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General Curvilinear Ocean Model (GCOM): Enabling Thermodynamics

M. Abouali,1, C. Torres, R. Walls, G. Larrazabal, M. Stramska, D. Decchis1, and J.E. Castillo1.*
1San Diego State University
** Corresponding author: mabouali@sciences.sdsu.edu*

Abstract: General Curvilinear Ocean Model (GCOM) is a coastal ocean model curvilinear in 3 dimensions, developed by Carlos Torres et al. The model uses a direct numerical simulation (DNS) approach to solve the primitive Navier-Stokes equations and uses boundary fitted Curvilinear coordinates; therefore, it is possible to use GCOM in various topographies and meshes. GCOM is currently being coupled with heat and salinity conservation (or thermohaline dynamics) equations. GCOM is designed in a modular fashion, which will make it possible to easily add biogeochemical submodels to study different phenomenon, such as phytoplankton blooms, transport of pollution, or biogeochemical cycling. In a separate project an Atmosphere-Ocean Interaction model is developed, which will be coupled later to GCOM.

Keywords: Ocean Model, General Curvilinear Coordinate, Navier-stokes, Thermodynamics

1 Introduction

About two third of the Earth is covered with oceans and seas. Beside the complex ecosystem existing inside the ocean and being on of the main sources of food for human being, the ocean is interacting with the Earth Atmosphere and controlling its properties. Hence, indirectly affecting our other sources on lands. Furthermore, the oceans are one of the main sources of solid particles required in the formation of the clouds.

The huge specific heat capacity of the ocean has made it a big natural heat and energy storage. During the summer, while there exist excess of solar energy, the heat energy is absorbed and stored in the ocean and during the winter this excess of energy is released and helps to have a milder weather. Not just throughout the year, the ocean also helps to distribute the energy from those regions of the earth, which receive more energy, such as the equator, to those regions where solar energy budget is much less; thus, keeping the weather from being too cold or too warm. It is well known that the thermohaline circulation in the ocean had affected the ice age in the past.

It is no further needed to emphasize the importance and how strongly the ocean is affecting us, the human, and other animals. There are couple of methods used to study the ocean and its impact which can be categorized as: (1) Observational, (2) Analytical, (3) Laboratory, and (4) Numerical studies. The focus of this paper is on the latest, i.e. numerical studies and numerical models.

In this paper we introduce the General Curvilinear Ocean Model, abbreviated as GCOM, which was originally developed by Prof. C. Torres and Prof. J.E. Castillo [6] [5]. GCOM uses full 3D general curvilinear coordinate system. This feature enables the model to adapt very well to the real physical boundary of the study area both in vertical and horizontal; unlike the sigma coordinate which is designed to adapt just to the bathymetry; hence, the vertical direction. GCOM currently uses non-dimensional primitive Navier-Stokes equations with Boussinesq approximation and it was previously used in many projects.

In next section, we will explain the main equations building the core of the GCOM model. Later, we explain the thermodynamic equations, which are under process of being added to the model. As the curvilinear equations are being used, we need to transfer both the grid and the underlying equations into a rectangular uniform grid to ease the implementation in the computer and applying the boundary conditions [1] [2]. Therefore, we explain the necessary steps and relations needed for this transformation in 3 dimensional spaces. At the end, the future of the model is explained.

2 Governing Equations

In this section we introduce the core equations used in GCOM, i.e. Navier-Stokes (NS) equations [7]. we focus on both dimensional and non-dimensional form of the NS equations [4] [3].

2.1 Momentum equation

The dimensional form of the Navier-Stokes equation is:

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -g\hat{k} - 2\omega \times u - \frac{1}{\rho}\nabla p + \nu\nabla^2 u \tag{1}$$

where u is the velocity, g is the acceleration due to gravity, ω is the coriolis, ρ_0 is the density, ν is the viscosity, and p is the pressure. Each term has its unique meaning. from left to right:

- **Term I:** is the storage of the momentum.
- **Term II:** is the advection.
- **Term III:** is the Effect of the gravity/
- **Term IV:** is the effect of Coriolis Forces, due to the rotation of the earth.
- **Term V:** is the pressure gradient forces.
- **Term VI:** is the influence of the viscous stresses.

