Faraday waves in Bose-Einstein condensates

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Collaborators/Links (I)

- Nonlinear Dynamical Systems @ SDSU: http://nlds.sdsu.edu/
  - Peter Blomgren (Numerical PDEs, image processing)
  - Ricardo Carretero (App. math., nonlinear lattices and waves)
  - Joe Mahaffy (Mathematical biology, delay differential equations)
  - Antonio Palacios (Applied mathematics, bifurcations, symmetries)
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  - Diana Verzi (Mathematical biology, Mathematical Physiology)
- Research Students involved in BECs/nonlinear waves
  - Manjun Ma (Postdoc).
  - Ron Caplan, Rafael Navarro, Jake Talley, Eunsil Baik (PhD, Comp. Sci.).
  - John Everts, Suchitra Jagdish (MS, Dyn. Syst.).
  - Recent departures: Mike Davis (2007), Chris Chong (2006) (MS, Dyn. Syst.).

Collaborators in Nonlinear Waves/Lattices, BECs (II)

- Solitons, Vortices and Vortex Lattices
  - Panos Kevrekidis = PGK (UMass)
  - Alex Nicolin = AIN (NBI)
  - Peter Engels (WSU)
  - David Hall (Amherst Coll.)
  - Brian Anderson (UoA)
  - W. Królikowski (CUDOS/ANU)
  - D. Frantzeskakis = DJF (Athens)
  - Boris Malomed = BAM (Tel Aviv)
  - Faustino Palmero (Sevilla)
  - Jesus Cuevas (Sevilla)
  - Mason Porter (Caltech)
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  - Keith Promislow (SFU/MSU).
  - Lincoln Carr (Col. Sch. Mines)
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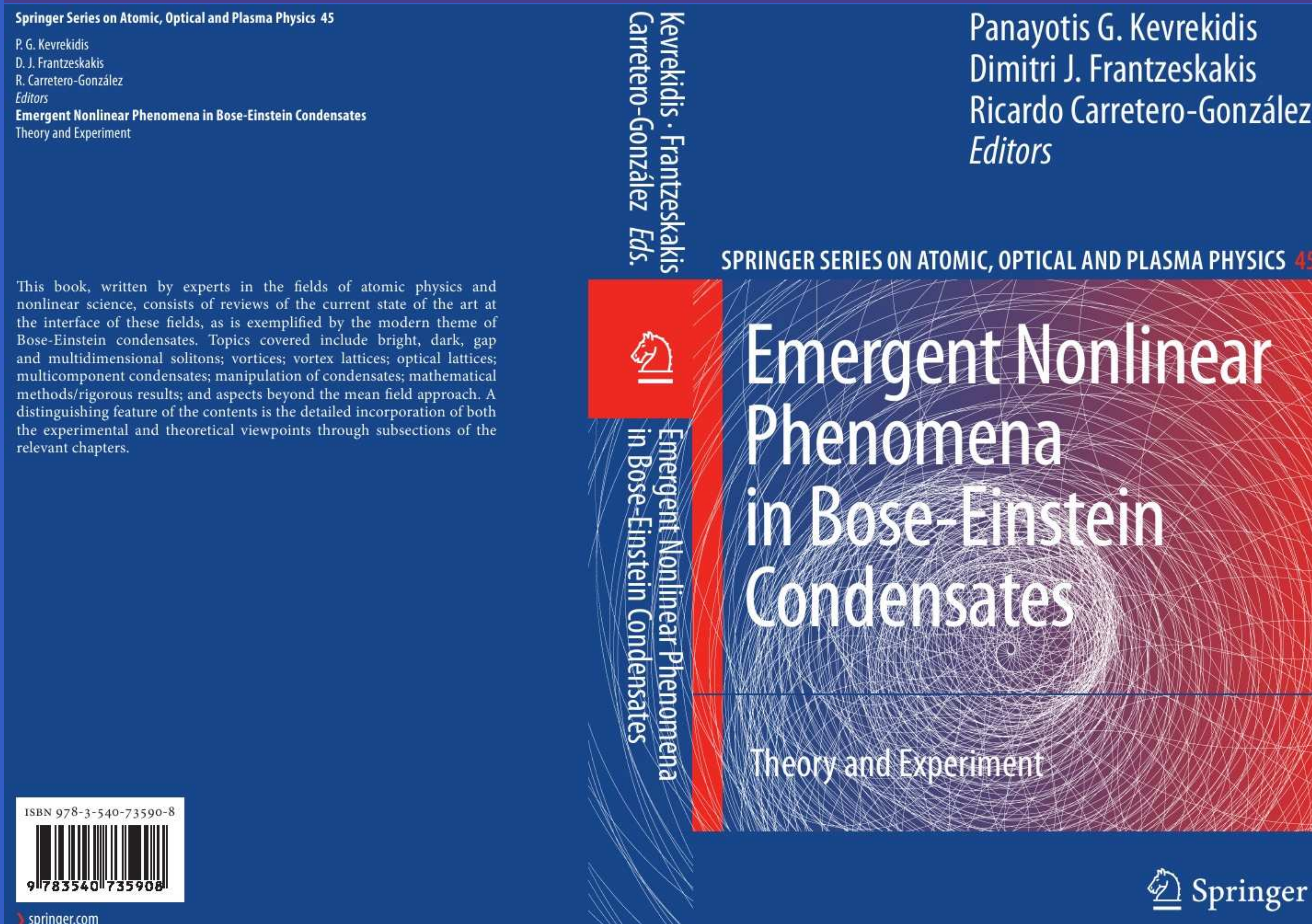
  - Enam Hoq (WNEC)
  - Nathan Kutz (UoW)
  - Jared Bronski (UI-UC)
  - Yannis Kevrekidis (Princeton)
  - Dimitri Maroudas (UMass)
  - Greg Herring (UMass)
  - George Theocharis (Athens)
  - Hector Nistazakis (Athens)
  - Alan Bishop (LANL)
  - Hadi Susanto (UMass)
  - Yaroslav Kartashov (ICFO)
  - Lluis Torner (ICFO)
  - Victor Vysloukh (UA, Mexico)

New Book — BECs: Theory and Experiment.



Road map

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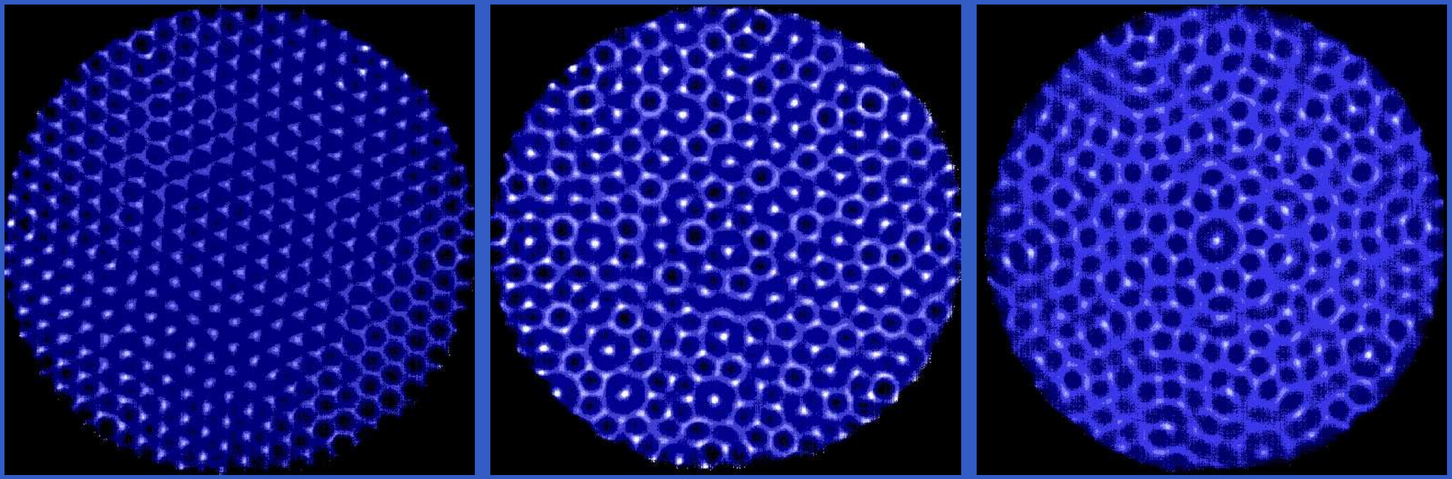
  - full 3D: numerics vs. experiment
- Outlook & to do's

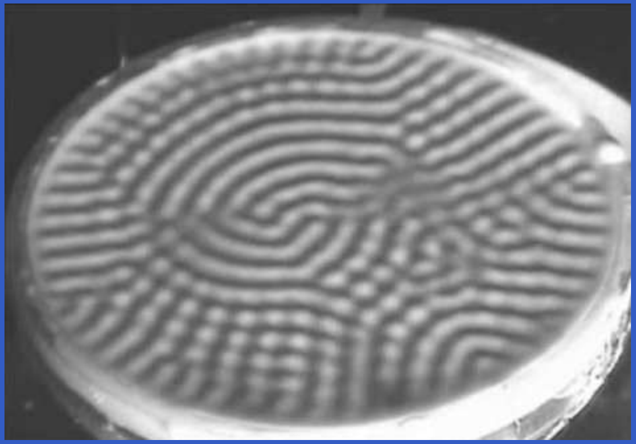
Faraday patterns: prelims

- Some of the oldest pattern formation phenomena
- M. Faraday, Philos. Trans. R. Soc. London **121**, 299 (1831).
- Dynamics of fluids in contact with vibrating surfaces.



- Recent experiments of Faraday waves in non-Newtonian fluids (cornstarch): F. Merkt, R.D. Deegan, D. Goldman, E. Rericha, and H.L. Swinney Persistent holes in a fluid, Phys. Rev. Lett., **92** 184501 (2004).

