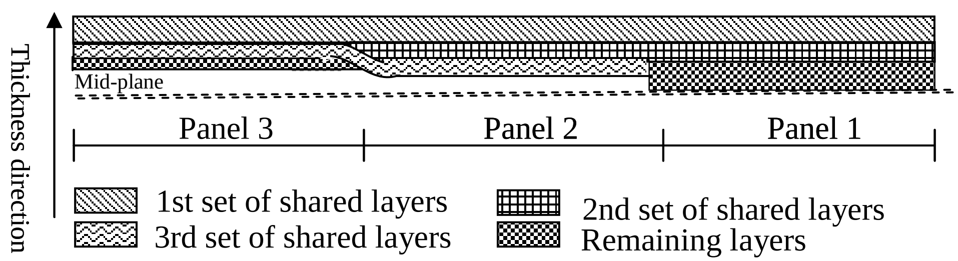
- Blending with LPBM
  1) The first set of shared layers is $n_0/n_{45}/n_{90} = 29/6/7$ for all three panels.
  2) Panel 3 is the first (thinnest) panel.
  3) Shuffling first set of shared layers to match the lamination parameters for panel 3.
  4) The second shared set for the panels 1 and 2 is $n_0/n_{45}/n_{90} = 6/8/0$
  5) Shuffling second set of shared layers to match the difference in the lamination parameters for panel 2.
  6) Local blending between panel 2 and panel 3: $n_0/n_{45}/n_{90} = 0/0/2$
  7) Shuffling third set of shared layers to match the difference of lamination parameters for panel 2.
  8) Remaining layers for panel 1 and 3 are added and shuffled.

Panel number	3	2	1
Number of plies ($n_0/n_{45}/n_{90}$)	29/6/12	35/14/9	40/17/7
Current remaining layers	0/0/5	6/8/2	11/11/0
Current remaining layers	0/0/5	0/0/2	5/3/0
Current remaining layers	0/0/3	0/0/0	5/3/0



With blending requirement

Calculation of lamination parameters in the blending scheme

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Determining stacking sequence of plies at local level

1) Stacking sequence for the first set of shared layers is obtained by the permutation GA to match the lamination parameters for the thinnest panel that came from the top level optimization subject to layup rules.
2) Following step 1, the values of lamination parameters corresponding to the first set of shared layers in each of the remaining panels are calculated.
3) Second set of shared layers is determined by the same blending scheme.
4) Stacking sequence of the second set of shared layers is determined by matching the lamination parameters for the next thinnest panel that came from the top level optimization minus the values already calculated for first set of shared layers.
5) Repeating the above procedure until the last set of shared layers is considered.
6) Summing up lamination parameters that contributed from the sets of shared layers for each panel.
7) Stacking sequence of remaining layers in each panel is determined by minimizing the difference between the lamination parameters from the top level and the ones summed up in the blending scheme.

