Symmetry in Complex Systems

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http://nlds.sdsu.edu
Department of Mathematics
San Diego State University

Computational Science Research Center, 2009

Outline

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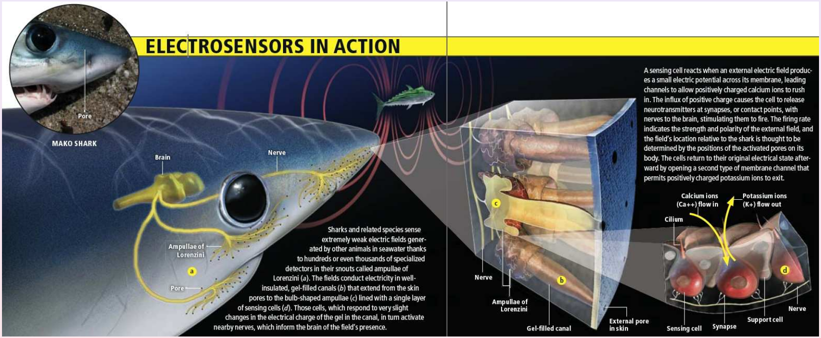
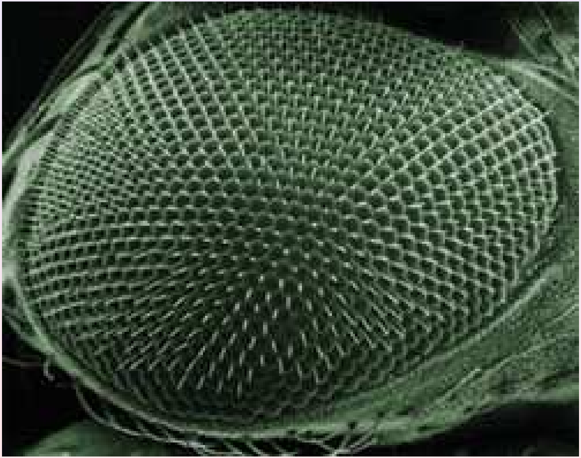
Smell: Meet Yogi

