BI-OBJECTIVE RELIABILITY BASED DESIGN OPTIMIZATION (RBDO) INCORPORATING STATISTICAL DATA UNCERTAINTY

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Objective

- The objective of this research is to develop methods to incorporate statistical data uncertainty in reliability estimation and optimization

  -Develop methods for estimating the confidence bounds of inverse reliability measures

  -Formulate and solve a bi-objective optimization to minimize design weight and sensitivity to data uncertainty with reliability constraints.

Introduction

- Large structural systems such as space vehicles and aerospace structures, have typical component level failure probabilities of the order of 10^{-4} to 10^{-7}.
- Accurately estimating such low failure probabilities requires very high accuracy, high fidelity analyses, and accurate statistical data.
- Quantification of risk using reliability based approaches relies on availability of statistics of the random variables that affect the response.
- Uncertainty is introduced from the use of small sample sizes to estimate the distribution parameters (mean and variance – location and scale) of a chosen distribution function.
- Uncertainty in the statistical parameters and distribution functions introduces uncertainty into the reliability estimates.

Uncertainty in Failure Probability (P_f)

- Calculating probability of failure (P_f) requires evaluating the following multidimensional integral

$$P_f = \int_{g(\mathbf{x})<0} f_{\mathbf{x}}(\mathbf{x})d\mathbf{x}$$

  - $f(x)$ is the joint distribution function of the random variables
  - $g(x)$ is the limit state or design constraint function

- The probability of failure due to uncertainties in design variables and distribution parameters that describe the random variables is

$$\tilde{P}_f = \int_{g(\mathbf{x})<0} f_{\mathbf{x}}(\mathbf{x})d\mathbf{x} = \int_{g(\mathbf{x})<0} \left[\int_{S_{\boldsymbol{\theta}}} f_{\mathbf{x}|\boldsymbol{\theta}}(\mathbf{x}|\boldsymbol{\theta}) f_{\boldsymbol{\theta}}(\boldsymbol{\theta}) d\boldsymbol{\theta} \right] d\mathbf{x}$$

  - Where $f(x|\theta)$ is the condition joint probability function of random variables and $f(\theta)$ is the joint distribution function of random parameters.

Confidence Bounds for Probability of failure

- Data uncertainty is a epistemic (reducible) uncertainty resulting from insufficient information or knowledge
- We estimate effect of statistical data uncertainty by calculating confidence bounds (interval measures) for the estimated failure probability

  -An α confidence bound is given by $[(P_f)_{1-\alpha}, (P_f)_\alpha]$

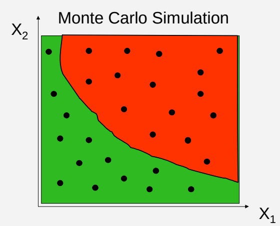
  - where

$$(P_f)_\alpha : \Phi\left(P_{f|\boldsymbol{\theta}}\right) = \alpha$$

$$(P_f)_{1-\alpha} : \Phi\left(P_{f|\boldsymbol{\theta}}\right) = (1-\alpha)$$

- The confidence bound calculated separates the effect of data (epistemic) uncertainty from variability (aleatoric uncertainty) in reliability estimation
- Estimating confidence bounds requires obtaining the cumulative distribution of failure probability

Sampling Based Methods (Monte Carlo Simulation)

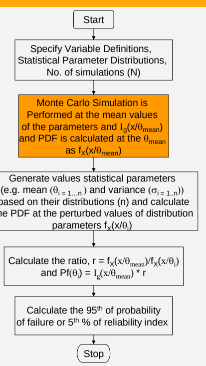


Probability of failure (P_f) using Monte Carlo Simulation

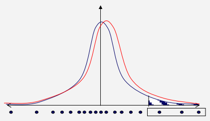
$$P_f = \frac{1}{N}\sum_{i=1}^{N} I_g(\mathbf{x})$$

$$I_g(x) = \begin{cases} 0 \; if \; g(x) > 0 \\ 1 \; if \; g(x) \le 0 \end{cases}$$

Monte Carlo Simulation with Ratio Method



Probability of failure calculated at the perturbed distribution parameters using a ratio method



$$\tilde{P}_f(\theta_i + \Delta\theta_i) = \frac{1}{N}\sum_{j=1}^{N} I_g(x) \frac{f(\mathbf{x}^{(j)}|\theta_i + \Delta\theta_i)}{f(\mathbf{x}^{(j)}|\theta_i)}$$

- Cannot use MCS results directly for optimization because it is very expensive and noisy
- Require surrogate (meta) models like response surface approximations to filter noise and reduce computational effort
- Fitting accurate response surface models for probability of failure is difficult because it changes several orders of magnitude in design space.