Blending: Stack repair

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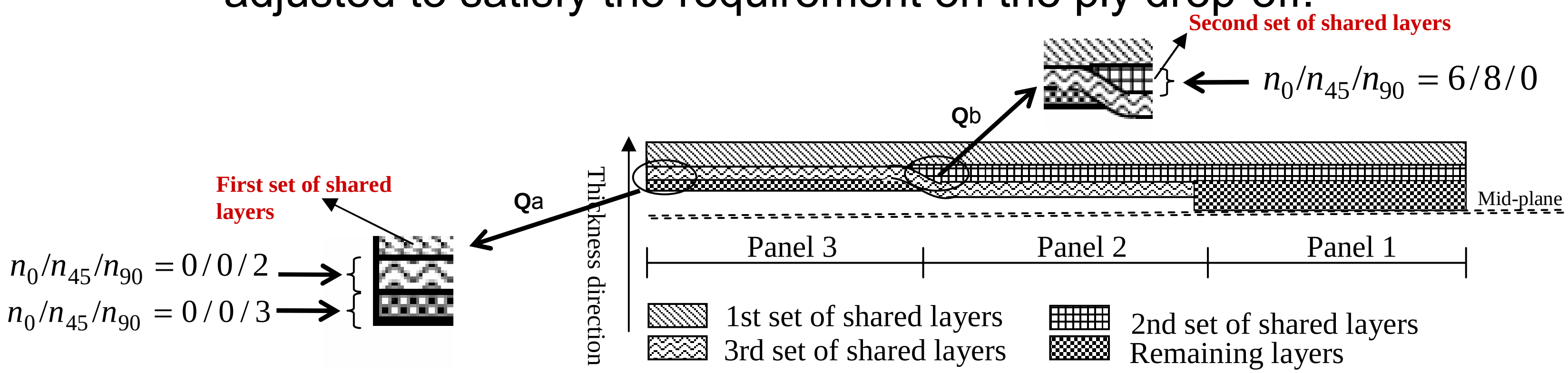
- Problem stack and its repair
  - Problems
    - The group of remaining layers in the panel 3 consists only of five 90 degree plies that violates the ply composition rule.
    - The total number of plies in the second set of shared layers truncated between the adjacent panels 2 and 3 can be considered too large ($n_0/n_{45}/n_{90} = 6/8/0$, 22 plies).
  - Solutions
    - Reserving some layers from the first set of shared layers will help avoiding more than four plies of the same angle placed together.
    - The number of plies in the second set of shared layers can be re-adjusted to satisfy the requirement on the ply drop-off.



Wing box example

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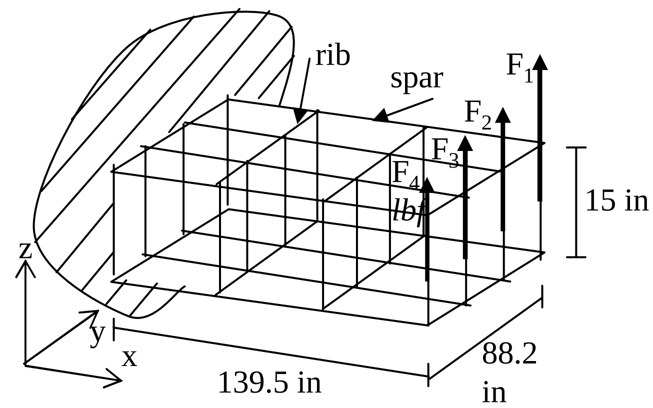
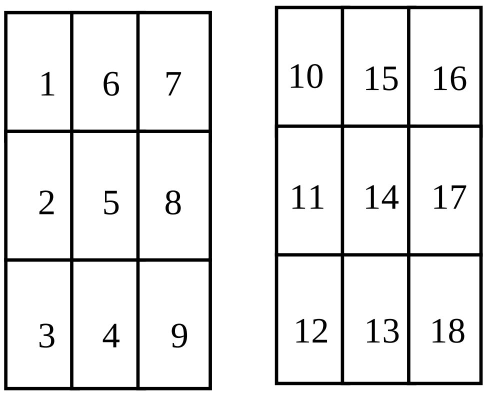
- Geometry
- Material properties
- Boundary conditions
  - Four point loads at free end
  - Fixed at the root of the wing
- Three optimization problems



Geometry of the wing box

Material properties for graphite-epoxy: T300/N5208.

Material properties	Values
Young's Modulus in direction1, E_1	127.56 GPa
Young's Modulus in direction2, E_2	13.03 GPa
Shear Modulus, G_{12}	6.41 GPa
Poisson's ratio, v_{12}	0.3
Material density, ρ	1577.76 kg/m^3
Ply thickness, t	0.127 mm
Allowable strain in fiber direction ε_{1a}	0.08
Allowable strain in transverse direction ε_{2a}	0.029
Allowable shear strain γ_{12}	0.015
Safety factor	1.5



Bottom and top skin panels

Problem with two designable substructures

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- Smeared stiffness-based method
- Lamination parameter-based method

