Equation of State for Neutron Star Matter

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1 Abstract

Neutron stars are among the most enigmatic objects in the Universe. They possess the mass of our sun but are several billion times smaller than our sun. The matter in the cores of neutron stars is therefore compressed to densities that are several times higher than the density of atomic nuclei. Under such extreme physical conditions the conventional building blocks of matter as we know them (atoms, protons, electrons) give way to new and widely unexplored states of matter, such as superconducting quark matter and novel particle condensates searched for in the most powerful terrestrial collider experiments. In this paper we study the thermal evolution of neutron stars in order to explore the properties of ultradense matter and the inner workings of neutron stars. The calculations are performed in the framework of Einstein's theory of general relativity, since neutron stars curve the geometry of space-time so strongly that classical Newtonian theory of gravity fails to describe their properties.

2 The Model

In this model the strong interaction is described by interacting baryons through the exchange of a medium range attracting σ meson and a short range repulsive ω meson. To describe such a model we must take a Relativistic Mean Field approach.

2.1 The Field Equations

The field equations derived from the Lagrangian Density functions at the mean field level are given by

$$m_\omega^2\omega_0 = \sum_B g_{\omega B} n_B\,. \tag{1}$$

$$\rho_{03} = \frac{g_\rho}{m_\rho^2}\sum_B I_3 n_B, \tag{2}$$

$$m_\sigma^2\sigma_0 + g_3\sigma_0^2 + g_4\sigma_0^3 = \sum_B g_{\sigma B} S(m_{eff,B}, k_{F,B}) \tag{3}$$

The function $S(m_{eff,B}, k_{F,B})$ is expressed with the use of the integral

$$S(m_{meff,B}, k_{F,B}) = \frac{2J_B+1}{2\pi^2}\int_0^{k_{F,B}} \frac{m_{eff,B}}{\sqrt{k^2+m_{eff,B}^2}}k^2dk \tag{4}$$

where J_B is the spin projections of Baryon B, $k_{f,B}$ is the Fermi momentum of type B.

n_B is the particle number density in units of fm^{-3}

$$n_B = \frac{\gamma_B k_{F,B}^3}{6\pi^2}, \tag{5}$$

γ_B stands for the spin-isospin degeneracy factor which equal 4 for nucleons.

$m_{eff,B}$ is the effective baryon mass generated by the baryon and scalar field interaction and can be defined as

$$m_{eff,B} = m_B - g_{\sigma B}\sigma_0\,. \tag{6}$$

2.2 Energy and Pressure of the System

Now that we have acquired a solution for the effective mass of the system.The energy density and pressure can be obtained. The energy density is given by

$$\epsilon = \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\sigma^2\sigma_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 + \epsilon_B \tag{7}$$

where ϵ_B is defined as

$$\epsilon_B = \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F,B}} k^2dk\sqrt{k^2+m_{eff,B}^2} + \frac{1}{3}\sum_\lambda \frac{1}{\pi^2}\int_0^{k_\lambda} k^2(k^2+m_\lambda^2)^{1/2}\,. \tag{8}$$

The total pressure of the system is given by

$$p = \frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\sigma^2\sigma_0^2 + p_B + \frac{1}{2}m_\rho^2\rho_{03}^2 \tag{9}$$

where p_B is defined to be

$$p_B = \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{k_{F,B}} k^2dk\frac{k^2}{\sqrt{k^2+m_{eff,B}^2}} + \frac{1}{3}\sum_\lambda \frac{1}{\pi^2}\int_0^{k_\lambda} \frac{k^4}{(k^2+m_\lambda^2)^{1/2}}\,. \tag{10}$$

It is important to note that our Lagrangian contains 6 parameters. They are the meson-nucleon coupling constants g_ω and g_σ; the meson masses m_ω and m_σ; and the meson-meson self coupling constants g_3 and g_4. However, for our purposes we only looked at the linear terms and neglected any non-linear effects due to the g_3 and g_4 meson-meson self coupling constants.

2.3 Neutron Star Matter

Since we are dealing with a two particle many body interaction it is convenient to introduce a term δ that accounts for an asymmetry in the number density between protons and neutrons.