[movie]



Bose-Einstein condensates (BEC)

1925 Bose & Einstein predicted that a gas at very low temperature undergoes quantum “freezing”.

- $T \downarrow \Rightarrow$ vel. $\downarrow \Rightarrow$ de Broglie: $\lambda = h/p \Rightarrow \lambda \uparrow \Rightarrow$ coherence $\Rightarrow$ all atoms enter the SAME quantum state
- BEC is to matter what laser is to light (coherent)
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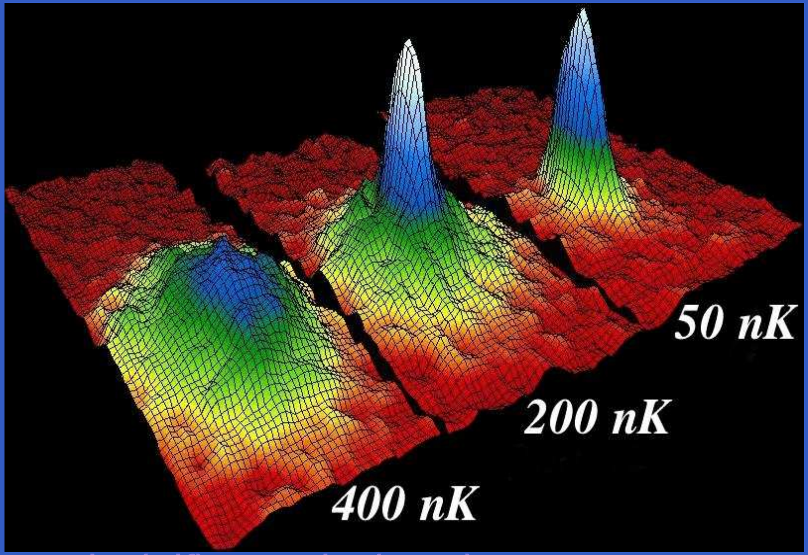
1995 Cornell + Wieman + others @ JILA + NIST + UC) achieved temperatures < 1/170$\overline{M}$ ^{o}K to produce a BEC (rubidium) for the 1st time.



movie (gif), movie (mov)

2001 Cornell + Ketterle + Wieman got the Nobel Prize in Physics for BECs.

2003 Abrikosov + Ginzburg + Leggett got the Nobel Prize in Physics for superconductors and superfluids.

2007 BEC count: Some 60 different BEC experiments.

BECs close to $T = 0$: Many-body Hamiltonian $\rightarrow$ GPE:

- Many-body Hamiltonian for interacting bosons confined in $V_{\rm ext}(\mathbf{r})$

$$\hat{H} = \int d\mathbf{r} \hat{\Psi}^{\dagger}(\mathbf{r}) \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{r}) \right] \hat{\Psi}(\mathbf{r}) + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \hat{\Psi}^{\dagger}(\mathbf{r}) \hat{\Psi}^{\dagger}(\mathbf{r}') V_{\rm 2B}(\mathbf{r}-\mathbf{r}') \hat{\Psi}(\mathbf{r}') \hat{\Psi}(\mathbf{r}),$$

  - $V_{\rm ext}(\mathbf{r})$: external potential (this is the shaker)
  - $\hat{\Psi}(\mathbf{r})$ and $\hat{\Psi}^{\dagger}(\mathbf{r})$ annihilation and creation operators
  - m : mass of bosons
  - $V_{\rm 2B}(\mathbf{r} - \mathbf{r}')$: two-body interatomic potential.

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- Heisenberg equation $i\hbar(\partial \hat{\Psi}/\partial t) = [\hat{\Psi}, \hat{H}]$ gives the dynamics:

$$i\hbar \frac{\partial}{\partial t} \hat{\Psi}(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\rm ext}(\mathbf{r}) + \int d\mathbf{r}' \hat{\Psi}^\dagger(\mathbf{r}', t) V_{\rm 2B}(\mathbf{r}' - \mathbf{r}) \hat{\Psi}(\mathbf{r}', t) \right] \hat{\Psi}.$$

- Binary hard-sphere collisions only: $V_{\rm 2B}(\mathbf{r}' - \mathbf{r}) = g\delta(\mathbf{r}' - \mathbf{r}) \rightarrow$ GPE:

$$i\hbar \frac{\partial}{\partial t} \Psi(\mathbf{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V_{\rm ext}(\mathbf{r}) + g|\Psi(\mathbf{r}, t)|^2 \right] \Psi(\mathbf{r}, t).$$

Gross-Pitaevskii Eq.:

- Close to $T = 0$ BEC $\rightarrow$ Gross-Pitaevskii Eq.:

$$i\hbar\frac{\partial\psi}{\partial t} = \left[-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{r}) + gN\left|\psi\right|^2\right]\psi, \quad (1)$$

  - $\psi(x,y,z,t)$: BEC wavefunction (normalized to unity),
  - $|\psi(x,y,z,t)|^2$: atom density,
  - N: number of atoms,
  - nonlinear coeff: $g = 4\pi\hbar^2 a_s/m$,
  - a_s scattering length:
    - a_s>0 : repulsive : (^{23}Na, ^{87}Rb, H, ^{4}He, ^{85}Rb)
      $\rightarrow$ [dark solitons, vortices]
    - a_s<0 : attractive : (^{7}Li, ^{85}Rb)
      $\rightarrow$ [bright solitons, Bose Nova]
  - $V_{\rm ext}(\mathbf{r}) = V_{\rm ext}(x,y,z)$: external confining potential

BECs inside magnetic trap:

- External (confining) magnetic potential:

$$V_{\text{ext}}(r,z) \quad = \quad \frac{1}{2}m\omega_r^2 r^2 + \frac{1}{2}m\omega_z^2 z^2,$$

  - $r^2 = x^2 + y^2$: transverse dimension
  - $\omega_r = \omega_r(t)$: time dependent transverse trapping
  - ω_z: the longitudinal trapping
  - $\omega_z \ll \omega_r$: Quasi-1D BEC (cigar shaped).

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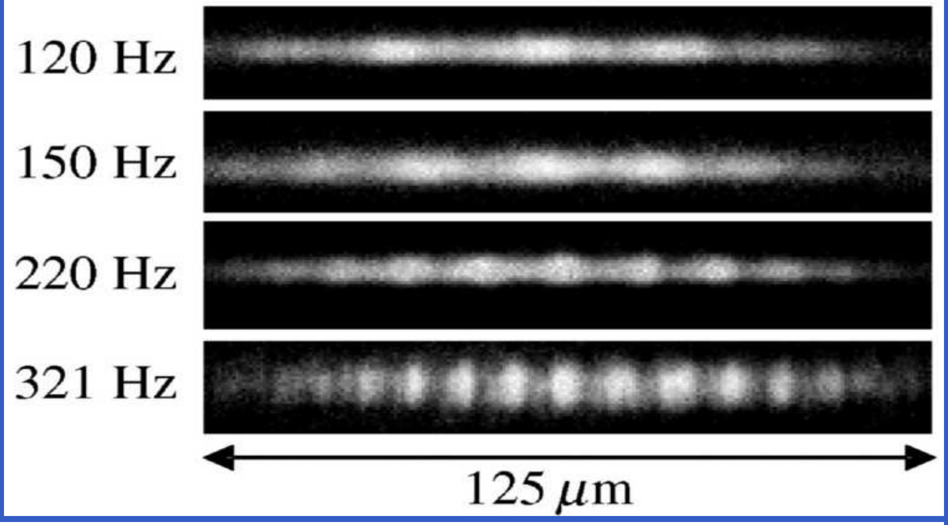
- Let us shake the condensate:
  - Periodically modulate the transverse confinement

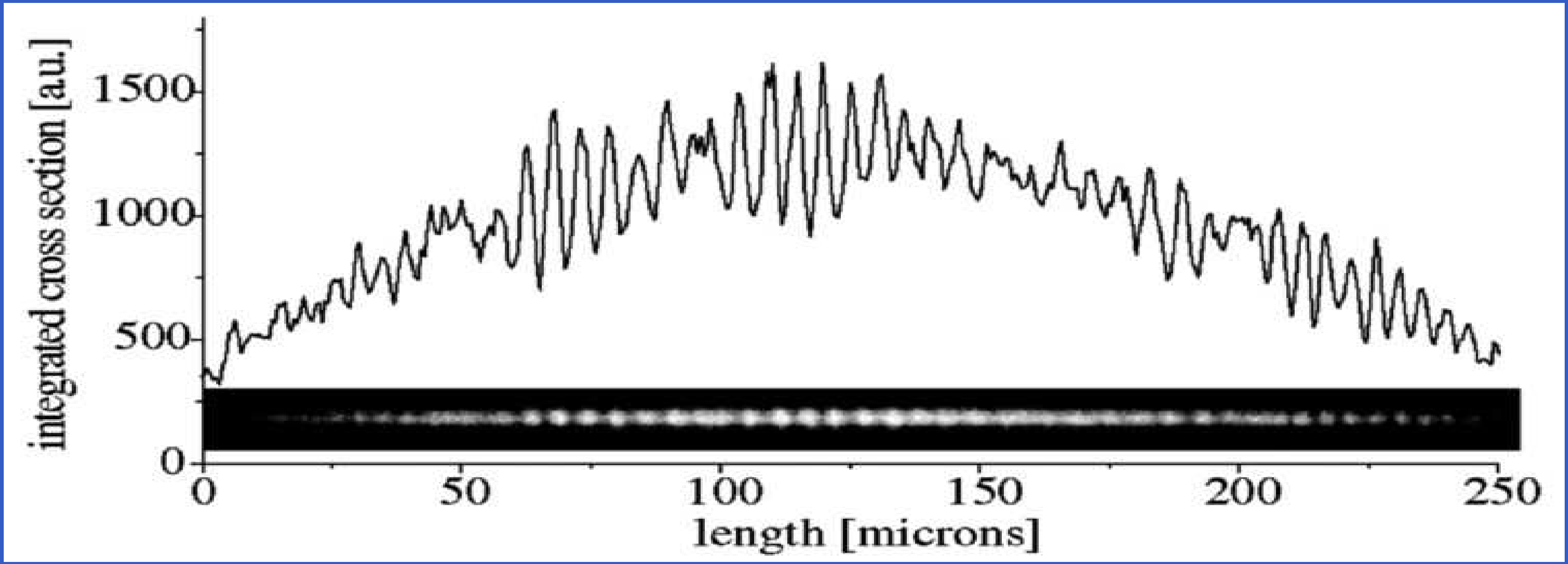
$$\omega_r(t) = \omega_{r,0} \cdot [1 + \epsilon \sin(\omega t)]$$

  - ω is the frequency of the driver (shaker)
  - As in a normal fluid $\Rightarrow$ Faraday patterns!