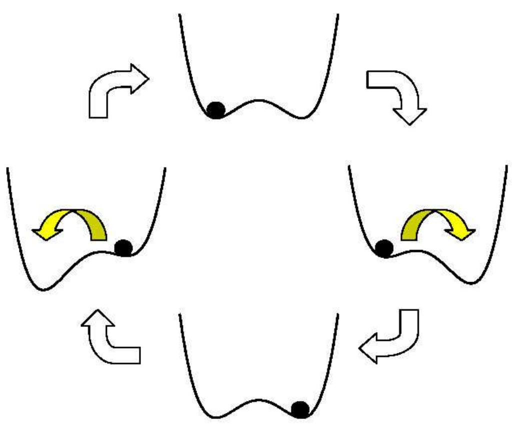
Electrosensors: Sharks



Sight: Fly Eye



Bistability



PSD Detection

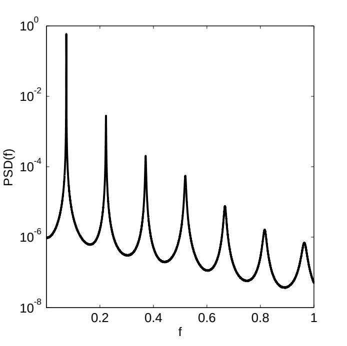


Figure: Without External Signal

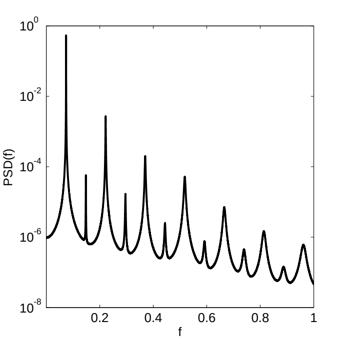
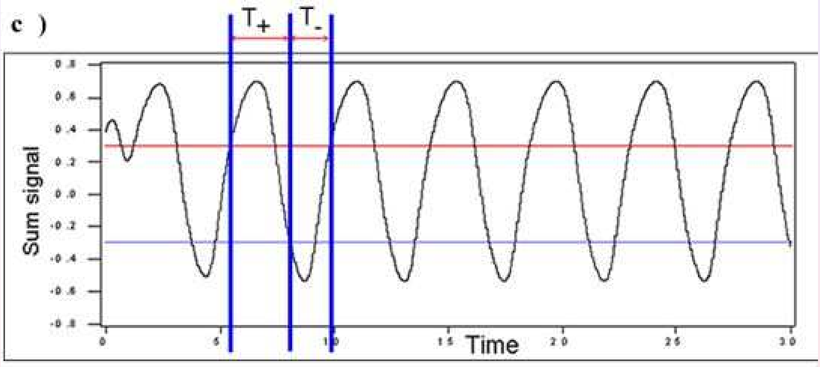


Figure: With External Signal

Residence Times Detection



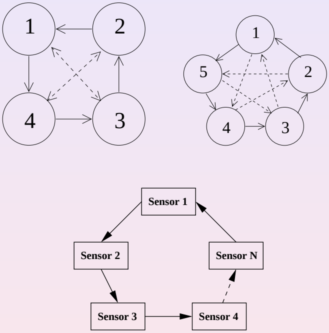


Figure: Complex Systems

Definition

"A complex system is a system composed of interconnected parts that as a whole exhibit one or more properties (behavior among the possible properties) not obvious from the properties of the individual parts." (Wikipedia).

General Approach

A general approach for the analysis of complex systems has been to derive a detailed model of the individual parts, connect the parts and note that the system contains some sort of symmetry, then attempt to exploit this symmetry in order to simplify numerical computations.

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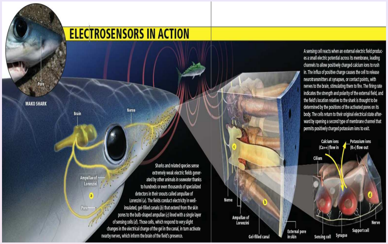
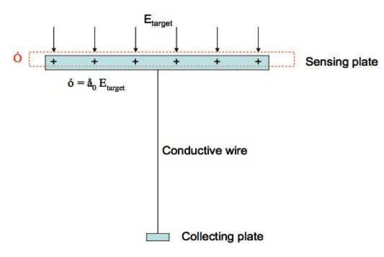




Model Equations

$$\tau \dot{x}_i = a x_i - b x_i^3 + \lambda(x_i - x_{i+1}) + \varepsilon, \qquad i = 1, 2, \ldots, N \mod N$$

One-Parameter Bifurcation Analysis

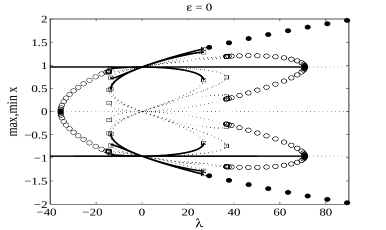


Figure: Bifurcation Diagram

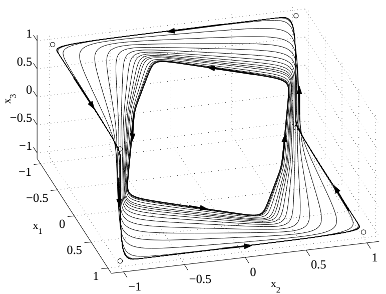


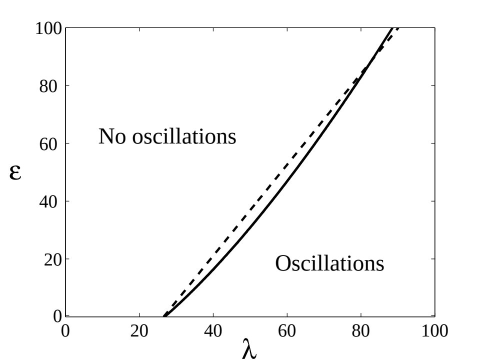
Figure: Phase-Space

Two-Parameter Bifurcation Analysis

Critical Coupling

Assuming ε is a dc electric-field.