Using the Reynolds number, i.e. $R_e = \frac{VL}{\nu}$, Froude Number, i.e. $F_r = \frac{V}{\sqrt{gL}}$, and Rossby Number, i.e. $R_o = \frac{V}{Lf}$ the above equation can be written in its non-dimensional form as follow:

$$\frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{\rho}{F_r^2 \hat{k}} + \frac{1}{R_o}\left(v\hat{i} - u\hat{j}\right) - \nabla p + \frac{1}{R_e}\nabla^2 u \tag{2}$$

where V is the reference velocity and L is the reference length, chosen by the user to non-dimensionalize the equations.

2.2 Continuity Equation

The continuity is given as:

$$\frac{\partial \rho}{\partial t} + \rho \cdot \nabla u = 0 \tag{3}$$

2.3 Poisson Equation

The Navier-Stokes equation does not provide any relation for the pressure. Thus, we need to find a method to calculate the pressure. One of the commonly used method is to apply a divergence operator on the momentum equation, 2, and then by the use of continuity equation, 3, derive a simplified equation for the pressure. Although we used the term simplified, but it does not mean that the the resulting equation is easy to solve. As the matter of fact, the resulting equation is called the poisson equation for the pressure and is one of the expensive part of the numerical model to solve. The resulting poisson equation for pressure is given in the 4. It has to be noted that this equation is mostly due to numerical purposes and does not have a physical meaning.

$$\begin{aligned} \nabla^2 p \quad = \quad & -\tfrac{1}{F_r^2}\nabla \cdot \left(\rho\hat{k}\right) - \nabla \cdot \left[(u \cdot \nabla)\, u\,\right] \\ & + \tfrac{1}{R_e}\nabla^2 D - \tfrac{\partial D}{\partial t} + \tfrac{1}{R_o}\nabla \cdot \left(v\hat{i} - u\hat{j}\right) \end{aligned} \tag{4}$$

where we have:

$$D = \nabla \cdot u \tag{5}$$

3 Transformation

As mentioned earlier, GCOM uses curvilinear coordinate system. Therefore, it is possible to use GCOM in the variety of the bathymetry and grids. But both the grids and the equation are needed to be transformed to a rectangular uniform grid as in 1.

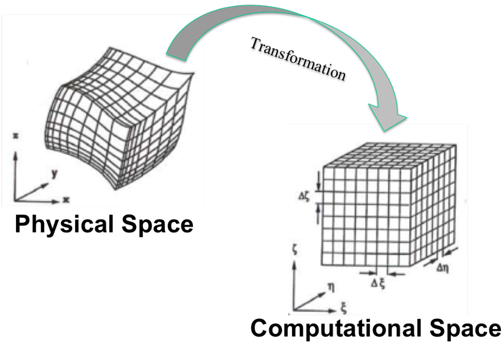

Figure 1: Transforming the grid

Once the grid is transformed we also need to transform the equations. As we have just transformed the grid, this means that just the spatial derivative are needed to be transformed and there is no need to transform the time derivative. The transformed spatial derivative can be easily obtained using the chain-rule in derivation as follow:

$$\begin{aligned} \frac{\partial}{\partial x} &= \xi_x \frac{\partial}{\partial \xi} + \eta_x \frac{\partial}{\partial \eta} + \zeta x \frac{\partial}{\partial \zeta} \\ \frac{\partial}{\partial y} &= \xi_y \frac{\partial}{\partial \xi} + \eta_y \frac{\partial}{\partial \eta} + \zeta y \frac{\partial}{\partial \zeta} \\ \frac{\partial}{\partial z} &= \xi_z \frac{\partial}{\partial \xi} + \eta_z \frac{\partial}{\partial \eta} + \zeta z \frac{\partial}{\partial \zeta} \end{aligned} \tag{6}$$

where ξ_x, η_x, $\cdots$ are called the metrix of the transformation and can be obtained using:

$$\begin{vmatrix} x_\xi & x_\eta & x_\zeta \\ y_\xi & y_\eta & y_\zeta \\ z_\xi & z_\eta & z_\zeta \end{vmatrix} \cdot \begin{vmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{vmatrix} = I \tag{7}$$