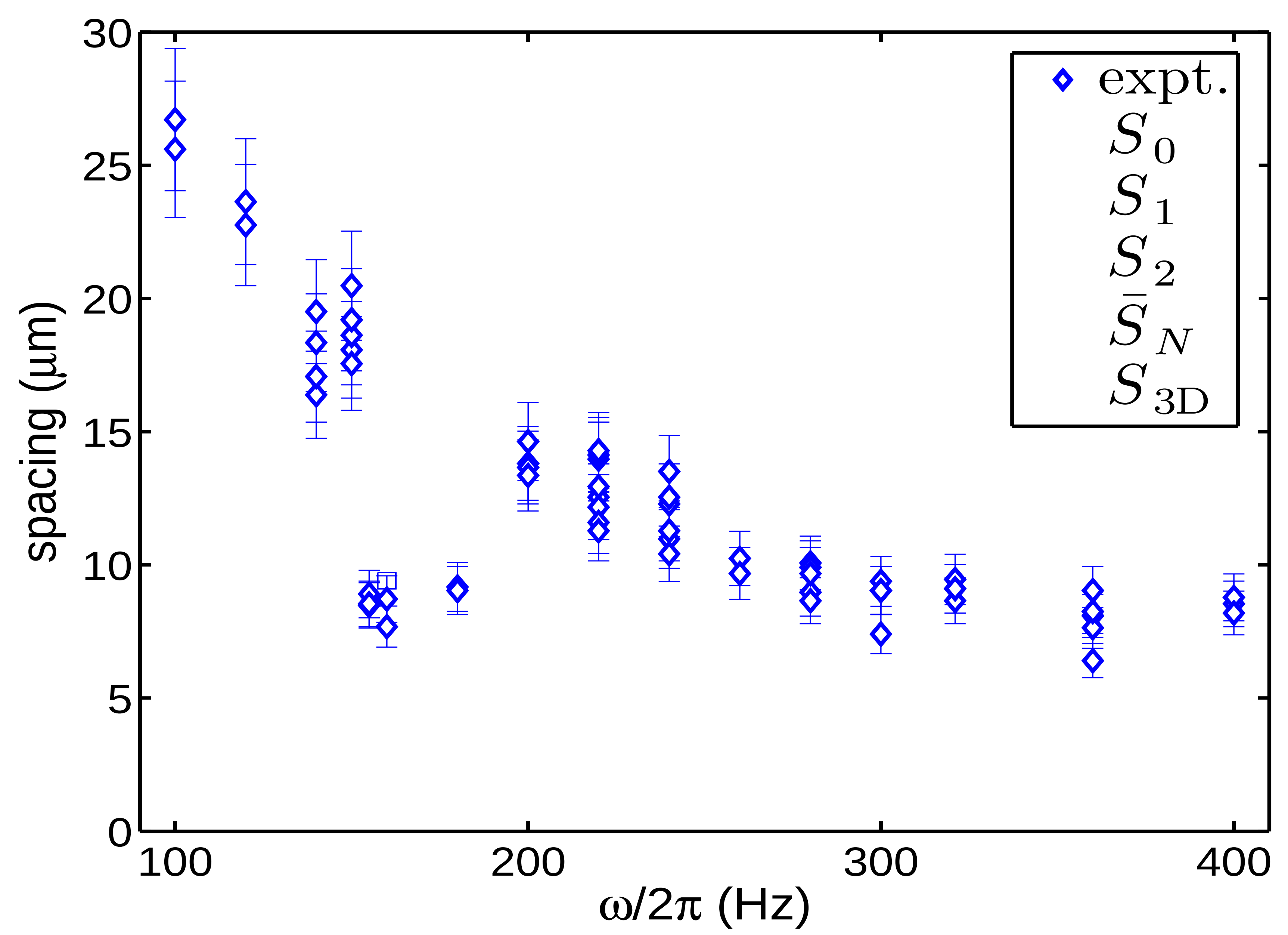
Faraday waves: BEC Experiment

- Engels+Atherton+Hoefer, PRL **98**, 095301 (2007)
- Experimental setup:
  - $N = 500,000$ ^{87}Rb atoms
  - $(\omega_z, \omega_r)/(2\pi) = (7, 160.5)$ Hz.
  - $\omega/(2\pi) \in [100, 400]$ Hz.
  - $\epsilon \in [4\%, 40\%]$, $t \in [0, 30]$ ms.



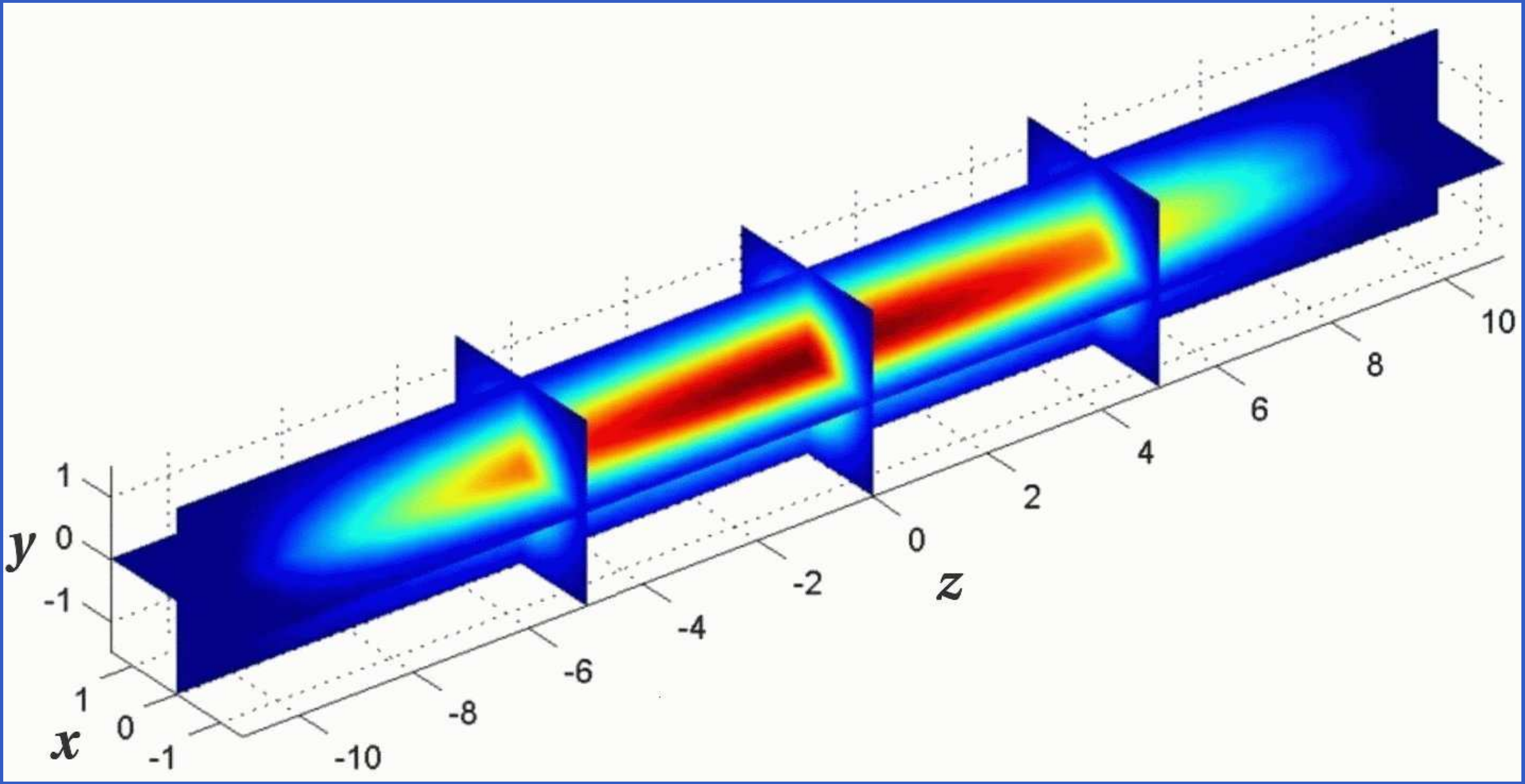


BEC Faraday Exp.: spacing vs. driving frequency



Strong transverse confinement: 3D $\rightarrow$ 1D

- If $\omega_r \equiv \omega_x = \omega_y \gg \omega_z$ the BEC behaves effectively 1D:



- Thus the 3D GPE $\rightarrow$ 1D GPE

Longitudinal dyn. → non-polynomial Schrödinger Eq:

- Separate radial (transverse) and longitudinal dynamics:

$$\psi(\mathbf{r},t) \;=\; \phi(r,t;\sigma(z,t)) \;\cdot\; f(z,t),$$

  - ϕ is taken at its ground state (Gaussian, mmm... not entirely true):

$$\phi(r,t;\sigma(z,t)) = \frac{e^{-r^2/2\sigma^2}}{\pi^{1/2}\,\sigma}$$

    where $\sigma = \sigma(z,t)$ is the z- and t-dependent transverse width.
  - VA [Salanasnich *et al.*, PRA **65**, 043614 (2002)] → NPSE:

$$\left\{\begin{array}{lcl} \sigma^2 &=& \dfrac{\hbar}{m\omega_r}\sqrt{1+2a_s N|f|^2} \\ i\hbar\dfrac{\partial f}{\partial t} &=& \left[-\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial z^2} + \hbar\omega_r \dfrac{1+3\,a_s N\,|f|^2}{\sqrt{1+2\,a_s N\,|f|^2}}\right] f. \end{array}\right. \tag{1}$$

- 2nd eq. (NPSE) does not depend explicitely on σ.
- Remember: $\omega_r(t) = \omega_{r,0}\cdot[1+\epsilon\sin(\omega t)]$ where ω is the driving freq.

Modulational Instability (MI) of the NPSE:

- Assume, for now, that $\omega_z = 0$ (i.e., homogeneous space in z). Latter we will relax this by appropriate averaging.

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- NPSE homogeneous state:

$$f_0(t) = A\exp\left[-ic\left(t - \epsilon\frac{\cos(\omega t)}{\omega}\right)\right],$$

where $c = \frac{\omega_{r,0}\left(1+3\,a_s N A^2\right)}{\sqrt{1+2\,a_s N A^2}}$, $A = \sqrt{1/2L}$, and domain=$[-L : L]$.

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- Modulational Instability (MI) for $f_0(t)$:

$$f(t) = f_0(t)\left\{1 + [u(t) + iv(t)]\cos(kz)\right\}.$$

where k is the wave number of the modulation

- MI: find the most unstable k

MI of the NPSE ⇒ Mathieu equation

- NPSE (after linearizing) ⇒ Mathieu Eq.:

$$\frac{d^2u}{d\tau^2} + \left[a(k,\omega) + b(k,\omega)\sin(2\tau)\right]u = 0.$$

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- Stability and Floquet-Bloch Theorem:
  - $u(\tau) = e^{i\mu\tau} g(\tau) = e^{i\mu_1\tau} e^{-\mu_2\tau} g(\tau)$
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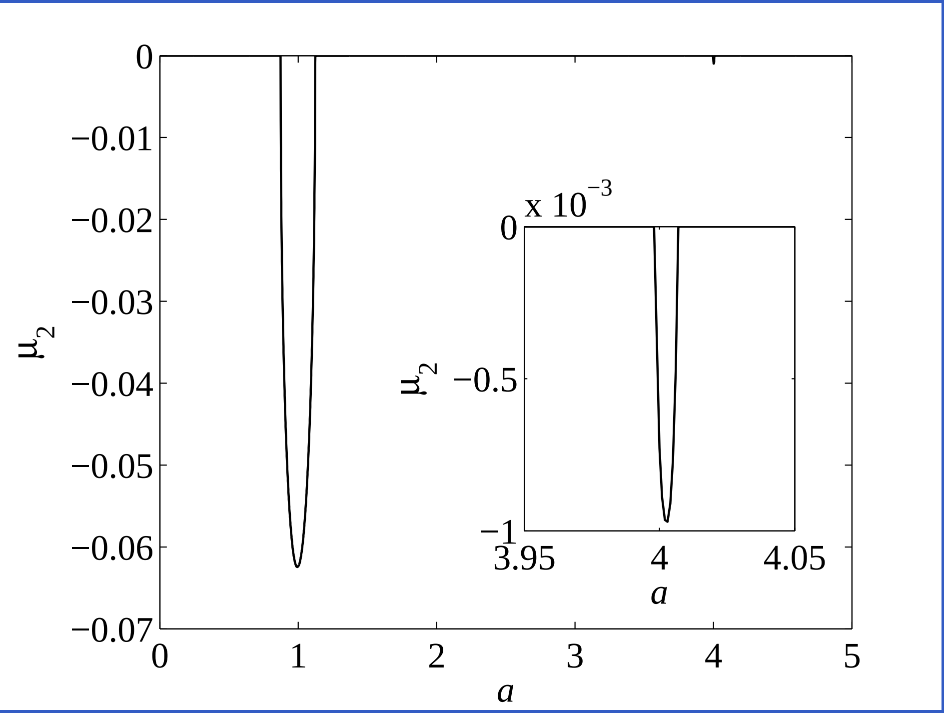

- Instability regions:
  Most unstable is for $a \approx 1$
  (this is a life safer, see next slide)

MI: most unstable mode for the Mathieu Eq.:

- Mathieu Eq. Coefficients:

$$a(k,\omega) = \frac{k^2}{2\hbar\pi m^2\omega^2}\,\frac{6\pi a_s^2\hbar^2 mN^2\omega_{r,0}+a_sN2\hbar^3k^2\pi\sqrt{L^2+La_sN}+2\hbar^3k^2\pi\sqrt{L^4+L^3a_sN}+2gLm^2N\omega_{r,0}}{a_sN\sqrt{L^2+La_sN}+\sqrt{L^4+L^3a_sN}},$$

$$b(k,\omega) = \frac{k^2\omega_{r,0}\epsilon N}{2\hbar\pi m\omega^2}\,\frac{2gmL+6a_s^2\hbar^2N\pi}{a_sN\sqrt{L^2+La_sN}+\sqrt{L^4+L^3a_sN}}.$$

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- Since k is of the order of microns $\Rightarrow k^4 \ll k^2$
- Then: $a \approx 1 \Rightarrow k \propto \omega$ (... sigh of relief ...)

