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Eigenvalue spectra of spatial dependent networks

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Abstract

The spectral density of spatial ER and scale-free networks is under investigation. An asymmetryic spectrum is found to be a universal property of spatial networks. We quantify the asymmetry by looking at the skewness and find that it results from an increase in odd moments of the spectrum. This work shows that the spectrum can be used as a detection tool for spatial dependence in complex networks.

1. Introduction

Most studies on complex networks are on systems that live in abstract dimensionless network space. However in real-life networks it is observed that the geographical position of the nodes is often important and that links between closer edges are favored. Spatial dependence is vital in describing examples such as the road network, the internet, neural networks, friendship networks, and networks on disease spreading [1].
The most commonly studied characteristics for studying the topology of a network are the degree distribution, the clustering coefficient, and the path length. Also the spectral density $\rho(\lambda)$:

$$\rho(\lambda) = \frac{1}{N}\sum_{j=1}^{N}\delta\left(\lambda - \lambda_j\right) \qquad (1)$$

obtained by calculating the eigenvalues λ_j of the adjacency matrix of a network, is used to classify the topology of the network. In this work we investigate how the spectral density changes for a spatial network.
Thereto we construct spatial ER and scale-free networks and study their spectral density. The models are discussed in section 2. The spectra are shown in section 3 and discussed in section 4.

2. Construction of spatial dependent toy models

. In the spatial dependent ER network, l links are created with the probability depending on the distance between the nodes

$$p_{i,j} \propto d_{i,j}^{\;-\alpha}. \tag{1}$$

Here α is a measure for the spatial dependence. For α=0 the regular ER network is found where connections are made completely at random. For higher α, closer edges are formed, and for $\alpha=\infty$ the network with the closest possible links is formed.

A scale-free network distinguishes itself from an ER network in 2 ways: i) it is a growing network: nodes are added all the time and ii) it shows preferential attachment. In a spatial dependent scale-free network the probability for connecting a newly added node j to an existing active node i is proportional to both the current degree of the active nodes k_i and the distance between i and j:

$$p_{i,j} \propto d_{i,j}^{\;-\alpha}\left(k_i + 1\right). \tag{2}$$

Every node that is added will be connected to m existing nodes. The exact details of the used method are described elsewhere [1].
All constructed networks have $<k>=10$, $N=1000$, $l=5000$ and are averaged over 100 configurations.

3. Results and discussion

The resulting spectra for the ER networks are shown in Figure 1a. As expected, for α=0 we find the semicircle. For higher α, we see that the peak increases and shifts to the left while the right tail becomes heavier. For the spectra of the scale-free network in Figure 1b without spatial dependence, we recover the symmetric trianglelike spectral density that is characteristic for the scale-free network [1]. For the spatial dependent scale-free networks, we see the peak shifting to the left and again the right tail becoming heavier. In both spatial networks we observe that the result of spatial dependency is an asymmetric spectrum.

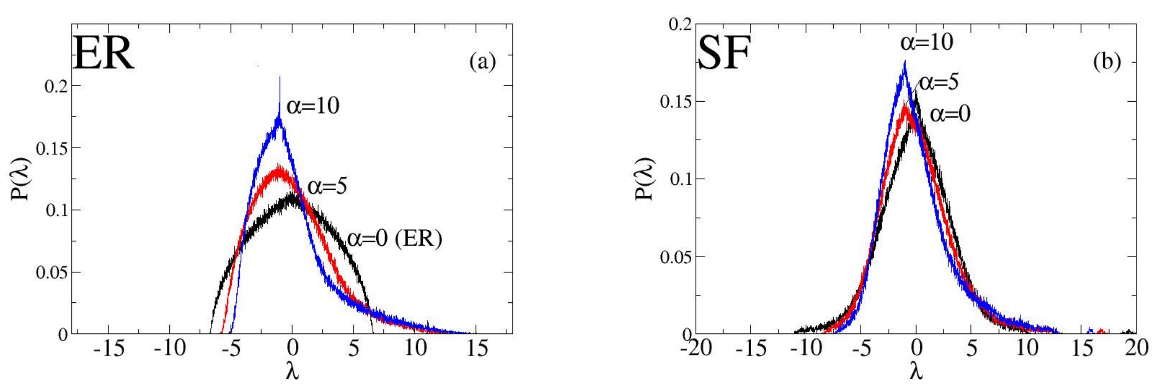

Figure 1: Spectral density (a) for ER network and (b) for scale-free network.