The three dimensional transformed Laplacian operator is:

$$\begin{aligned} \nabla^2(f) = \quad & L(f) \\ & -L(x)\,[\xi_x \frac{\partial(f)}{\partial \xi} + \eta_x \frac{\partial(f)}{\partial \eta} + \zeta x \frac{\partial(f)}{\partial \zeta}\,] \\ & -L(y)\,[\xi_y \frac{\partial(f)}{\partial \xi} + \eta_y \frac{\partial(f)}{\partial \eta} + \zeta y \frac{\partial(f)}{\partial \zeta}\,] \\ & -L(z)\,[\xi_z \frac{\partial(f)}{\partial \xi} + \eta_z \frac{\partial(f)}{\partial \eta} + \zeta z \frac{\partial(f)}{\partial \zeta}\,] \end{aligned} \tag{8}$$

where we have:

$$\begin{aligned} L(f) = \quad & a\frac{\partial^2(f)}{\partial \xi^2} + b\frac{\partial^2(f)}{\partial \eta^2} + c\frac{\partial^2(f)}{\partial \zeta^2} + \\ & 2\,[d\frac{\partial^2(f)}{\partial \xi \partial \eta} + e\frac{\partial^2(f)}{\partial \zeta \partial \eta} + d\frac{\partial^2(f)}{\partial \xi \partial \zeta}\,] \end{aligned} \tag{9}$$

and the coefficients in 9 are defined as follow:

$$\begin{aligned} a =& \quad \xi_x^2 + \xi_y^2 + \xi_z^2 \\ b =& \quad \eta_x^2 + \eta_y^2 + \eta_z^2 \\ c =& \quad \zeta_x^2 + \zeta_y^2 + \zeta_z^2 \\ d =& \quad \xi_x \eta_x + \xi_y \eta_y + \xi_z \eta_z \\ e =& \quad \zeta_x \eta_x + \zeta_y \eta_y + \zeta_z \eta_z \\ q =& \quad \xi_x \zeta_x + \xi_y \zeta_y + \xi_z \zeta_z \end{aligned} \tag{10}$$

4 Thermodynamic Equations

The original GCOM code was not coupled with the thermodynamic equations; thus, its usage was limited to the neutral condition. As the development, the thermodynamics equations are being coupled with the model. In equation, the amount of potential temperature and salinity are two important factor. These two scalar variable, together with pressure define what the density is in the certain location of the ocean. The equation which relates the pressure, salinity, and the potential temperature to the density is called Equation of State:

$$\rho = f\,(p, \theta, S) \tag{11}$$

4.1 Conservation of Heat

The heat equation is given by:

$$\frac{\partial \theta}{\partial t} + u \cdot \nabla \theta = \nu_\theta \nabla^2 \theta + Q_\theta \tag{12}$$

where ν_θ is the thermal diffusivity and Q_θ is the sink or source term and it is invariant to coordinate transformation. Different model has to be executed to account for incoming solar radiation, amount of evaporation, or any other interactions between the ocean and the atmosphere.

4.2 Salinity Equation or Conservation of Heat

The conservation of Salinity can be written as:

$$\frac{\partial \S}{\partial t} + u \cdot \nabla S = \nu_S \nabla^2 S + Q_S \tag{13}$$

where ν_S is the salinity diffusivity and Q_S is the sink or source term and it is invariant to coordinate transformation. As an example, if we have precipitation, which is of different salinity than the ocean, It must be take care of in this term.

5 Solution Method

In previous sections different equations that are used in GCOM were discussed. But the order that these equations are executed is as follow:

- **Step 1:** First all fields are initialized and the model parameters are read from a text file.
- **Step 2:** The Poisson equation for the pressure is solved and the new pressure field is obtained.
- **Step 3:** The momentum equation is solved and the new velocity field is obtained.
- **Step 4:** Using the updated velocity field the Thermodynamic equations, both conservation of heat and conservation of salinity, are solved.
- **Step 5:** Using the equation of the state a new density field is recalculated.
- **Step 6:** Different fields are prepared for the next iteration. Required fields for post processing are written to text file.
- **Step 7** The time is advanced by d_tI and f the desired time interval has not reached, we continue on step 2.

6 Future of the GCOM

GCOM was originally developed in FORTRAN 77 in a single file. It is more desired to change the code in to a modular fashion, by using different subroutines. The code is also rewritten in FORTRAN 90 using its parallel and vector operation capabilities.

Furthermore, the GCOM is currently using second order central finite difference. It has been shown that higher order mimetic orders behave much better in such a highly non-linear and complex system of equations. Therefore, the numerical scheme of the code is also being changed to the mimetic method developed by Prof. Castillo. This method replaces the divergence and the gradient operator with its equivalent discrete operator.

Currently the code solves the given equation directly. This requires a very fine mesh at the Kolmogorov scale. For high Reynolds number and real application, this requires a very fine mesh; hence, a huge amount of memory and computation time, which is not available even on the most powerful existing supercomputers, and it is believed that this amount of memory will not be available in near future either. Therefore, a closure model, or a sub-grid model is required to account for the energy and the effect of small scale eddies. This step is being currently studied and added to the code.

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