Probability Sufficiency Factor (PSF)

- A solution is to use and inverse reliability measure, such as ***Probabilistic Sufficiency Factor*** (*PSF*)
- ***PSF of a design is the minimum safety factor required to ensure that the any constraint violation has a probability less the specified target failure probability***

Safety factor for a constraint $g_r(\hat{\mathbf{x}}, \mathbf{d}) \le g_c(\hat{\mathbf{x}}, \mathbf{d})$ is $s(\mathbf{x}, \mathbf{d}) = \dfrac{g_c(\hat{\mathbf{x}}, \mathbf{d})}{g_r(\hat{\mathbf{x}}, \mathbf{d})} \ge s_r$

Probability of failure is then defined as $P_f = \int_{s<1} f_{\mathbf{x}}(\mathbf{x})$

Design reliability constraint requires $\text{Probability}(s \le 1) \le P_r$

An inverse measure PSF is proposed such that $\text{Probability}(s \le P_{sf}) = P_r$

PSF is the n^th smallest safety factor in a MCS

$$PSF = n^{th} \min_{i=1}^{N}\left[s_{cr}(\mathbf{x}_i)\right] \qquad n = NP_r$$

P_r is the target probability of failure.

No of samples	Safety Factor
1	0.78
2	0.80
3	0.82
4	0.87
5	0.93
9	0.99
10	1.04
99	2.03
100	2.14

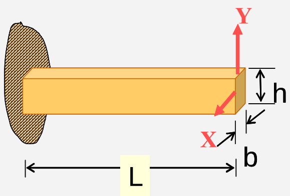
Confidence Interval Calculation of PSF

- A ratio method is proposed to calculate the PSF for perturbed values of the distribution parameters
- PSF is calculated at the distribution parameter PSF(θ) and also the perturbed value of PSF($\theta+\Delta\theta$), using the ratio of the joint distribution functions at MCS sampling points

$$PSF(\boldsymbol{\theta} + \Delta\boldsymbol{\theta}) = S(x^{(k)}) \quad \text{where} \quad S(x^{(1)}) < S(x^{(2)}) < \cdots < S(x^{(p)})$$

$$k = \max(p) \quad \text{such that} \quad \frac{1}{N}\sum_{j=1}^{p} \frac{f(\mathbf{x}^{(j)}|\theta + \Delta\theta)}{f(\mathbf{x}^{(j)}|\theta)} \le P_r$$

Test Problem



Variable	Mean	COV	Distribution
Load X (lb)	500	0.2	Normal
Load Y (lb)	1000	0.1	Normal
Modulus, E (psi)	29 x 10^6	0.05	Normal
Strength, S (psi)	40,000	0.05	Normal
Length, L (inch)	100	0	Fixed
Max deflection, D_0 (inch)	2.25	0	Fixed

Bi - Objective Function Formulation

- First Objective : ***Minimize volume of the cantilever beam***

$$f_V = wtL$$

  - Subjected to the constraints:

$$PSF_{Mean}^{Strength} \ge 1$$

$$PSF_{Mean}^{Dispalcement} \ge 1$$

- Second objective function: ***Minimize PSF confidence bound (effect of data uncertainty)***

$$f_{CB} = \left(PSF_{Mean}^{Strength} - PSF_{5\%}^{Strength}\right)^2 + \left(PSF_{Mean}^{Displacement} - PSF_{5\%}^{Displacement}\right)^2$$

  - Subjected to the constraints:

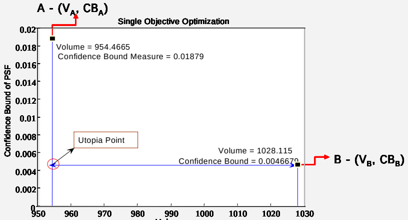
$$PSF_{Mean}^{Strength} \ge 1$$

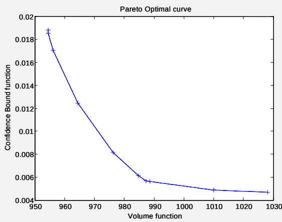
$$PSF_{Mean}^{Dispalcement} \ge 1$$

- Optimized the individual objective functions and obtained their respective values as shown in the figure below
- Normalize the two objective functions using the optimum values obtained from single objective optimization
- Used a weighted sum approach to perform optimization for the bi-objective case

$$f = (1-\alpha) f_V + \alpha f_{CB}$$

- Varied the weighting coefficient α from the 0 to 1 and repeated optimization to develop the pareto-optimum curve





Conclusions

- Developed methods to estimate the confidence bounds for the inverse probability measures (PSF).
- Performed robust reliability based design optimization.