To quantify the asymmetry, we look at the skewness, which is defined in terms of the central moment of the distribution m_s:

$$m_s = \frac{1}{N}\sum_{j=1}^{N}(\lambda_j - \mu)^s = \int(\lambda-\mu)^s \rho(\lambda)d\lambda \ . \tag{3}$$

with μ the mean eigenvalue. The physical meaning of the moments is that is determines $D_s = Nm_s$, the so called "number of directed paths of the graph that return to their starting vertex after s steps" [2]. The skewness is now defined as:

$$S = \frac{m_3}{m_2^{3/2}} = \frac{N^{-1}\sum_i \lambda_i^3}{\sigma^3}. \tag{4}$$

since μ=0 (because nodes are not allowed to form links with themselves and hence the adjacency matrix is traceless). The skewness is shown in Figure 2 as a function of α. It is observed that skewness increases strongly with spatial dependence.

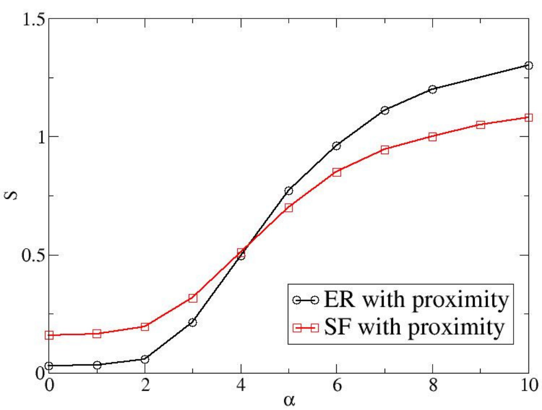

Figure 2: Skewness for ER network and scale-free network.

Discussion and conclusion

To understand the observed asymmetry, we rewrite Eq.(4) and find

$$S = \frac{D_3}{N\sigma^3} \tag{5}$$

We see that the skewness is thus directly dependent on the number of directed paths of 3 steps (triangles) in the system. D_s is shown in Fig.3 for a system without proximity ($\alpha = 0$) and with strong proximity ($\alpha = 8$). The main effect of the spatial dependence is an increase in the number of directed paths with odd length. This is a consequence of the increased number of triangles as a result of the spatial nature of the network. Future work consists of quantitatively explaining this increase of triangles in the spatial network and investigating the spectrum of a spatial small-world network [4].

In summary, we have shown that an asymmetric spectrum is a universal property for spatial networks. The asymmetry was quantified by the skewness and is a result of the spatial nature of the network, which leads to an increase in the number of triangles in the system. This result shows that the spectrum can be used as a detection tool for spatial dependence.

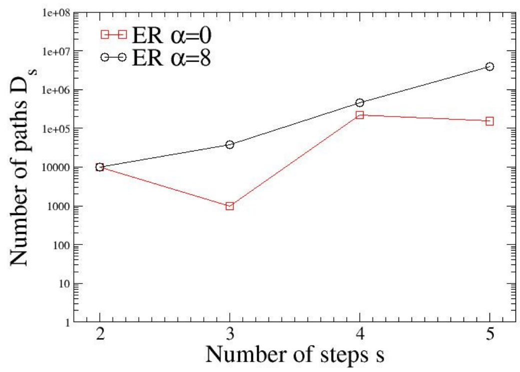

Figure 3: Number of directed paths of s steps for ER network with and without spatial dependence.

References

[1] M. Gastner and M. Newman, The European Physical Journal B, 49, 247–252 (2006).
[2] I. J. Farkas, I. Derényi, A.-L. Barabási, and T. Vicsek, Phys. Rev. E, 64, 026704 (2001).
[3] M. Barthélemy, Europhysics Letters, 63, 915–921 (2003).
[4] J. Billen, M. Wilson, A. Baljon, and A. Rabinovitch, (unpublished).