Top level optimization

	n_0 (Continuous)	n_{45}	n_{90}	n_0 (Rounded)	n_{45}	n_{90}	Active constraints
Top skin panels	40.95	10.45	16.39	41	11	16	Panel 16 (buckling)
Bottom skin panels	5.67	1.48	5.17	6	1	6	
Buckling load factor	0.990			1.022			
Total number of plies	184.11			186			
Total number of plies[5]	208.76			208			

Top level optimization

	n_0 (Continuous)	n_{45}	n_{90}	n_0 (Rounded)	n_{45}	n_{90}	V_1	V_2	V_3
Top skin panels	34.492	7.445	26.139	34	8	26	0.9434	1.0065	1.2108
Bottom skin panels	8.163	1.480	2.181	9	1	3	0.8944	1.0435	0.9710
Buckling load factor	1.0009			1.0183					
Total number of plies	177.65			180					
Total number of plies[5]	208.76			208					

Local level optimization

Panel no.	Stacking sequence
16	$[(\pm 45)_2/(0_4/90/45/0/-45)_3/(0_4/90/45/90/-45)_5/0_4/90/45/0/-45/90_2/0]_s$
7	$[\pm 45/0_3/90_3/0_2/90_3/0]_s$
Buckling load factor	1.020

Local level optimization

Panel no.	V_1	V_2	V_3	Buckling load factor
16	0.9434	1.0080	1.2107	1.0178
7	1.0131	1.0190	1.1730	

Stacking sequence:

16 $[(\pm 45)_2/90/0/45/90_2/-45/90_2/0_3/90/0/45/90/-45/90/0/45/90_2/-45/0/90/45/90_2/-45/0_3/90/0/90/45/90_2/-45/0/90/0_2/90/0_4/90/0_4/90/0/90/0_2/90_2/0_2/90/0/90/0_4/45/0_2/-45]_s$

7 $[\pm 45/(90/0)_3/0_6]_s$

[5] Liu, B., Haftka, R.T. and Akgun, M.A. Two-Level composite wing structural optimization using response surface, SMO, 20, 87-96, 2000

Problem with six designable substructures

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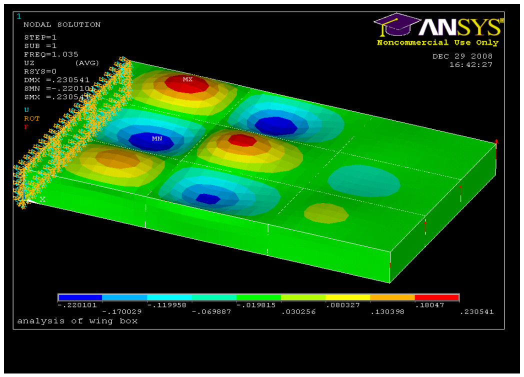
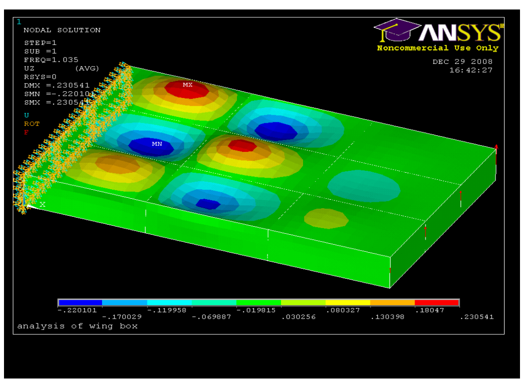
- Smeared stiffness-based method
  - Top level optimization



1st buckling mode for discrete optimal design

	n_0 (Continuous)	n_{45} (Continuous)	n_{90} (Continuous)	n_0 (Rounded)	n_{45} (Rounded)	n_{90} (Rounded)	Active constraints
Top skin panels							
Panel no.16	30.20	12.54	24.56	30	13	25	
Panel no.17	18.69	20.53	12.10	19	21	12	
Panel no.18	24.43	5.40	8.92	24	6	9	buckling
Bottom skin panels							
Panel no.7	1.50	1.32	1.45	2	1	2	
Panel no.8	2.38	1.01	1.32	3	1	2	
Panel no.9	7.81	3.06	3.35	8	3	4	
Buckling load factor	0.9960			1.0440			
Total number of plies	448.82			460			
Total number of plies[5]	465.63			464			