Most unstable mode: $a(k,\omega) = 1 \rightarrow$ Faraday spacing

- The most unstable wave number is:

$$k = \alpha\omega, \quad \alpha \equiv \frac{m^{1/2}}{\omega_{r,0}^{1/2}\hbar^{1/2}}(2a_s\rho)^{-1/2}\,(1+2a_s\rho)^{3/4}\,(4+6a_s\rho)^{-1/2}, \qquad (2)$$

  where $\rho = N/2L$ is the density of the condensate.
- $\Rightarrow$ the predicted spacing is (for spatially *homogeneous*): $\mathcal{S}_0 = 2\pi/k$.

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- Consider slow spatial variations (small ω_z) w.r.t. the Faraday pattern. Use the Thomas-Fermi (TF) approx. (large atom number limit):

$$\rho(z) \approx 3\frac{L^2 - z^2}{4L^3},$$

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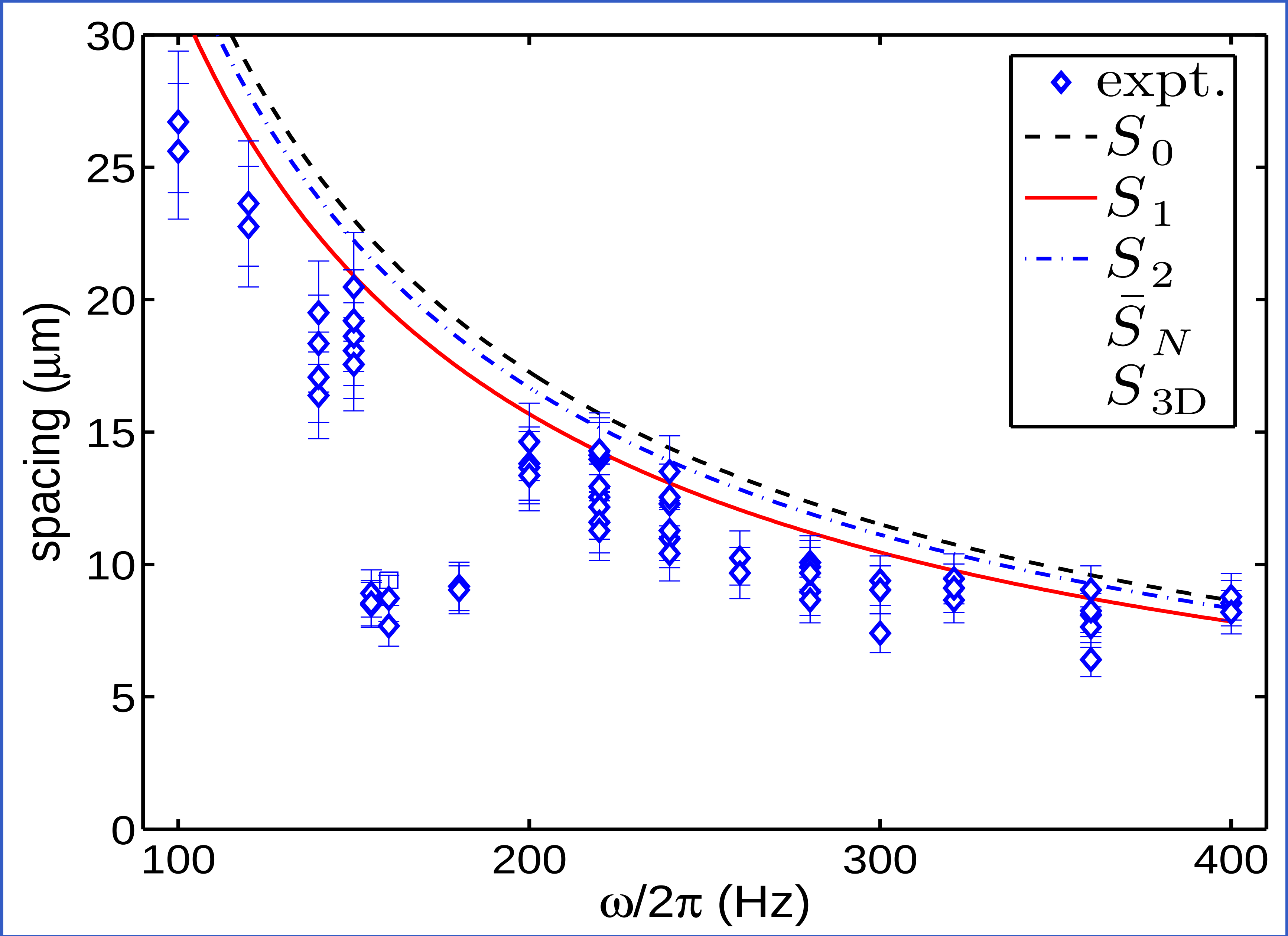
- Average spacing:
  - Averaging the k's:

$$\mathcal{S}_1 = 2\pi/\bar{k}, \quad \bar{k} = \frac{1}{2L}\int_{-L}^{L} k(z)dz$$

  - Averaging the spacings:

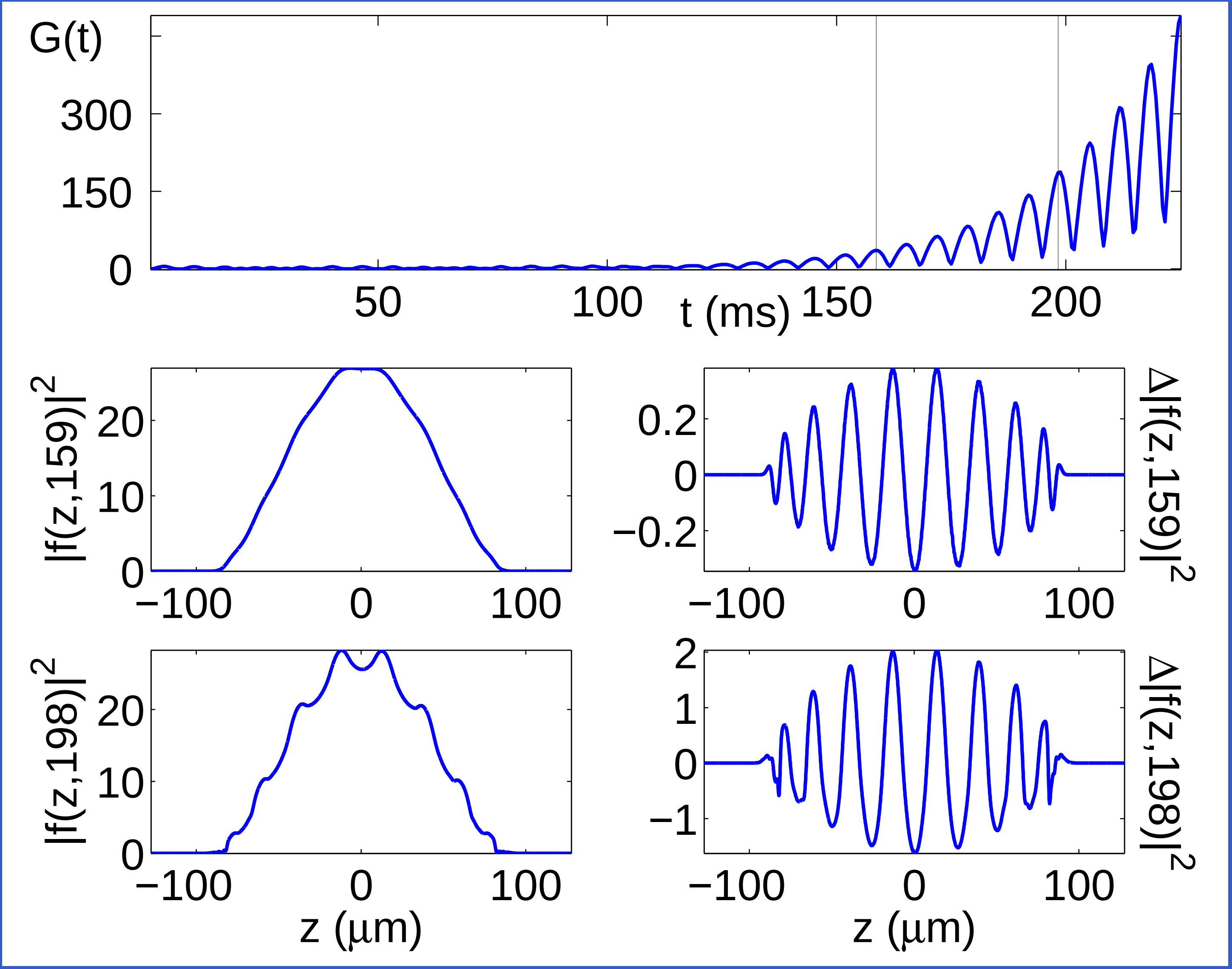
$$\mathcal{S}_2 = \frac{1}{2L}\int_{-L}^{L} 2\pi/k(z)dz,$$