$$\lambda_c = \frac{a}{2} + \frac{3}{2}\frac{\varepsilon}{1+\sqrt{\frac{2a}{b}}}$$



Time-Periodic Target Signal

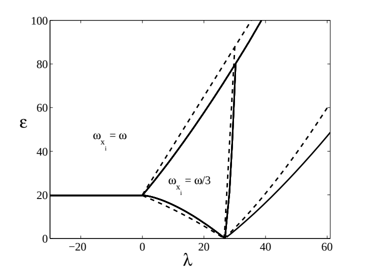
Model Equations

$$\tau \dot{x}_i = a x_i - b x_i^3 + \lambda(x_i - x_{i+1}) + \varepsilon \sin(\omega t).$$

Asymptotic Analysis via Airy Functions

$$\varepsilon_c = \frac{F_0 - \lambda e}{1 + \lambda f}$$

$$\varepsilon_c = \frac{2(F_0-\lambda e)(1+\lambda f)+k_1^3(1-\lambda f)\Omega^2}{2(1+\lambda f)^2} \pm \frac{\sqrt{k_1^3\Omega^2(8(F_0-\lambda e)(1+\lambda f)-k_1^3\Omega^2(1-\lambda f))}}{2(1+\lambda f)^2}$$



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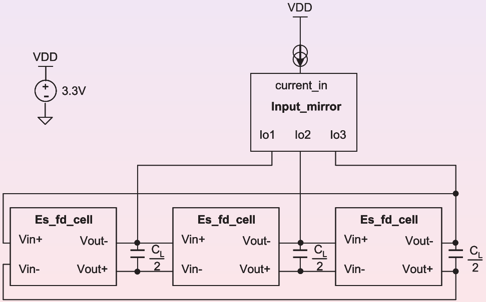
Hardware Dynamics

$$C_L \dot{V}_i = -gV_i + I_s \tanh[c_s V_i] + I_c \tanh[c_c V_{i-1}] - \varepsilon$$



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One-Parameter Bifurcation Diagrams

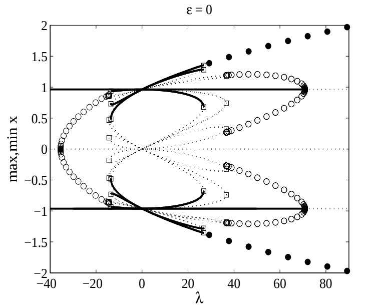


Figure: Theory

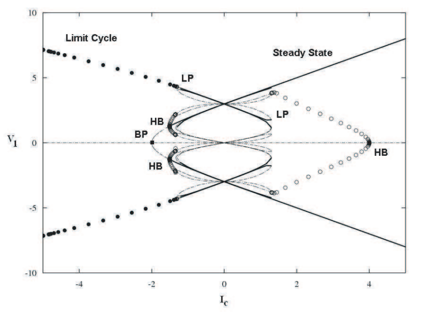
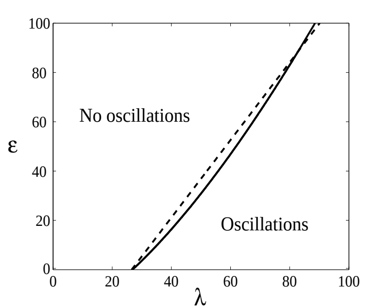


Figure: Experiment

Two-Parameter Bifurcation Diagrams



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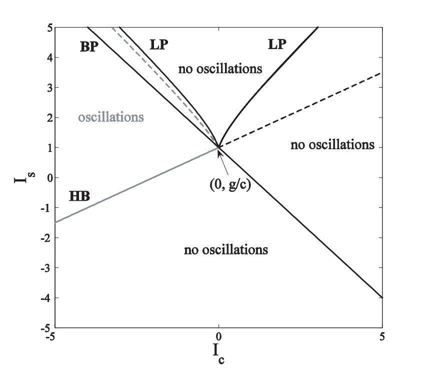


