ACSESS proceedings
2008

Parallel implementation of the CCBA for mesh generation

E.D. Batista, J.S. Otto and J.E. Castillo

CSRC an interdisciplinary gateway to the future
SAN DIEGO STATE UNIVERSITY

Parallel implementation of the CCBA for mesh generation

E.D. Batista*, J.S. Otto and J.E. Castillo
Computational Science Research Center, SDSU
dbatista@sciences.sdsu.edu

1.- Abstract

In this work we extend the CCBA to produce a parallel-iteratively mesh generator. The CCBA or Convex-Combination-Based-Algorithm (presented in ACSESS 2007) uses a mesh-size function and convex combinations on local polytopes to change the position of the nodes of an initial mesh. This process is repeated iteratively and stops when the max norm between two consecutives meshes is smaller than a given threshold. The parallelization is obtained by dividing the initial mesh into as many submeshes as processors available and dealing with each submesh in a different processor. This so-called mesh decomposition technique does not depend on the physical domain of the problem, which is the main difference and the main advantage over domain decomposition-like techniques for parallelization. We also present a way to reduce overhead while preserving the property of non-folding of the original sequential algorithm. Examples of 2D meshes show the potential of the technique for constructing non-uniform quad meshes.

2.- CCBA

We start with an initial grid constructed over the domain, Ω. Then, for each point, p, of the mesh we consider a convex polytope $V \subset \Omega$ that contains p. If V is generated from points v_j then, p is updated according to

$$\hat{p} = \sum_{j=1}^{n} \lambda_j(p) v_j \qquad (1)$$

where λ_j are estimated with the mesh-size function f, $f > 0$.

Since (1) defines a local convex combination mapping that maps V into a convex closed region, then (1) is one-to-one, and $\hat{p} \in V$. Moreover, $\hat{p} \notin \partial V$ because $f > 0$ and, in turn, $\lambda_j > 0$. This property guarantees that non-folding will be produced.

2.1.- The mesh-size function

The mesh-size function, f, is a positive function defined on Ω. Hence, f is independent of the numbers of points of the grid, which provides simplicity and versatility to the model. For regions of high density, the mesh-size function f has small values, whereas for regions of small density, f has bigger values. The mesh-size function can be obtained through different ways: given *a priori*, based on the geometry of Ω, based on errors from differential equations or combination of these, for mention just a few.

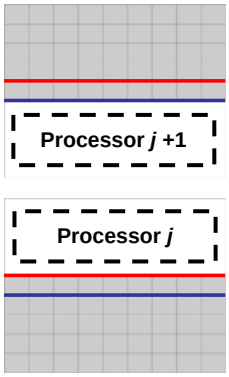
3.- Parallelization

Because calculations in the convex-combination based algorithm are done locally, the CCBA can be easily parallelized. We work with a SIMD (Singular Instruction Multiple Data) problem and we consider the following information as input values:

- P_t: total number of processors to be used,
- n: rows of the grid,
- m: columns of the grid.

P_t will be written as follow: $P_t = P + 1$, where P is the number of processors that will work modifying the mesh points, and the number 1 stands for the master processor.

First we do *mesh decomposition*, which consists of dividing the initial mesh in as many submeshes as processors available are. Then, we distribute the P pieces of the initial mesh among P processors where they will be modified. During mesh decomposition we also replicate certain rows. The last two rows of processor j will be the first two rows of processor $j+1$, with $j = 1, \ldots, P-1$. Each processor works independently and only interacts with other processors at the end of each iteration, to exchange the replicated rows. Processor j will update its last row (red line) with the second row of processor $j+1$; and processor $j+1$ will update its first row (blue line) with the row prior to the last row of processor j. By proceeding this way, each processor interacts with its two closest neighbors (processors) and no foldings are guaranteed. In addition, the duplication of rows reduces idle time and avoids doing extra computations. It is worth to say that communication, idle time, and extra computation are the three main sources of overhead in parallel computation.