Faraday waves: Experiment + Theory



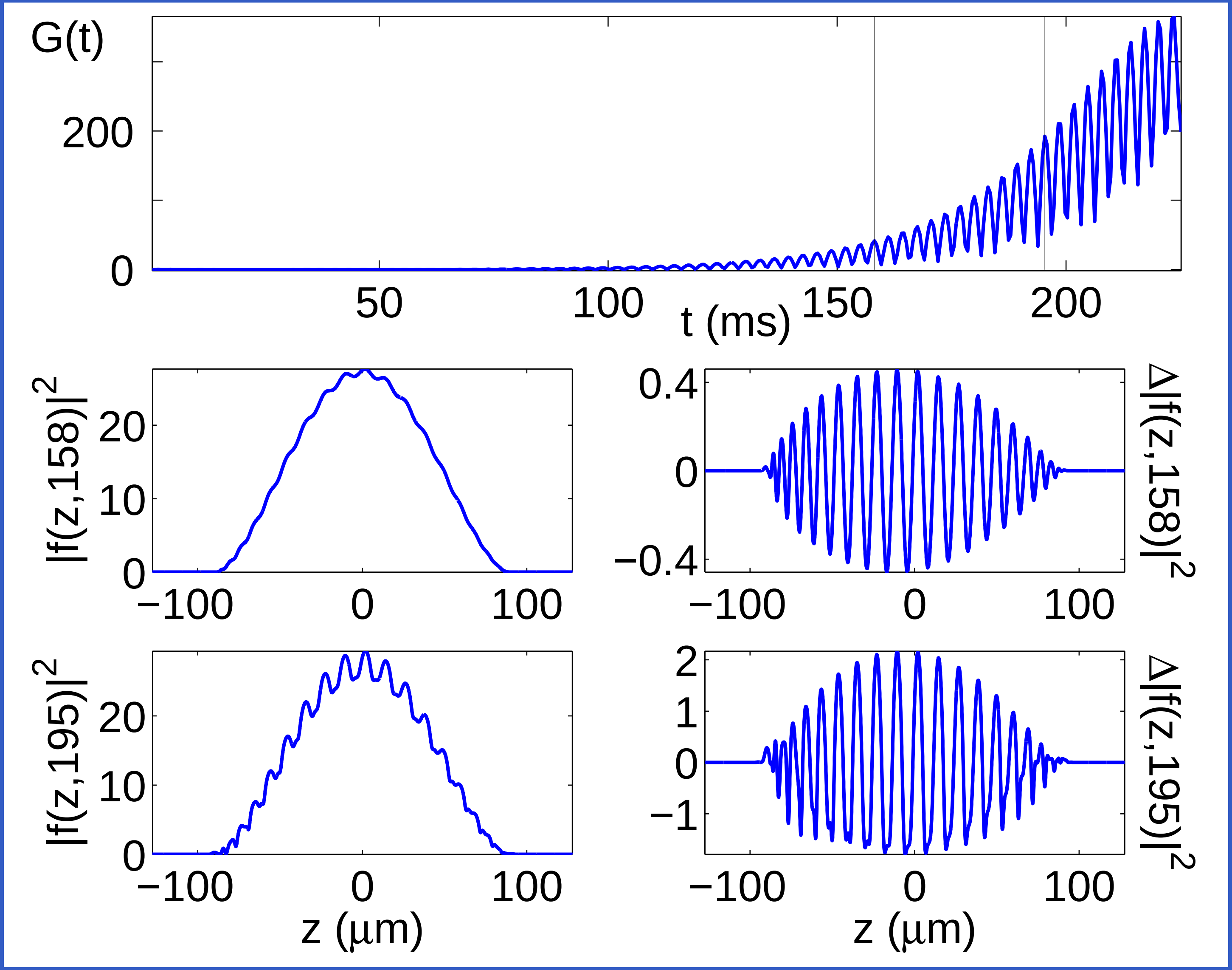
Faraday patterns: 1D NPSE Numerics

- 40% modulation with driving frequency $\omega/(2\pi) = 150$ Hz



Faraday patterns: 1D NPSE Numerics

- 20% modulation with driving frequency $\omega/(2\pi) = 321$ Hz



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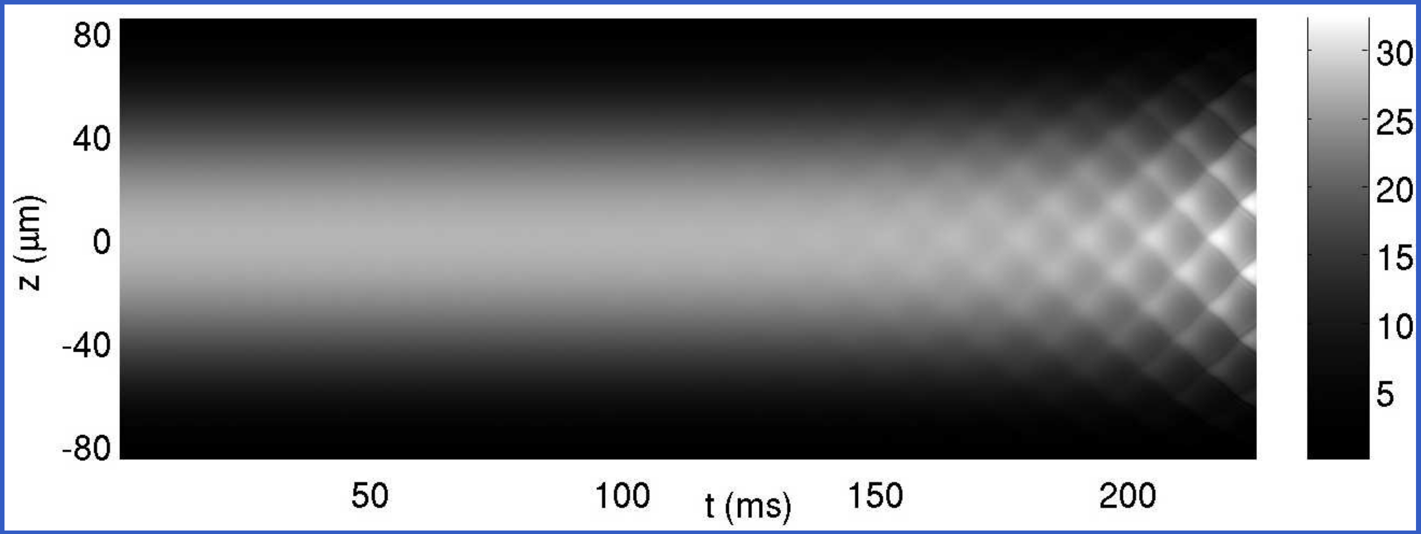

[movie]

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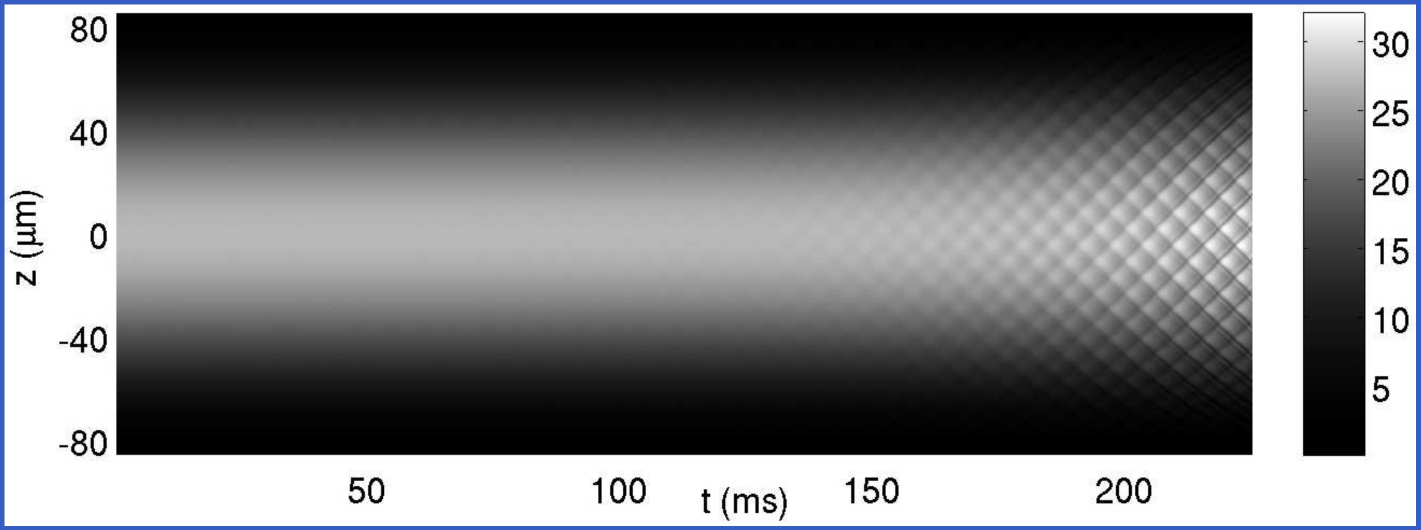
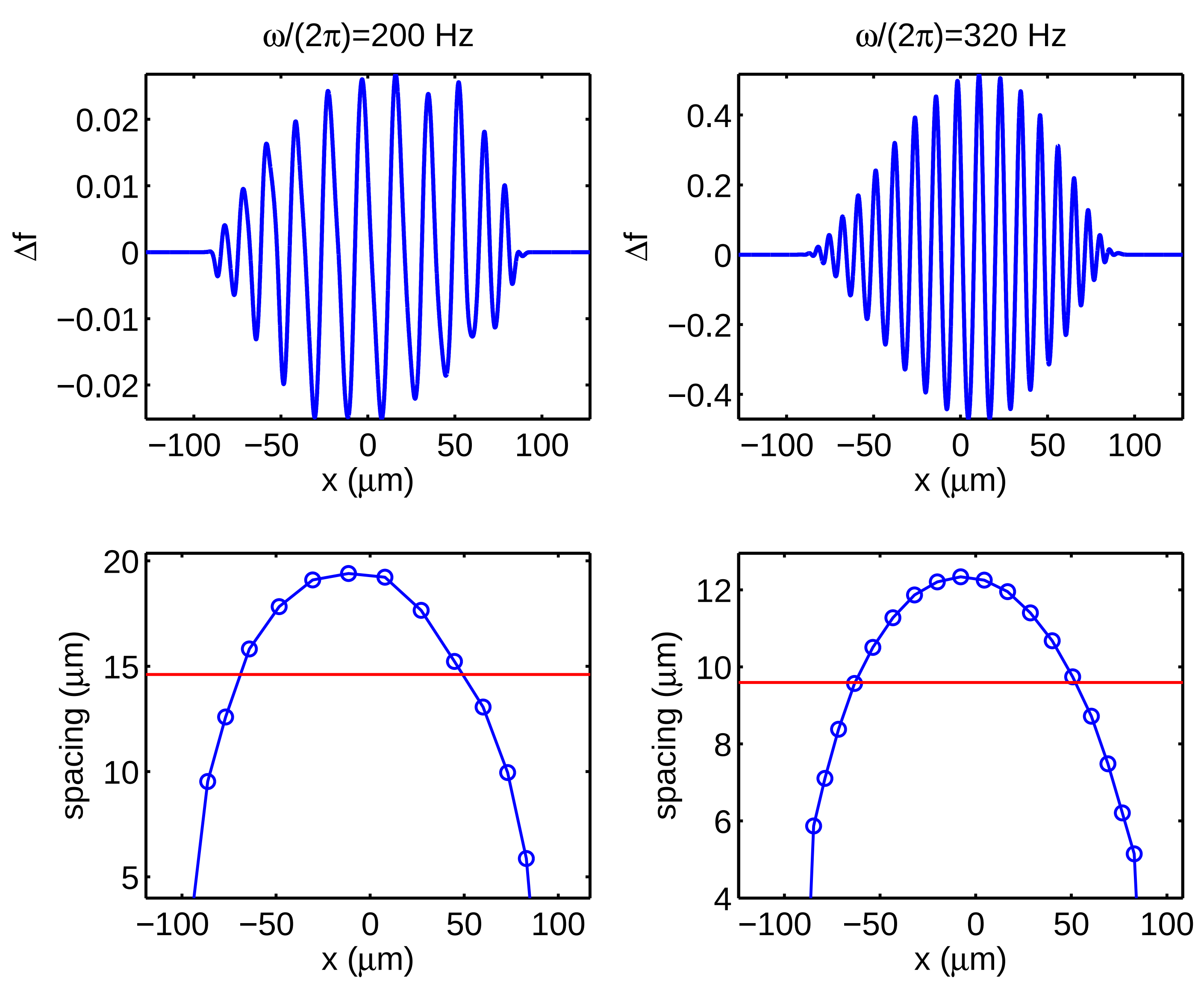