δ is given by

$$\delta = \frac{n_N - n_P}{n_N + n_P} \tag{11}$$

where n_N and n_P are the corresponding number densities for neutrons and protons.

Composition for Neutron Star matter

We show here the composition for the neutron star matter.

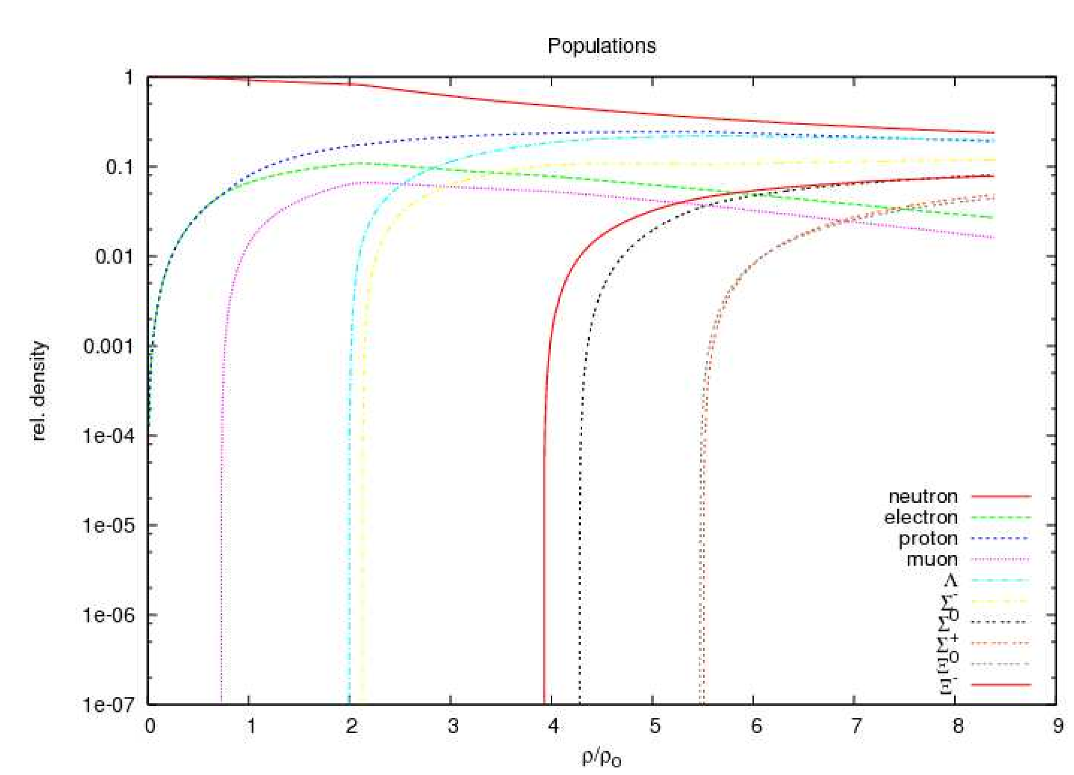

Figure 1: Composition for Neutron Star Matter.

Equation of State for Neutron Star Matter

Here we display some of the equations of state for nuclear matter for different parameter sets.