Figure: Experiment

Time-Series Oscillations

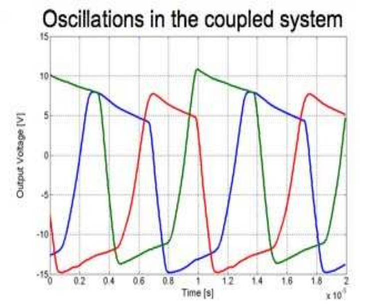


Figure: Theory

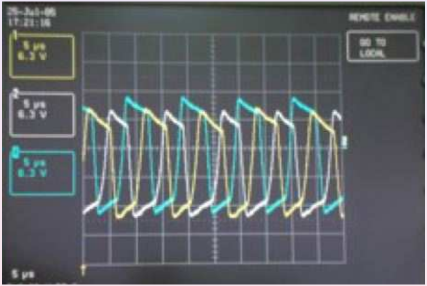
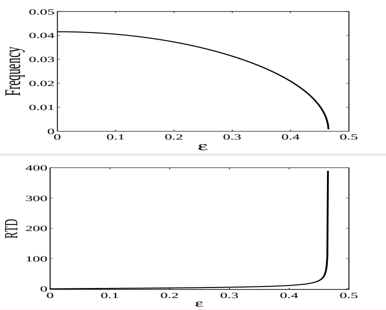


Figure: Experiment

Frequency and RTD Response



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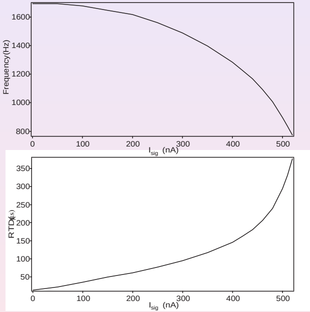
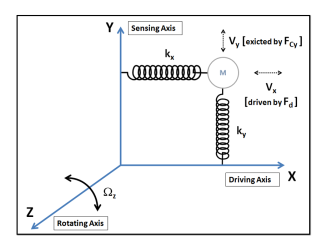


Figure: Experiment

Outline



Equations of Motion

$$
\begin{aligned}
m\ddot{x}_j + c\dot{x}_j + \kappa x_j + \mu x_j^3 &= A_d \sin w_d t + 2m\Omega_z \dot{y}_j + \sum_{k \to j} c_{jk} h(x_j, x_k) \\
m\ddot{y}_j + c\dot{y}_j + \kappa y_j + \mu y_j^3 &= -2m\Omega_z \dot{x}_j,
\end{aligned}
$$

Outline

Van der Pol Transformations

$$\begin{aligned} u_{1j} &= x_j \cos w\tau - \dot{x}_j/w \sin w\tau, \\ u_{2j} &= -x_j \sin w\tau - \dot{x}_j/w \cos w\tau, \\ u_{3j} &= y_j \cos w\tau - \dot{y}_j/w \sin w\tau, \\ u_{4j} &= -y_j \sin w\tau - \dot{y}_j/w \cos w\tau \end{aligned}$$

where $w_0 = \sqrt{k/m}$ and $\tau = w_0 t$.

Averaging

$$\frac{du}{d\tau} = \varepsilon G(u,\tau) \quad \xrightarrow{\int_0^T} \quad \frac{du}{d\tau} = \varepsilon \bar{G}(u)$$

Amplitude Equations

$$\dot{A}_j = \frac{\varepsilon}{2w}\left[-\rho A_j + \frac{3}{4}\mu|A_j|^2 A_j i + w\gamma B_j - A_d i - \lambda(A_{j+1} - 2A_j + A_{j-1})i\right]$$

$$\dot{B}_j = \frac{\varepsilon}{2w}\left[-\rho B_j + \frac{3}{4}\mu|B_j|^2 B_j i - w\gamma A_j\right].$$

where $\rho = cw_0 w + \Delta i$, $\varepsilon\Delta = w^2 - 1$, $\varepsilon = 1/(mw_0^2)$.

Outline

Definitions

Symmetry Set of transformations that leave an object unchanged.