3.1.- Mesh decomposition

The idea is to divide the initial mesh into sub-meshes and assign each of these pieces to a processor. The size of each sub-mesh will be proportional to the computational power of the corresponding assigned processor. Thus, we are able to adapt the algorithm to heterogeneous clusters of computers. Mesh decomposition has several advantages over domain decomposition. Domain decomposition, as its name suggests, means dividing the given domain, Ω, into several pieces. This can be a very difficult task depending on the complexity of Ω. Mesh decomposition, on the other hand, is geometry-independent. Since the initial mesh (and every structured quad mesh, actually) is stored in a matrix, dividing this mesh simply means selecting a number of rows and/or columns from the matrix. This is done automatically. For instance, example 2 presents a very complex non-convex domain. However, for this case mesh decomposition is as easy as it is for example 1, which has a very simple square-shape domain. After doing domain decomposition each piece of the decomposed domain remains unchanged during the mesh generation process; local meshes are adapted to the sub-domains. In contrast, in the mesh decomposition case, each sub-mesh adapts itself to the domain. They can change independently, facilitating the refinement procedure. We can appreciate this in example 1. Each sub-mesh is a uniform rectangular-shape mesh but, at the end of the mesh generation process,

each one presents a very different shape resulting from the mesh-size function used for mesh refinement.

4.- Numerical examples

4.1.- Example 1

For this example, we consider a square-shape domain with boundaries $x=-3$, $x=3$, $y=-3$, and $y=3$. The initial mesh is the simplest one, a uniform Cartesian grid with $n=m=60$. We have taken $P_t=6$, which means that we have the master processor that takes care of indicating when the mesh is ready based on the errors sent from other processors, and five processors working on building the mesh. The next step is mesh decomposition. Assuming that every processor has the same computational power, the initial mesh is divided into 5 submeshes with the same amount of points each (see figure **1**). Now, each processor will modify their submeshes interacting with the (up to) two closest neighbors after each iterations for updating the sub-boundaries' elements. Figure **1** shows the final submeshes obtained from each processor.