- Lamination parameter-based method
  - Top level optimization



1st buckling mode for discrete optimal design

	n_0 (Continuous)	n_{45} (Continuous)	n_{90} (Continuous)	n_0 (Rounded)	n_{45} (Rounded)	n_{90} (Rounded)	V_1	V_2	V_3
Top skin panels									
Panel no.16	30.20	12.54	24.56	28	16	22	1.1268	1.0102	1.2132
Panel no.17	18.69	20.53	12.10	26	13	19	1.1610	1.0086	1.3022
Panel no.18	24.43	5.40	8.92	22	6	14	1.2398	1.0098	1.0982
Bottom skin panels									
Panel no.7	4.39	1.30	1.28	5	1	1	1.3715	1.0579	0.7382
Panel no.8	3.92	1.20	2.06	4	1	2	1.1144	1.0576	0.7906
Panel no.9	7.48	1.72	2.68	8	2	3	0.8432	1.0485	0.9308
Buckling load factor	1.0039			1.0349					
Total number of plies	456.68			464					
Total number of plies[5]	465.63			464					

Problem with six designable substructures

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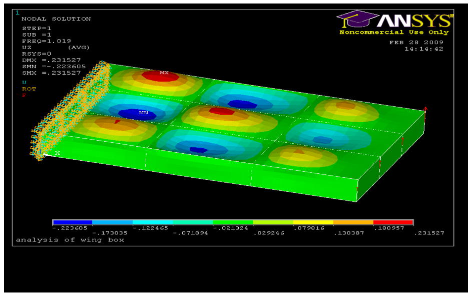
▲ Smeared stiffness-based method

- Local level optimization

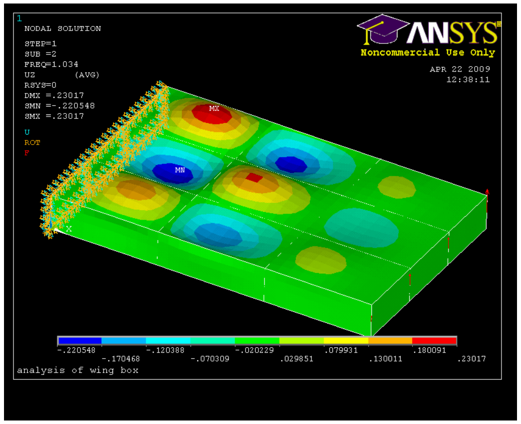


1st buckling mode for discrete optimal design

Panel no.	Stacking sequence
16	$[(\pm 45)_2/(0_4/90/45/0/\text{-}45)_2/(0_4/90/45/90/\text{-}45)_2/90_2/0/(\pm 45)_3/(90/(\pm 45)_4/(90_4/0_3)_3/90/0_2]_s$
17	$[(\pm 45)_2/(0_4/90/45/0/\text{-}45)_2/(0_4/90/45/90/\text{-}45)_2/90_2/0/(\pm 45)_3/\ (90/\pm 45)_4/(\pm 45)_8]_s$
18	$[(\pm 45)_2/(0_4/90/45/0/\text{-}45)_2/(0_4/90/45/90/\text{-}45)_2/90_2/0_4/90/0_2]_s$
7	$[\pm 45/90_2/0_2]_s$
8	$[\pm 45/90/0/90/0_2]_s$
9	$[\pm 45/90/0/90/0_2/(\pm 45)_2/0_4/90_2/0]_s$
Buckling load factor	1.019

▲ Lamination parameter-based method

- Local level optimization



2nd buckling mode for discrete optimal design

Panel no.	V_1	V_2	V_3	Buckling load factor
16	1.1582	1.0070	1.2177	
17	1.1801	1.0081	1.2030	
18	1.2398	1.0139	1.0987	1.0337 (2nd buckling factor)
7	1.2630	1.0547	0.8958	
8	1.2604	1.0547	0.8958	
9	1.2084	1.0296	1.1270	0.9614 (1st buckling factor)

Stacking sequence:

