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Abstract

Simple circular waveguides promise to be an ideal architecture for building high-precision matter-wave interferometers that exploit the coherent source atoms provided by Bose-Einstein condensates. Using finite difference methods, we perform numerical calculations of the time-dependent Gross-Pitaevskii equation in one and two dimensions to simulate gravity-induced quantum interference of condensates counterpropagating in a circular waveguide. The aim of this work is to understand the impact multimode excitations and nonlinear interactions have on the feasibility of interferometric measurements. Our results vividly illustrate many of the challenges to be expected in performing these types of experiments.

Introduction

The study of dilute atomic gas Bose-Einstein condensates (BECs) propagating in simple circular waveguides has garnered significant interest in recent years from the experimental and theoretical communities within the emerging field of coherent atoms optics [1–12]. These waveguides promise to be an ideal architecture for building high-precision matter-wave interferometers that exploit the coherent source of atoms provided by BECs. For more than a century, optical interferometers have successfully exploited the wave-like nature of light to enable both insights into fundamental scientific questions and high-precision metrology required for a variety of modern technological applications. Today, BEC-based atom interferometers are on the verge of revolutionizing the future interferometric measurement.

The ultracold atoms of a BEC, unlike photons, are extremely sensitive to environmental factors such as electromagnetic fields and intertial forces. This inate sensitivity to the world around us may revolutionize the future of interferometric measurements forever. By providing measurement sensitivities several orders-of-magnitude better than their optical counterparts [13–15], BECs offer the potential of fundamentally new types of measurements that are not possible with optical interferometers. Although a number of first generation experiments using simple circular waveguides have been performed [7, 8, 11], all have been met with mixed success. While technical challenges remain to be solved by the experimentalists, a deficit of theoretical knowledge remains with respect to how noise, multimode excitations, and nonlinear interactions will impact the future performance of high-precision matter-wave interferometers using BECs.

The principal aim of this work is to theoretically investigate the dynamics of Bose-Einstein condensates propagating within a simple harmonic oscillator ring. This circular waveguide approximates well many of the experimental systems that have been developed. Using finite difference methods, we perform numerical calculations of the time-dependent Gross-Pitaevksii equation (GPE) in one and two dimensions to simulate interferometric measurements of the quantum-mechanical phase shift induced when the ring is tilted at various angles with respect to a local gravitational potential. In focusing on multimode excitations generated by the condensate's propagation around the ring and the nonlinearity of the condensate's mean-field interaction, we have been able to illustrate many of the challenges these experiments will likely face in the near future, even when performed under the most ideal experimental conditions.

Gross-Pitaevskii Equation

A BEC, composed of several million atoms interacting with one another, is inherently a quantum-mechanical many-body problem. Although a set of equations to describe this complex system are rather straightforward and simple to derive from first principles, the equations quickly become intractable, even for modern supercomputers. Naturally, an approximation scheme is required to reduce the problem to a managable form.

At zero temperature, the dynamics of a dilute Bose-Einstein condensate are well described by a macroscopic wave function satisfying a nonlinear Schrödinger equation. Known as the Gross-Pitaevskii equation [16–18], this simple model has proven itself to be an invaluable theoretical description of BECs. In general form, the time-dependent Gross-Pitaevskii equation is given by [14]:

$$i\frac{\partial}{\partial t}\psi\left(\mathbf{r},t\right) = \left[-\frac{1}{2}\nabla^2 + V\left(\mathbf{r},t\right) + g|\psi\left(\mathbf{r},t\right)|^2\right]\psi\left(\mathbf{r},t\right),$$

where $V\left(\mathbf{r},t\right)$ is an external potential acting on the condensate. g is a nonlinear coupling constant that is linearly dependent upon the number of atoms in the condensate. ψ, the condensate wave function, describes the density or "classical atomic field" of the condensate atoms. Instead of describing each individial atom within the condensate, the GPE uses a mean-field approximation to model their collective motion as a whole.

Although the mean-field approach of the Gross-Pitaevskii equation simplifies considerably the mathematical complexity associated with the many-body physics of dilute atomic gas BECs, few analytic solutions of this nonlinear partial differential equation actually exist. While osccasionally variational and perturbative methods prove useful in the analysis of some problems, in general, numerical techniques are often the only tool available to study these systems. To simulate gravity-induced interference of BECs occuring within a simple harmonic oscillator ring, we perform numerical calculations of the GPE in one dimension using the semi-implicit Crank-Nicolson finite difference method and in two dimensions using the Peaceman-Rachford alternating direction implicit finite difference method on a polar grid.

Gravity-Induced Interference

When a two-dimensional simple harmonic oscillator ring is tilted in a local gravitational potentical, an additional term modifies the potential. This modified form is as follows:

$$V\left(\rho,\varphi\right) = \frac{1}{2}k_\rho^2\left(\rho-\rho_o\right)^2 + V_G\rho\sin\left(\vartheta\right)\cos\left(\varphi\right)$$

where V_G is the strength of the gravitational potential and ϑ is the tilt angle of the ring with respect to this potential. Because this term creates a slightly different potential for condensates depending upon which direction they propagate within the ring, it is an ideal test case in which to study the principle variables that may affect the performance of BEC-based interferometers (see Figure 1).