Invariance $f : \mathbf{C}^3 \rightarrow \mathbf{R}$ is invariant under a group Γ if $f(\gamma x) = f(x)$, for all $\gamma \in \Gamma$.

Equivariance $\frac{dx}{dt} = f(x, \lambda)$, $x \in R^n$, has Γ-symmetry if $f(\gamma x, \lambda) = \gamma f(x, \lambda)$, for all $\gamma \in \Gamma$, where $\Gamma \subset O(n)$.

Isotropy Subgroup The amount of symmetry of a solution x is given by: $\Sigma_x = \{\gamma \in \Gamma \ : \gamma x = x\}$.

Fixed Subspace $\text{Fix}(\Sigma) = \{x \in R^n \ : \sigma x = x, \ \forall \sigma \in \Sigma\}$.

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Let

1. $z = (z_d, z_s, A_d)$ be the state-variable of a single gyroscope, where $z_d = A$ and $z_s = B$.
2. $\Gamma \times T^1$ be the group of symmetries of the network.
3. T^1 act on $\mathbf{C}^3$ in the standard way:

$$\theta \dot{z} = (e^{i\theta} z_d, e^{i\theta} z_s, e^{i\theta} A_d).$$

Theorem

(a) A Hilbert basis for the T^1-invariant polynomials on $\mathbf{C}^3$ is:

$$u_1 = z_d\bar{z}_d, \quad u_2 = z_s\bar{z}_s, \quad u_3 = A_d\bar{A}_d,$$
$$v_1 = z_d\bar{z}_s, \ \bar{v}_1, \quad v_2 = z_d\bar{A}_d, \ \bar{v}_2, \quad v_3 = z_s\bar{A}_d, \ \bar{v}_3.$$

(b) The T^1-equivariant are generated by

$$\begin{bmatrix} z_d \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} z_s \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} A_d \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ z_d \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ z_s \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ A_d \\ 0 \end{bmatrix},$$

$$\begin{bmatrix} 0 \\ 0 \\ z_d \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ z_s \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ A_d \end{bmatrix}.$$

Normal Forms

$$\frac{dz}{dt} = (g_1(z, A_d), g_2(z, A_d), 0),$$

where $g_1 = p_1 z_d + p_2 z_s + p_3 A_d$, $g_2 = q_1 z_d + q_2 z_s + q_3 A_d$,, $p_i' s$ and $q_i' s$ are complex-valued functions of u_1, u_2, u_3, v_1, v_2, v_3.

Network Dynamics

$$\begin{aligned}
\dot{z}_{dj} &= g_1(z_{dj}, z_{sj}, A_d) + & h(z_{d,j+1} - z_{dj}, z_{dj} - z_{d,j-1}) + \\
& & h(z_{s,j+1} - z_{sj}, z_{sj} - z_{s,j-1}) \\
\dot{z}_{sj} &= g_2(z_{dj}, z_{sj}, A_d) + & h(z_{d,j+1} - z_{dj}, z_{dj} - z_{d,j-1}) + \\
& & h(z_{s,j+1} - z_{sj}, z_{sj} - z_{s,j-1}) \\
\dot{A}_d &= 0.
\end{aligned}$$

Global Symmetries

Let $\gamma \in \Gamma$. It can be shown that

$$\gamma \cdot (z_1, \ldots, z_n) = (z_{\gamma^{-1}(1)}, \ldots, z_{\gamma^{-1}(n)}).$$

Combining *local* and *global* symmetries:

$$(\zeta, e^{i\theta})(z_1, \ldots, z_n) = (e^{i\theta} z_{\zeta(1)}, \ldots, e^{i\theta} z_{\zeta(n)}), \quad \text{where } \zeta \in \mathbf{D}_n.$$