16	$[(\pm 45)_2/(0_2\,/45/0_2/\text{-}45)_2/0_2/45/90/\text{-}45/(0/90)_2/0_2/90_3/0/90/0_3/90_3/0/90/45/90/\text{-}45/0/(0/90)_2/90_2/(90/0)_3/45/0/\text{-}45/(\pm 45)_6/90_2/45/90/\text{-}45/(45/0/\text{-}45)_2]_s$
17	$[(\pm 45)_2/(0_2\,/45/0_2/\text{-}45)_2/0_2/45/90/\text{-}45/(0/90)_2/0_2/90_3/0/90/0_3/90_3/0/90/45/90/\text{-}45/0/(0/90)_2/90_2/(90/0)_3/45\ /0/\text{-}45/(\pm 45)_6]_s$
18	$[(\pm 45)_2/(0_2\,/45/0_2/\text{-}45)_2/0_2/45/90/\text{-}45/(0/90)_2/0_2/90_3/0/90/0_3/90_3/0/90/45/90/\text{-}45/0/(0/90)_2]_s$
7	$[\pm 45/0_4/90/0]_s$
8	$[\pm 45/0_4/90_2]_s$
9	$[\pm 45/0_4/90_2/45/90/\text{-}45/0_4]_s$

Problem with nine designable substructures

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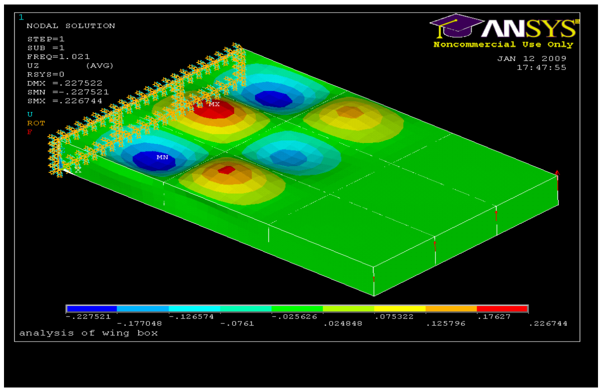
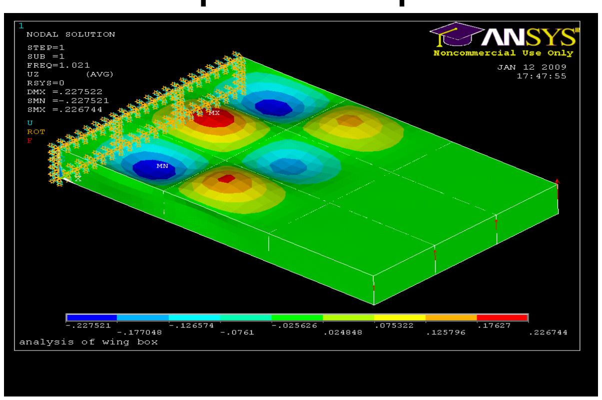
- Smeared stiffness-based method
  - Top level optimization



1st buckling mode for discrete optimal design

Panel no.	n_0	n_{45}	n_{90}	n_0	n_{45}	n_{90}	Active constraints
	(Continuous)			(Rounded)			
10	26.21	15.25	15.61	26	16	16	
11	21.90	15.49	13.46	22	16	13	
12	21.21	6.79	9.22	21	7	9	buckling
13	25.40	7.12	6.86	25	8	7	buckling
14	30.71	16.24	13.91	31	17	14	
15	34.46	18.38	17.70	34	19	18	
16	28.57	15.48	16.46	29	16	16	
17	29.83	11.65	15.11	30	12	15	
18	24.95	5.45	10.72	25	6	11	
Buckling load factor	0.9967			1.0032			
Total number of plies	1171.9			1192			