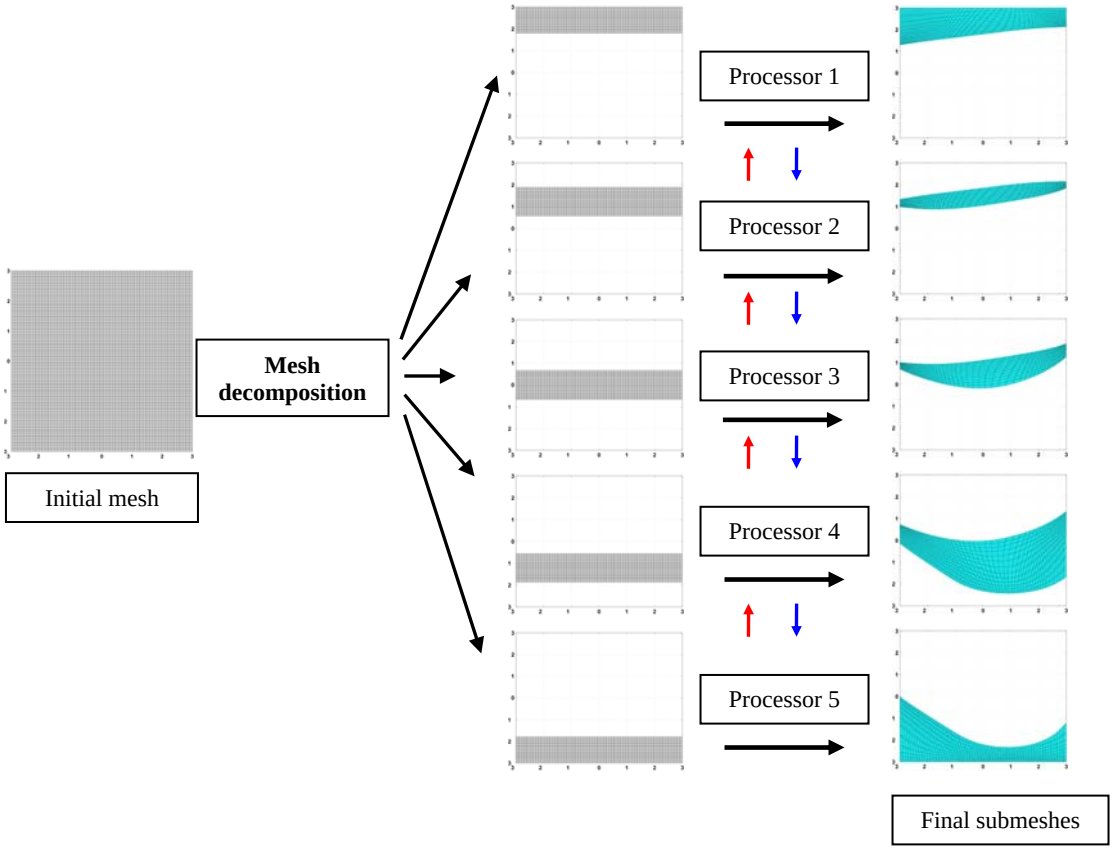


Fig. 1 Mesh decomposition and final submeshes obtained after several iterations.

We note how each submesh evolves independently and changes its shape in order to attain the desired refinement, given by the mesh-size function *f*. This is one of the differences between

mesh decomposition and domain decomposition, since for the latter the individual subdomains does not change their shapes. For this example, *f* is obtained from

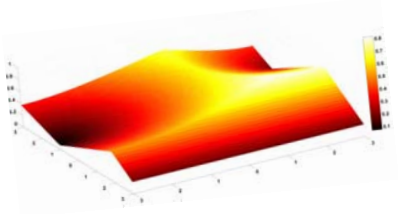
$$f(x,y) = \min\{ g_1(x,y), g_2(x,y), g_3(x,y) \}$$

where

$$g_1(x,y) = .3 \times |y+3| + .2$$

$$g_2(x,y) = .2 \times \sqrt{(x-3)^2 + (y-2)^2} + .2$$

$$g_3(x,y) = .15 \times \sqrt{(x+3)^2 + (y-1)^2} + .07$$



Mesh-size function *f*

The final mesh is obtained by putting together the different pieces, being careful with the repeated sub-boundaries.

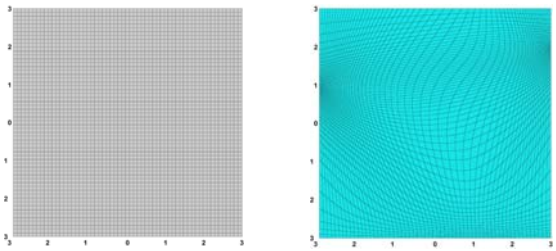
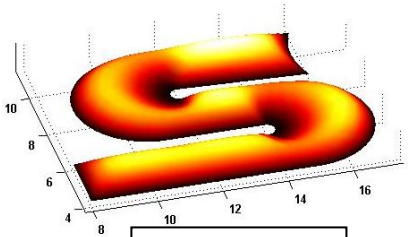


Fig. 2 Initial Cartesian grid, left, and final smooth mesh, right.

4.2.- Example 2

In this case, we have an S-shape domain. The initial mesh, again, is obtained the simplest way, just straight lines across the domain. After this, we proceed with mesh decomposition. As mentioned before, mesh decomposition is as easy to do as it was for the former convex example, since this is a domain-independent technique. Figure **3** shows the results. For this example we have used 7 processor with equal computational power. The mesh-size function, *f*, has been obtained by applying the R-function technique explained in [3]. *f* takes into account the curvature of the domain, producing small elements in regions close to sections of the boundary with high curvature.



Mesh-size function

5.- Conclusions and future works

The technique produces meshes of high quality that are smooth and can be adapted to different domains and different levels of refinement. The implementation of a mesh-size function offers flexibility to control the refinement of the mesh. The parallelization of the original CCBA can be done very easily by using a geometry-independent mesh decomposition technique, instead of

using domain decomposition. The duplication of the boundaries of the sub-meshes, on the other hand, guarantees that unfolded meshes will be produced.
We expect to develop a deeper analysis of the speed up and scalability of this parallel mesh generator. Also, we will study the possibility of extending the algorithm for constructing 3D meshes.