Predictions

Isotropy subgroup Σ	Fixed-Point Subspace Solution
$\mathbf{D}_m(k) = \langle\{\sigma^k, k\}\rangle$ $\mathbf{D}_m(k\sigma) = \langle\{\sigma^k, k\sigma\}\rangle$ $\mathbf{Z}_m = \langle\{\sigma^k\}\rangle$	In Phase In Phase In Phase
$\mathbf{D}_m(+-) = \langle\{\sigma^{k-1}k, 1), (k\sigma, -1)\}\rangle$ for m even $\mathbf{D}_m(--) = \langle\{\sigma^{k-1}k, 1), (k\sigma, -1)\}\rangle$ for k even	Standing Wave Standing Wave
$\mathbf{Z}_m(p) = \langle\{\sigma^k, w^{pk}\}\rangle$ where $p \in \{1, \ldots, [m/2]\}$	Traveling Wave

Table: Predictions for a $\mathbf{D}_n$-symmetric CINS network.

Lattice of ISotropy Subgroups

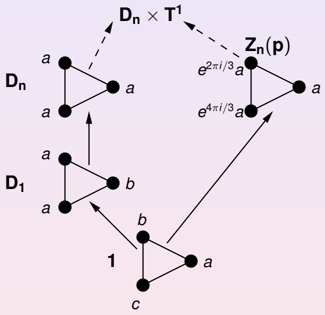
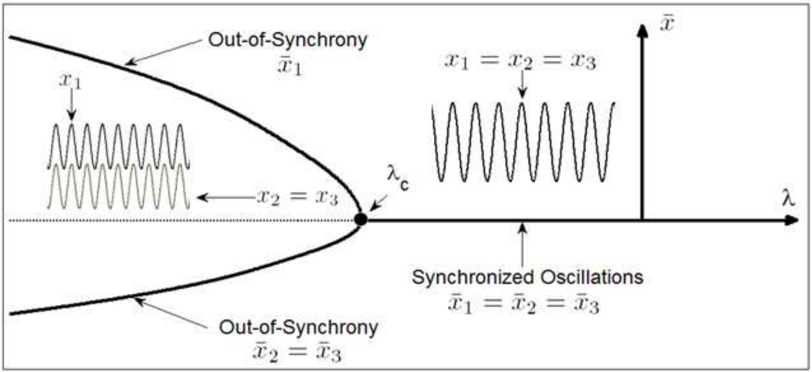


Figure: Lattice of isotropy groups and patterns of oscillation for a CINS ring with $\mathbf{D}_n$-symmetry, case study $n = 3$.

One-Parameter Bifurcation



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- Susan Guyler, M.S. Applied Math.
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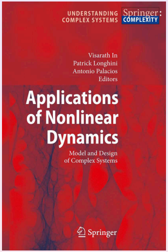

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Model and Design of Complex Systems

This edited book is aimed at interdisciplinary device-oriented applications of nonlinear science theory and methods in complex systems. In particular, applications directed to nonlinear phenomena with space and time characteristics. Examples include: complex networks of magnetic sensor systems, coupled nano-mechanical oscillators, nano-detectors, microscale devices, stochastic resonance in multi-dimensional chaotic systems, biosensors, and stochastic signal quantization. "Applications of nonlinear dynamics: model and design of complex systems" brings together the work of scientists and engineers that are applying ideas and methods from nonlinear dynamics to design and fabricate complex systems.... *more on http://springer.com/978-3-540-85631-3*

► Devoted specifically to the applications of complex nonlinear dynamic phenomena to real systems and device applications ► Applies new fields of nonlinear dynamics (e.g. stochastic resonance, experimental chaos and neural coding) to make devices operate more efficiently and cheaply ► Brings together researchers from physics, engineering, biology and chemistry ► Includes coherent treatise of the International Conference on Applications of Nonlinear Dynamics held at Poipu Beach, Koloa (Kauai), Hawaii, from September 24-27 in 2007

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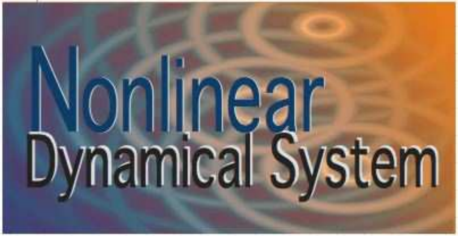

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