- Lamination parameter-based method
  - Top level optimization



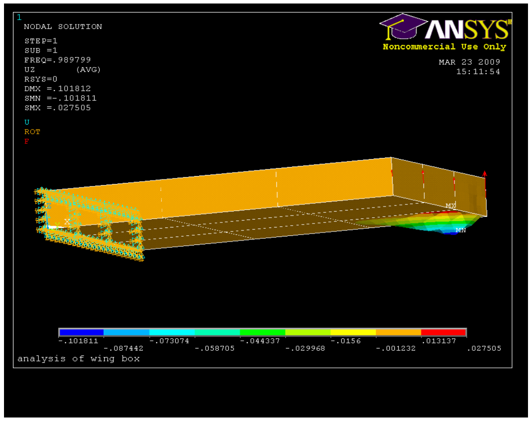
1st buckling mode for discrete optimal design

Panel no.	n_0	n_{45}	n_{90}	n_0	n_{45}	n_{90}	V_1	V_2	V_3
	(Continuous)			(Rounded)					
10	27.07	14.44	21.40	27	15	21	1.0978	1.0094	1.2446
11	25.34	12.85	19.08	25	13	19	1.1261	1.0086	1.2905
12	20.73	5.67	12.84	21	6	13	1.2319	1.0089	1.0736
13	20.70	5.66	12.84	21	6	13	1.2311	1.0087	1.0745
14	25.35	13.24	19.28	25	14	19	1.1189	1.0083	1.2596
15	27.66	15.70	22.04	28	16	22	1.0947	1.0096	1.2001
16	27.48	15.81	22.07	27	16	22	1.0987	1.0102	1.2013
17	25.56	13.49	19.36	26	14	19	1.1224	1.0082	1.2492
18	20.99	6.05	13.05	21	7	13	1.2243	1.0071	1.0460
Buckling load factor	1.0014			1.0213					
Total number of plies	1177.32			1192					

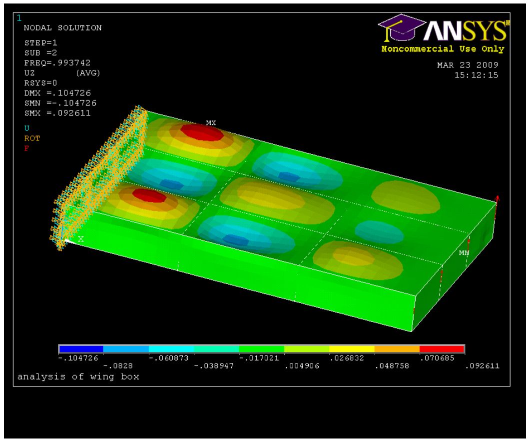
Problem with nine designable substructures

- Smeared stiffness-based method
  - Local level optimization

Panel no.	Stacking sequence
10	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_4/(\pm 45)_2/90/(90/\pm 45)_4/(90/0_2)_2/(\pm 45)_2/(\pm 45/90)_2]_s$
11	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_4/(\pm 45)_2/90/(90/\pm 45)_4/90/(\pm 45)_4]_s$
12	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_3/\pm 45/90_2]_s$
13	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_4/\pm 45/0_3/\pm 45]_s$
14	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_4/(\pm 45)_2/90/(90/\pm 45)_4/(90/0_2)_2/(\pm 45)_2/0_2/(\pm 45/0)_3]_s$
15	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_4/(\pm 45)_2/90/(90/\pm 45)_4/(90/0_2)_2/(\pm 45)_2/(\pm 45/90)_2/0_3/(\pm 45)_2/0_3/45/0/-45/90_2/0]_s$
16	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_4/(\pm 45)_2/90/(90/\pm 45)_4/(90/0_2)_2/(\pm 45)_2/\pm 45/90/\pm 45/0/90/0_2]_s$
17	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_4/(\pm 45)_2/90/(90/\pm 45)_4/(90/0_2)_2/0_2/90/0_2]_s$
18	$[(\pm 45)_2/(0_4/90/45/0/-45)_2/(0_4/90/45/90/-45)_2/90/0_4/90_3/0_3/90]_s$
Buckling load factor	0.990 (1st buckling of panel 9) 0.994 (2nd buckling of panel 16)