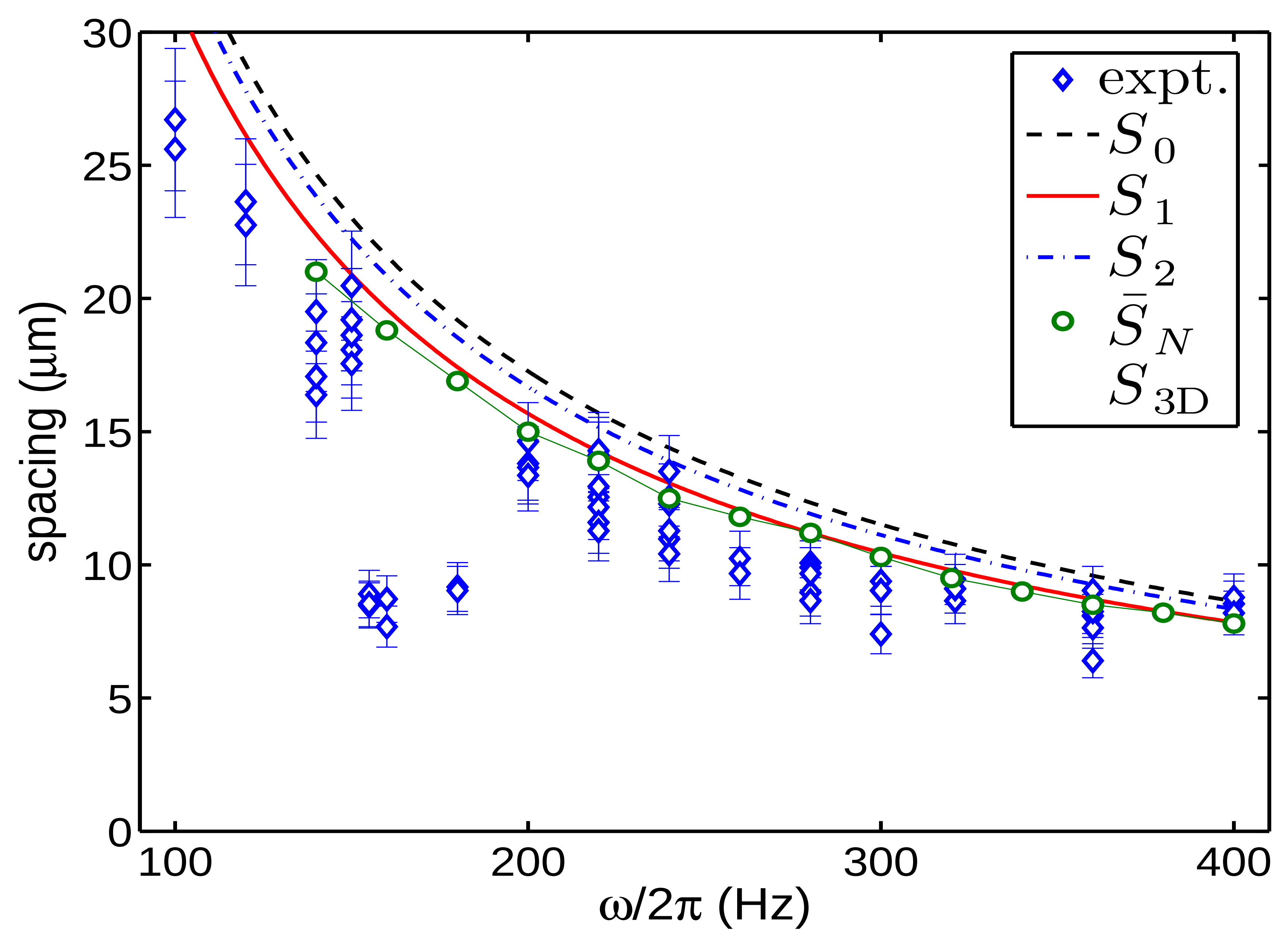
[movie]

Faraday spacing from 1D NPSE Numerics

- Spacing is not homogeneous → average spacings:



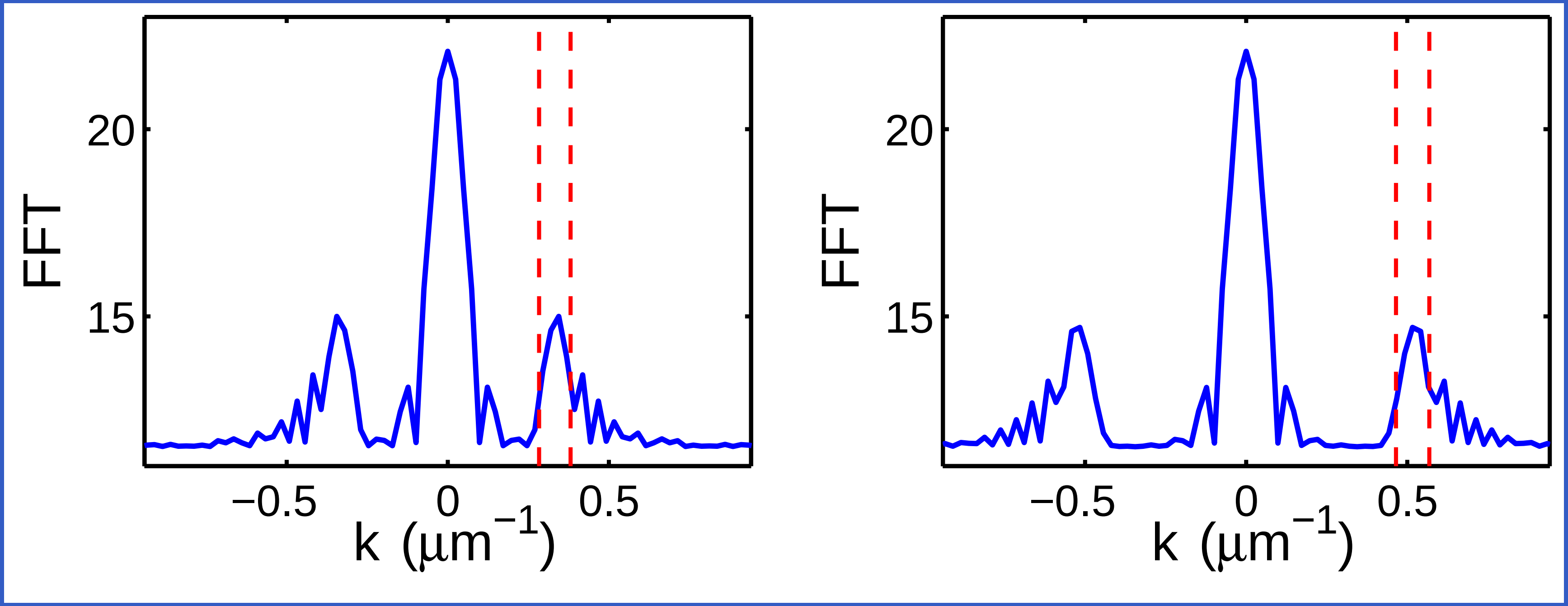
Faraday waves: Exp. + Theory + 1D Numerics



Faraday spacing: estimating error-bars using FFT:

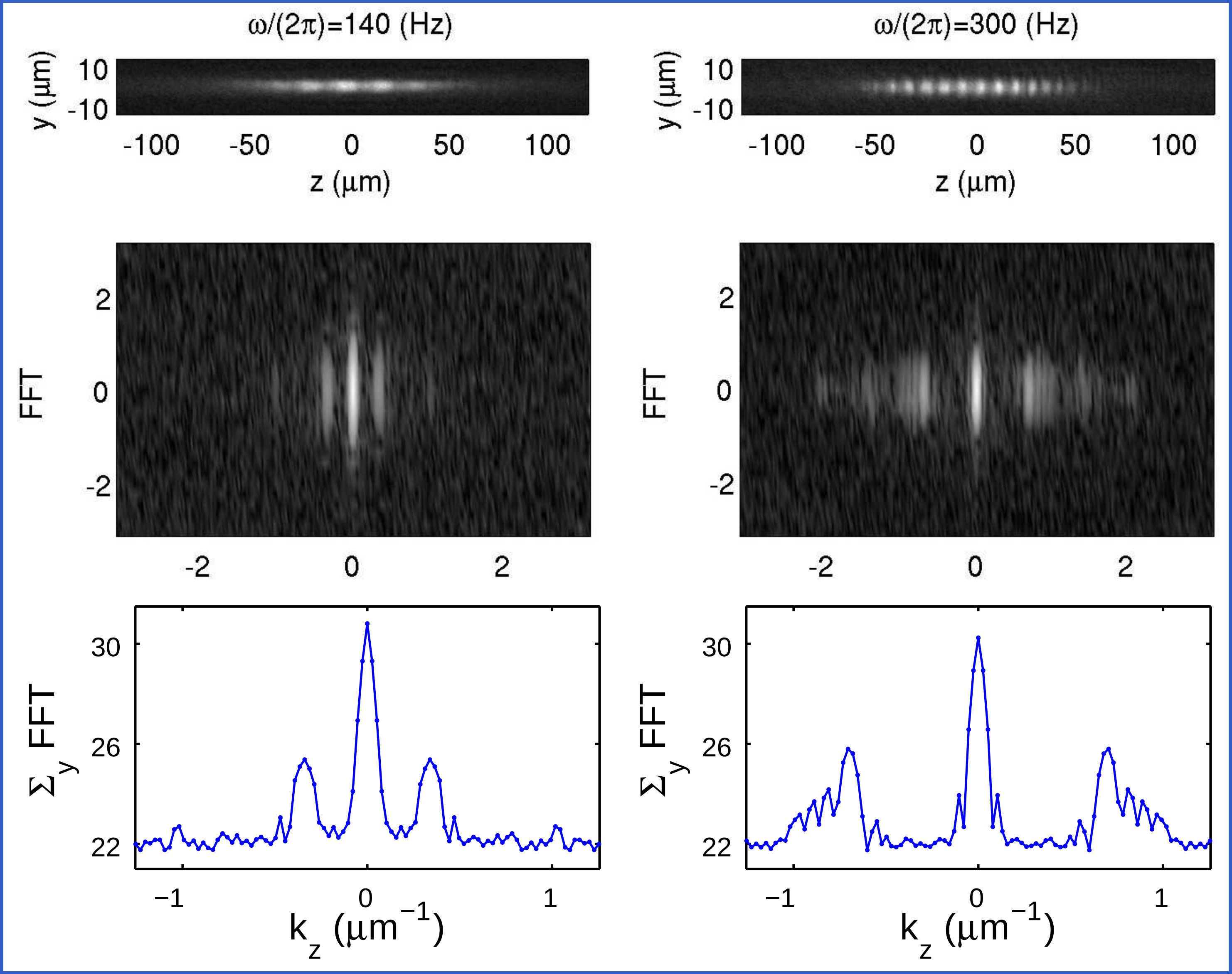
- 1D NPSE numerics:

$\omega/(2\pi) = 200$ Hz $\qquad\qquad$ $\omega/(2\pi) = 320$ Hz



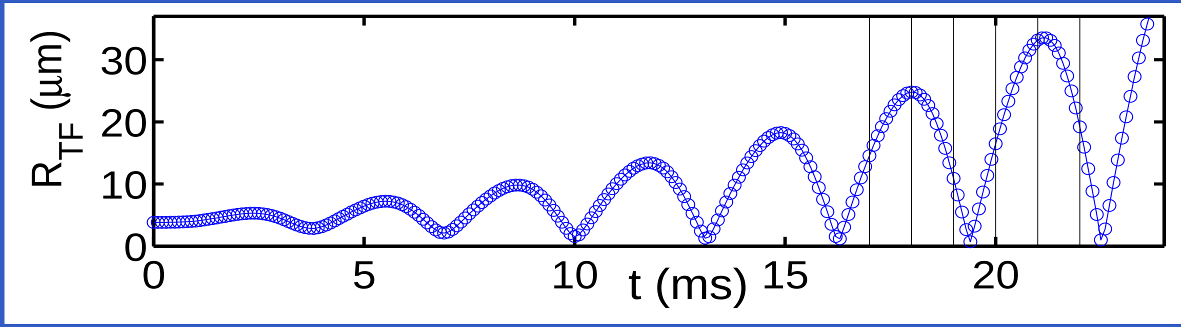
Faraday spacing: estimating error-bars using FFT:

- Experiment:



Faraday waves: 3D Numerics

- Extremely hard numerical problem:
  - Dimensionality (3D)
  - Impact oscillator dynamics for the radial size of BEC:



    - mass concentration at $r = 0 \Rightarrow$ fine resolution close to $r = 0$
    - wavefunction accelerates in the radial direction $\Rightarrow$ fine resolution at the periphery of the cloud.