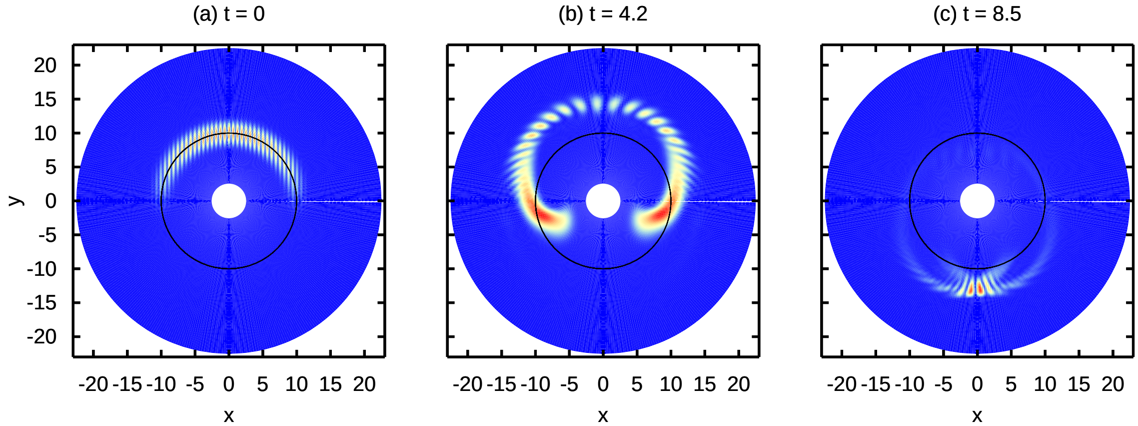

Figure 1: (a) At $t = 0$, interference fringes are visibile across the initial extext of the condensate. This is an initial laser pulse used to coherently split the condensate into two counterpropagating momentum states. (b) As time evolves, relative phase differences between the two counterpropagating waves are accumulated as a consquence of their distinct paths in the local gravitational potential. (c) When the waves recombine, a slightly asymmetric interference pattern is generated. Stored within this asymmetry is information about the local gravitational potential.

To simplify the analysis of the differential phase-shift observed in the interference patterns when the ring is tilted at various angles, we reduce the two-dimensional data sets to one dimension by integrating across the radial dimension of the system. Effectively, this results in a profile of the interference pattern along the circumference of the ring's potential minimum, where most of the phase information is stored. This reduced, one-dimensional interference pattern allows for spectacular visualization of the interference process by aggregating these profiles into a spacetime plot (see Figure 2).

Conclusion

At present, we are developing robust analysis algorithms to extract the phase information from our reduced, one-dimensional interference patterns. As such, we have yet to compare these reduced interference patterns to those produced in our one-dimensional calculations of the Gross-Pitaevskii equation. Our hope is that this comparative analysis will provide key insights into how the interplay between multimode excitations and the nonlinear mean-field interaction of BECs will impact the feasibility of future interferometry experiments.

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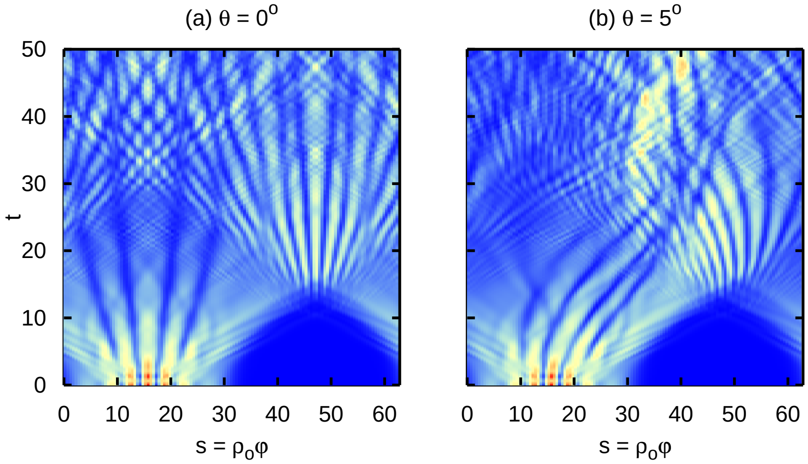

Figure 2: A spacetime representation of the reduced, one-dimensional interference pattern for a BEC when the ring is tilted at (a) $\vartheta = 0^o$ and (b) $\vartheta = 5^o$ in the local gravitational potential. In these spacetime figures, the x-axis corresponds to the position along circumference of the ring's potential minimum, while the y-axis represents time. At $t = 0$, the applied laser pulse coherently splits the condensate centered about $s = 5\pi$ into two counterpropagating momentum states. This is depicted as the outgoing flow of the density to the right and left in the spacetime plots. At approximately $t = 15$, these waves recombine on the other side of the ring at $s = 15\pi$ to produce an interference pattern in the wake of their collision. The asymmetry in the interference pattern induced by the gravitational potential is clearly seen in (b) when the ring is tilted.

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