1st buckling mode for discrete optimal design



2nd buckling mode for discrete optimal design

Problem with nine designable substructures

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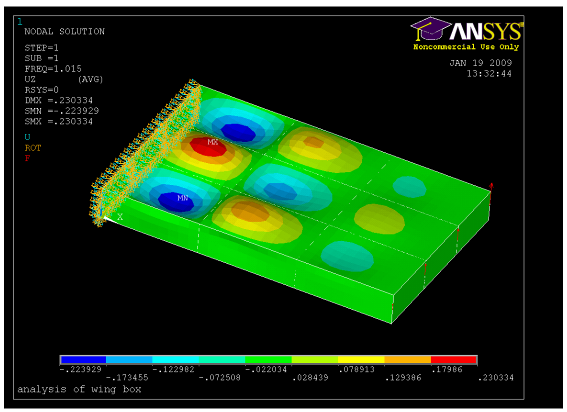
- Lamination parameter-based method
  - Local level optimization



1st buckling mode for discrete optimal design

Panel no.	V_1	V_2	V_3	Buckling load factor
10	1.1527	1.0080	1.1901	
11	1.1685	1.0089	1.1774	
12	1.2319	1.0143	1.0736	
13	1.2319	1.0143	1.0736	
14	1.1643	1.0086	1.1816	
15	1.1457	1.0076	1.1934	
16	1.1474	1.0077	1.1927	1.015
17	1.1623	1.0086	1.1833	
18	1.2266	1.0135	1.0856	

Stacking sequence

10 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2/45/90_2/\text{-}45/90_4/0_2/45/0_2/\text{-}45/(\pm 45)_6/0/90_2/0/\pm 45]_s$

11 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2/45/90_2/\text{-}45/90_4/0_2/45/0_2/\text{-}45/(\pm 45)_5]_s$

12 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2]_s$

13 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2]_s$

14 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2/45/90_2/\text{-}45/90_4/0_2/45/0_2/\text{-}45/(\pm 45)_6]_s$

15 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2/45/90_2/\text{-}45/90_4/0_2/45/0_2/\text{-}45/(\pm 45)_6/0/90_2/0/\pm 45/90/0/\pm 45]_s$

16 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2/45/90_2/\text{-}45/90_4/0_2/45/0_2/\text{-}45/(\pm 45)_6/0/90_2/0/\pm 45/90/\pm 45]_s$

17 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2/45/90_2/\text{-}45/90_4/0_2/45/0_2/\text{-}45/(\pm 45)_6/0]_s$

18 $[(\pm 45)_2/0/(0_2/45/0/\text{-}45)_2/90/0_2/45/0_2/\text{-}45/(0/90)_2/90/0/90_2/45/90_2/\text{-}45/0_2/90/0_3/90_2/(90/0)_2/\pm 45]_s$

Discussion for smeared stiffness-based method

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- Advantage
  - Avoiding stack optimization at local (bottom) level by performing quicker post-processing function of ply shuffling.
- Disadvantage
  - Ply shuffling can lead to a (slight) violation of buckling constraint, particularly when shuffling is performed many times in blending procedure as described earlier.

Discussion for lamination parameters-based method

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- Advantage
  - No need to check whether strength or buckling constraints have been violated as long as lamination parameters obtained after local level optimization match the given lamination parameter values that came the top level optimization.

- Disadvantage
  - Difficulty in matching lamination parameters from the top level while considering ply continuity for a small number of plies in the laminated composite structure (as in an example shown earlier).

Conclusions

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- Smeared stiffness-based method
  - Uses an assumption of homogenous laminates in the top level optimization.
  - A ply shuffling technique HyperShuffle used at the local level without a need for solving an optimization problem.
  - Manufacturing and general composite layup requirements considered in the ply shuffling procedure.
  - A manual adjustments (adding layers) can prevent buckling if it occurs.
- Lamination parameter-based method
  - No numerical simulation within the local level optimization, only dealing with the lamination parameter values calculated by simple formulae.
  - No buckling analysis needs if the target values of the lamination parameters, passed from the top level, were kept.
  - Difficulty in matching the lamination parameters from the top level while considering ply continuity for the laminated structure with a small number of plies.