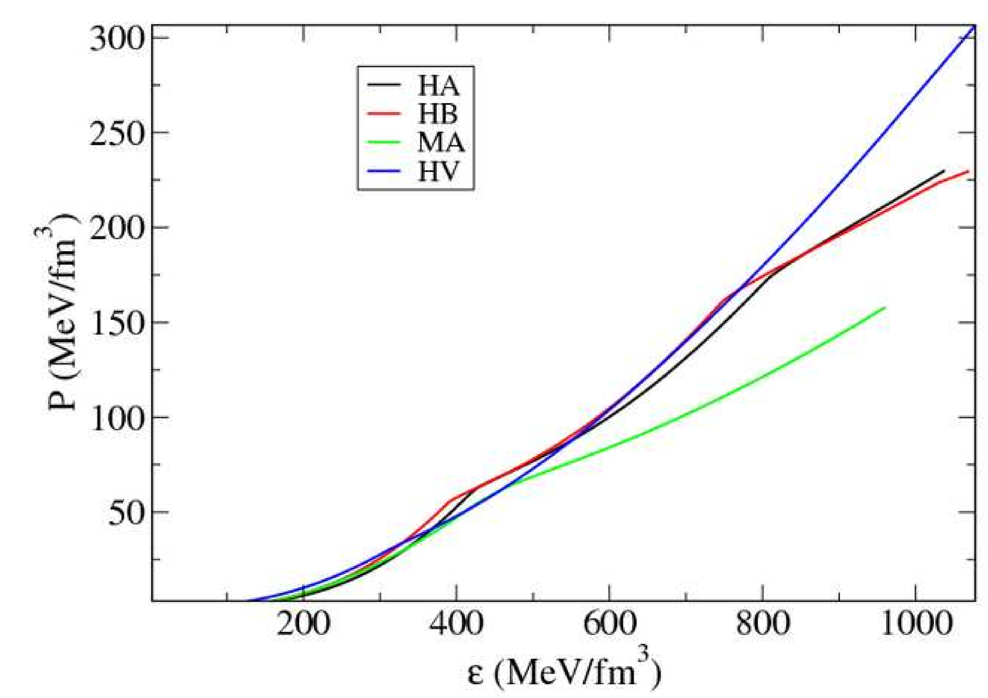

Figure 2: Equation of State for Neutron Star Matter

3 Sequence of Stars

We use the previous equations of state as an input for a code the solves the General Relativistic hydrostatic equilibrium equations and obtain the star structure. Here we plot some of the families of stars obtained.

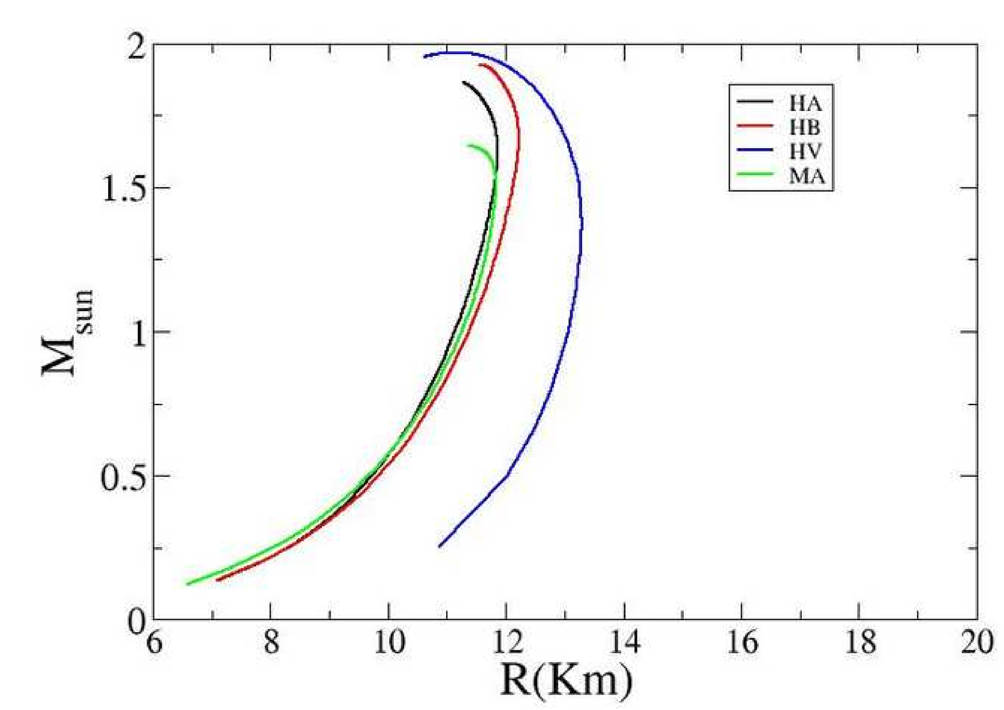

Figure 3: Mass Radius Diagram

4 Thermal Evolution of Neutron Stars

The thermal evolution of a Neutron Star is strongly dependent on its composition, because of that understanding how a Neutron tar cools down might help us to constrain further the equation of state. The General Relativistic Thermal Evolution equations of a Neutron Star are:

$$\frac{\partial \tilde{L}_r}{\partial M_r} = -\frac{\tilde{Q}_\nu}{\rho\sqrt{1-\frac{2GM_r}{c^2 r}}} - \frac{C_v}{\rho\sqrt{1-\frac{2GM_r}{c^2 r}}}\frac{\partial \tilde{T}}{\partial t}, \tag{12}$$

$$\frac{\partial \ln \tilde{T}}{\partial M_r} = \tilde{\nabla}\frac{\partial \ln P}{\partial M_r}, \tag{13}$$

We solved these equations for the equations of state studied in this work and obtained

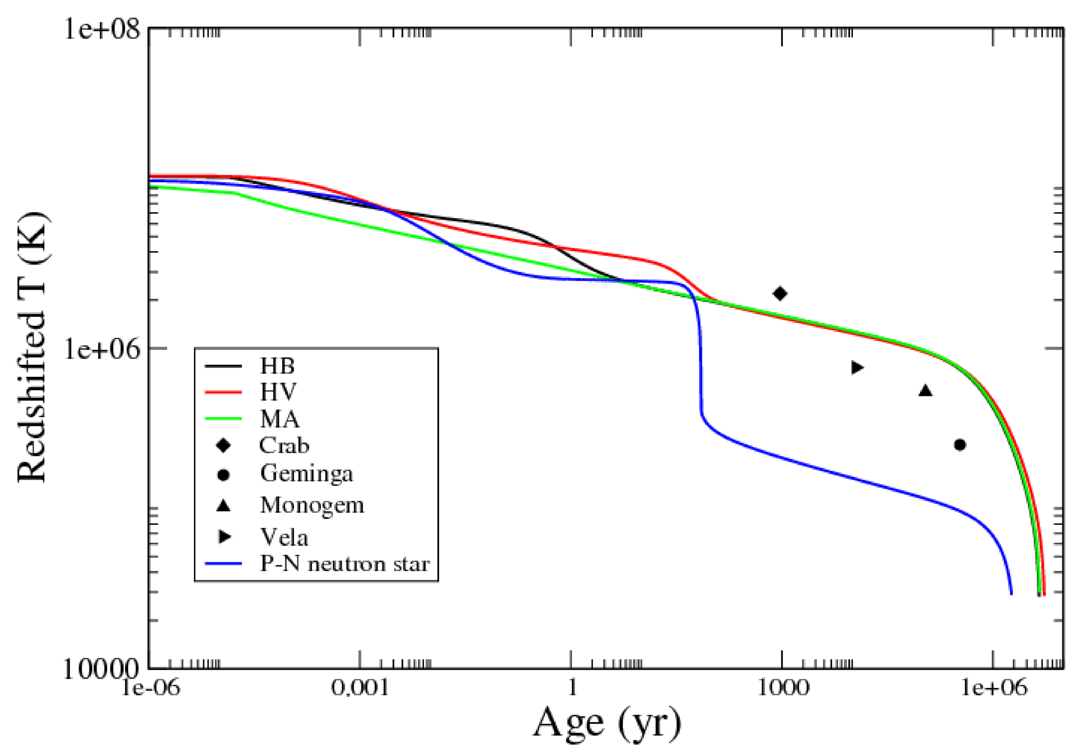

Figure 4: Thermal Evolution

5 Conclusions

In this project we have studied several different equations of state. We have tried several parameters set, always constrained by nuclear matter properties. As expected the neutron stars properties strongly depend on the equation of state.

One of the objectives of this work was to perform a threefold investigation: we standed studying the neutron star matter and composition, moved to neutron star structure and ended by studying the thermal evolution of the star. All of these processes are extremely important in order to build a more precise theory of Neutron Stars. By comparing theoretical predictions with observed data we can constrain the equation of state for these objects, which will allow us to understand better the fundamental physics that govern our universe.

6 References

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