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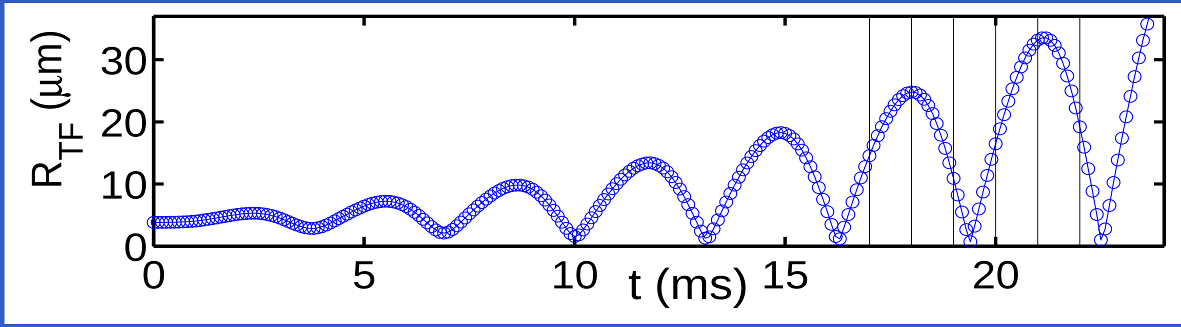

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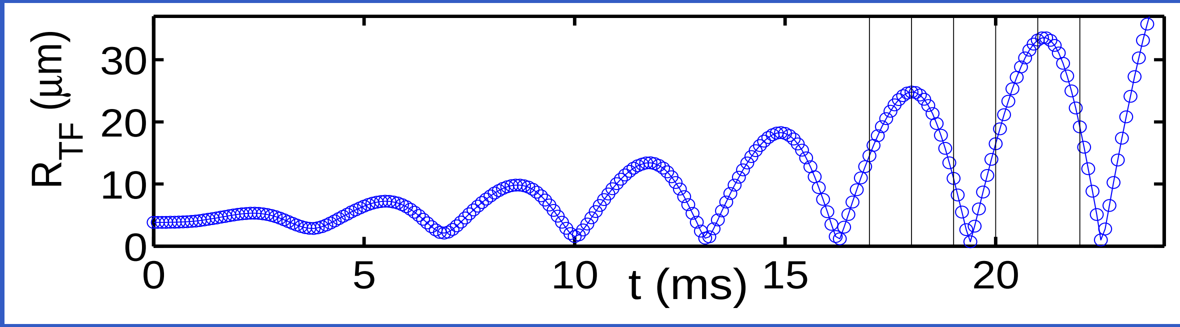

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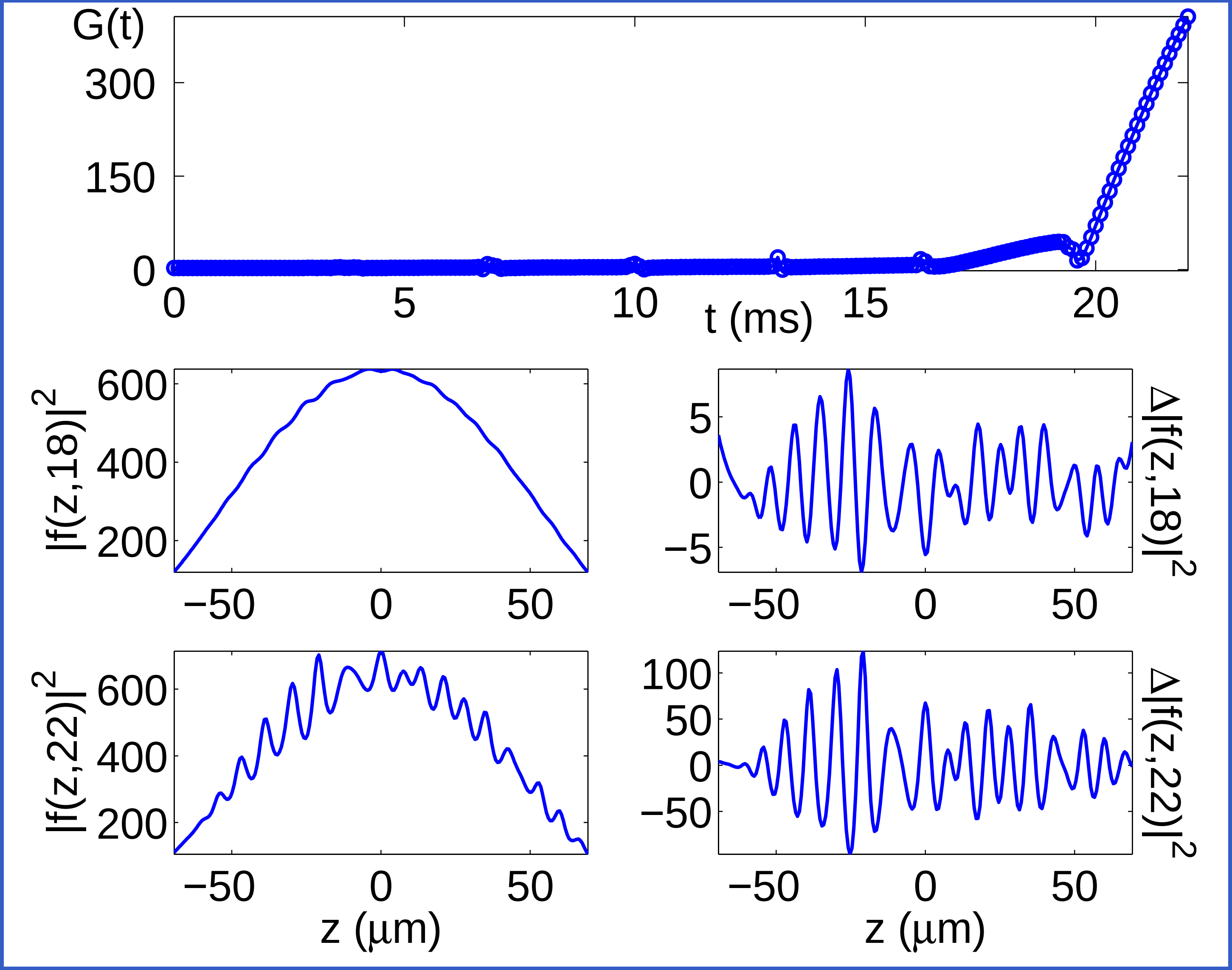
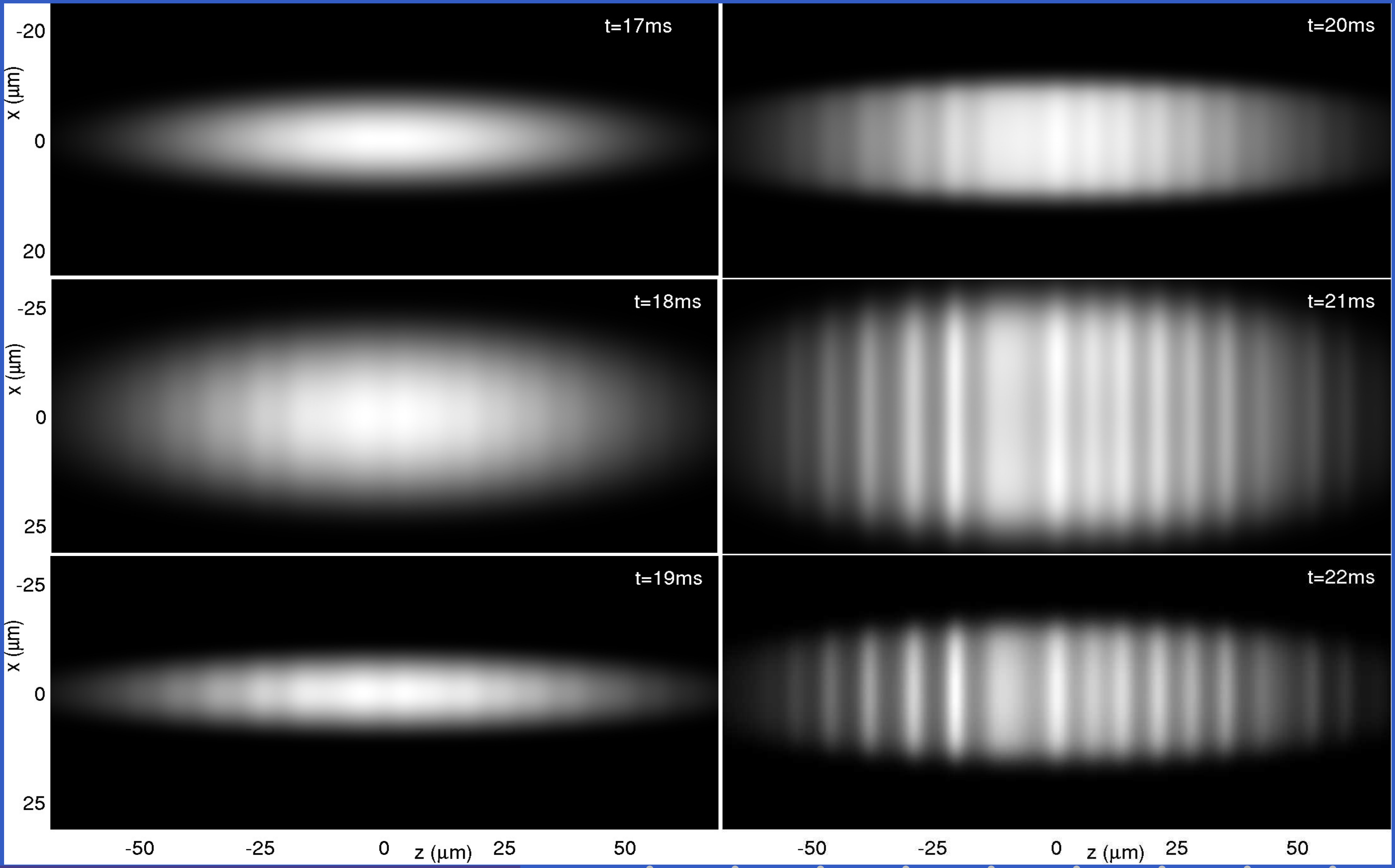
  - Extremely fine grid: <u>2001</u>$\times$401 points in (r, z)-plane
- Only few impact oscillations captured before numerics collapse
- Ok since we observe the seeding of the Faraday pattern (spacing)