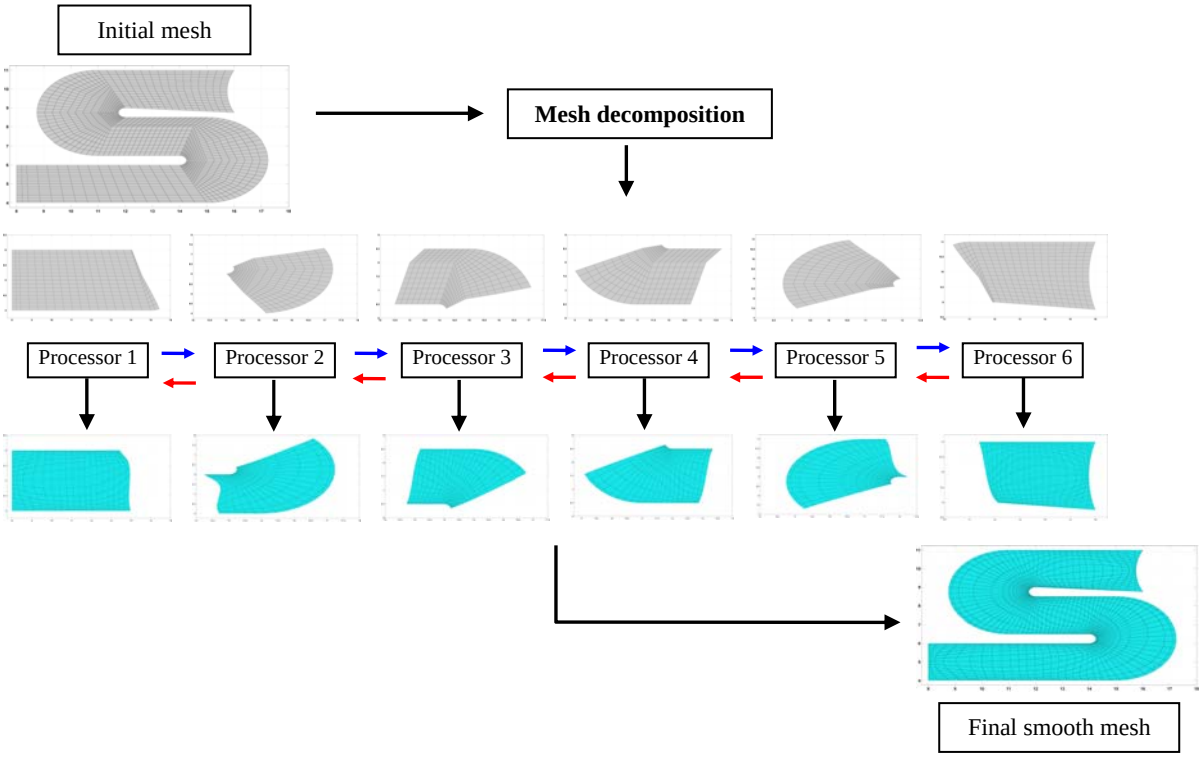


Fig. 3 Numerical results for the S-shape domain.

6.- References

1. E.D. Batista, *A convex-combination based algorithm for grid generation*, poster presentation at the Applied Computational Science and Engineering Student Support (ACSESS), San Diego State University, USA, March 7, 2007.
2. P.S. Pacheco, *Parallel programming with MPI*, Morgan Kaufmann Publishers, Inc. San Francisco, California, 1997.
3. V.L. Rvachev, T.I. Sheiko, V. Shapiro, I. Tsukanov, *Transfinite interpolation over implicitly defined sets*, Computer Aided Geometric Design, 18, 4, 195-220, 2001.
4. Castillo J.; Mathematical aspects of numerical grid generation; SIAM; (1991).

7.- Acknowledgements

Thanks very much Carny Cheng and Jennifer Winn for translating the original CCBA MATLAB code to C.