Faraday waves: 3D Numerics : 1D r-integrated view

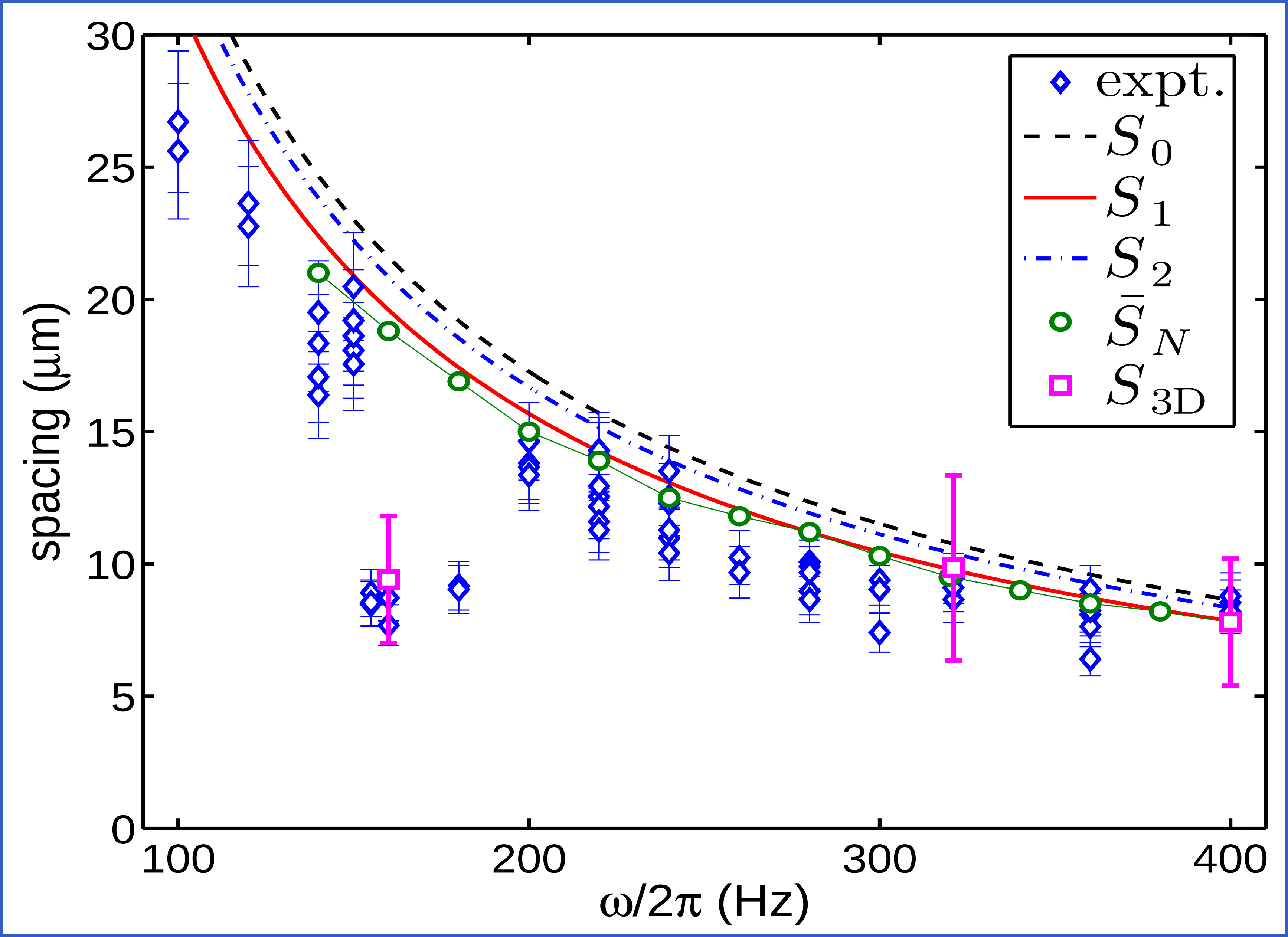
- 20% modulation with $\omega/(2\pi) = 321$ Hz for $N = 5 \times 10^5$ ^{87}Rb atoms:



Faraday waves: 3D Numerics : [movie]

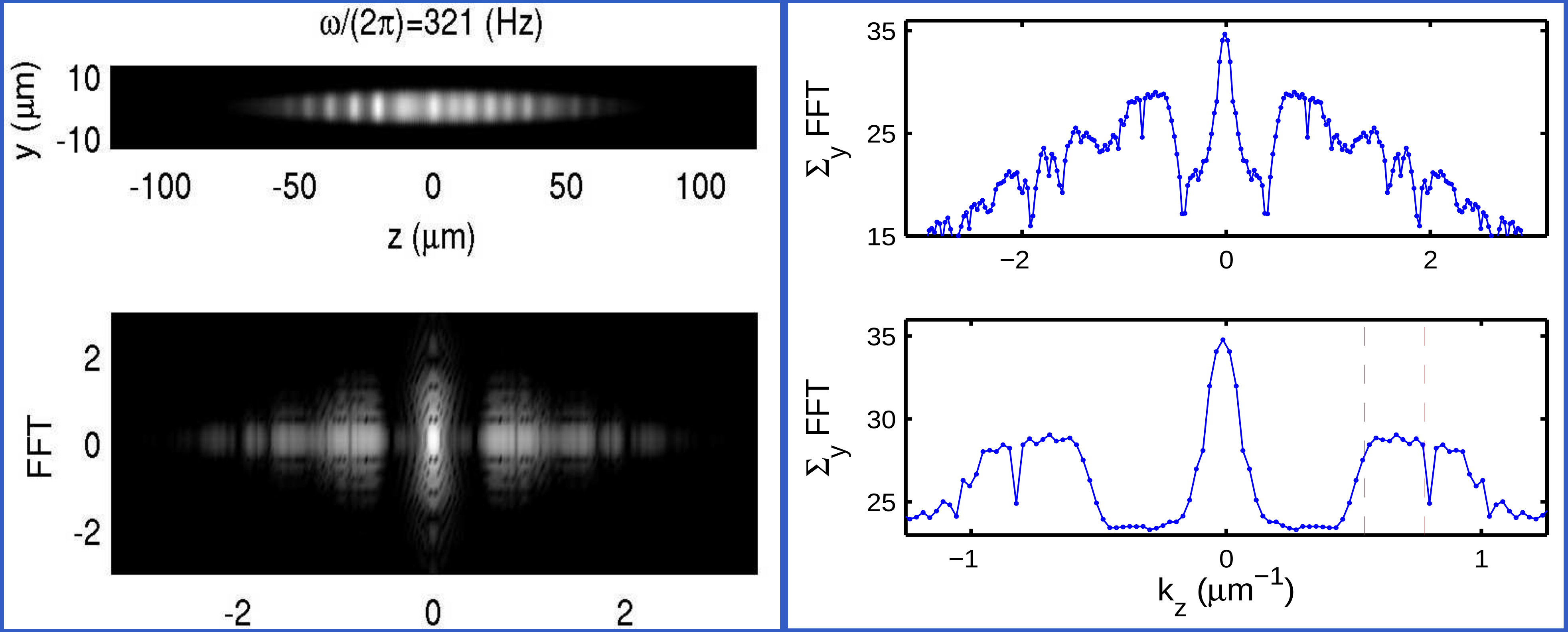


Faraday waves: Exp. + Theory + 1D & 3D Numerics



Faraday spacing: estimating error-bars using FFT:

- 3D numerics:



Conclusion, outlook & to do's

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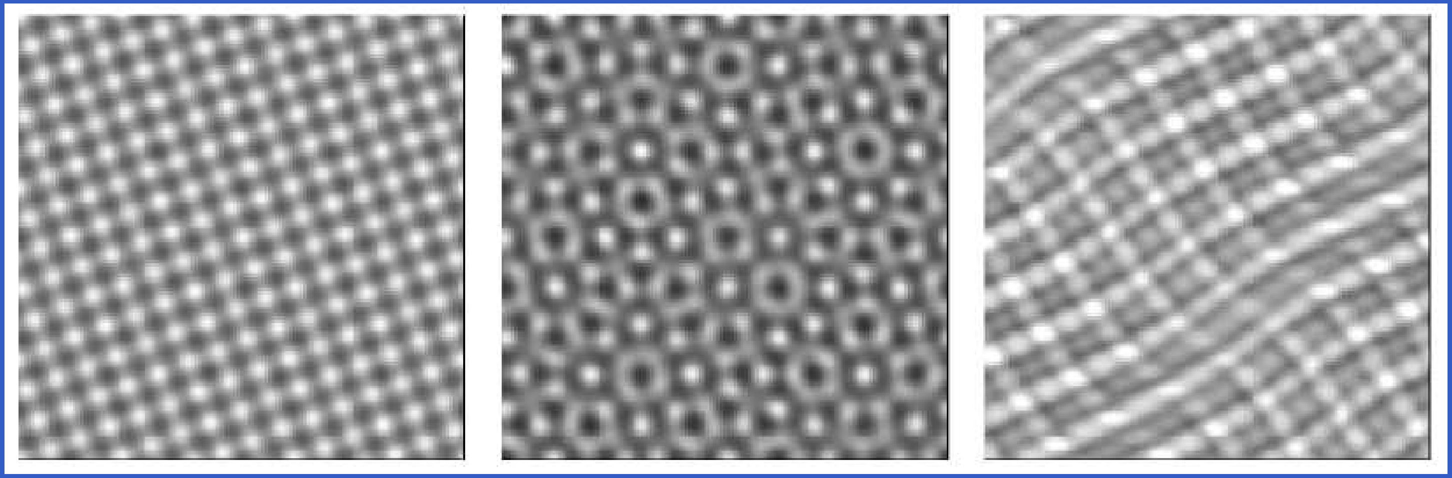
Conclusion, outlook & to do's

- We were able to predict the Faraday patterns in elongated BECs using NPSE reduction.
- The radial profile is not quite Gaussian
  $\longrightarrow$ a better ansatz is needed.
- Perform a similar analysis for 2D Faraday patterns
  $\longrightarrow$ In principle possible using an extension of the NPSE in 2D



- Maybe 3D?

NLDS: Nonlinear Dynamical Systems @ SDSU

http://nlds.sdsu.edu/ [Graduate Programs]

MS/PhD in Appl. Mathematics with concentration in Dynamical Systems.

- Fall 2008:
  - MATH-537 : Advanced Ordinary Differential Equations
  - MATH-538 : Dynamical Systems & Chaos I
  - MATH-636 : Mathematical Modeling
- Spring 2009:
  - MATH-531 : Advanced Partial Differential Equations
  - MATH-639 : Nonlinear Waves
  - MATH-638 : Dynamical Systems & Chaos II
- Fall 2009:
  - MATH-635 : Pattern Formation
  - MATH-693A : Advanced Numerical Analysis
  - MATH-797 : Research
- Spring 2010:
  - MATH-799A : Thesis – Project