Image Smoothing and Edge Detection by Nonlinear Diffusion and Bilateral Filter

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Abstract

We propose a new image smoothing and edge detection technique that employs a combination of nonlinear diffusion and bilateral filtering. The model is based upon two very well established methodologies in the image processing community, which makes the model easy to understand and implement. Our numerical experiments show that the proposed model is capable of achieving more accurate reconstructions from noisy images, as compared to two other popular nonlinear diffusion models in the literature. We also propose a new and simple diffusion stopping criterion, based on the second derivative of the correlation between the noisy image and the filtered image. This indirect measure allows stopping the diffusion process very close to the point of maximum correlation between the noise-free image and the reconstructed image, in the absence of the former. The stopping criterion is sufficiently general to be applied with most nonlinear diffusion methods normally used for image noise removal.

Introduction

Analysis of image features in early vision presents two almost mutually exclusive requirements. On the one hand, it is desirable to smooth homogeneous regions of the image, and on the other hand, we wish to preserve the location of the boundaries or edges accurately. In order to achieve both goals, the classical multiscale analysis theory due to Marr and Hildreth [1980] later formalized by Witkin [1983], Koenderink [1984] and Canny [1986], uses low-pass filtering obtained by convolving the image with Gaussians of increasing variance. This convolution of the image with a Gaussian at each scale is equivalent to the solution of the heat equation with the image as initial state [Koenderink 1984].

One of the first attempts to derive a model that incorporates local information from an image within a PDE framework was conducted by Perona and Malik [1990]. They proposed a nonlinear diffusion model (which they called "anisotropic") in order to avoid the blurring of edges and other localization problems presented by linear diffusion models. The model accomplishes this by applying a process that reduces the diffusivity in places having higher likelihood of being edges. This likelihood is measured by a function of the local gradient.

The results obtained by Perona and Malik were visually very impressive. Many of today's PDE-based image processing and edge detection models stem from the classic Perona-Malik nonlinear diffusion method.

Test Models

In order to compare the performance of the proposed model we implemented the three models below using finite difference, and a simple performance measure based on the correlation between the noise-free image and the three filtered images. Model 1 is the classic Perona-Malik model:

$$\partial_t u - \nabla \cdot \left(g\left(\|\nabla u\|^2 \right) \nabla u \right) = 0$$

$$\partial_{\mathbf{n}} u = 0, \qquad u(0,\mathbf{x}) = u_0(\mathbf{x}),$$

$$g\left(\|\nabla u\|^2 \right) = \frac{1}{1 + \|\nabla u\|^2 / \lambda^2}, \qquad \lambda = 10^{-2}.$$

Parameter $\lambda = 10^{-2}$ was estimated as an average of the "robust scale" proposed in [Black et al. 1998, Black and Sapiro 1999], using the initial state of the images employed in our tests. Model 2 is the Perona-Malik variant by Catté, Lions, Morel and Coll [1992]:

$$\partial_t u - \nabla \cdot \left(g\left(\|\nabla u_\sigma\|^2 \right) \nabla u \right) = 0$$

$$\partial_{\mathbf{n}} u = 0, \qquad u(0,\mathbf{x}) = u_0(\mathbf{x}),$$

$$g\left(\|\nabla u\|^2 \right) = \frac{1}{1 + \|\nabla u_\sigma\|^2 / \lambda^2}, \qquad \lambda = 10^{-2},$$

$$u_\sigma = G_\sigma * u, \qquad \sigma = 1,$$

where G_σ is a Gaussian kernel, and $*$ denotes a convolution. It has been shown [Mrázek 2001] that $\sigma = 1$ is sufficient for a large interval of noise variances provided that the noise in neighboring pixels is uncorrelated and that the grid size is one. Model 3 is the proposed model:

$$\partial_t u - \nabla \cdot \left(g\left(\|\nabla u_{BF}\|^2 \right) \nabla u \right) = 0$$

$$\partial_{\mathbf{n}} u = 0, \qquad u(0,\mathbf{x}) = u_0(\mathbf{x}),$$

$$g\left(\|\nabla u\|^2 \right) = \frac{1}{1 + \|\nabla u_{BF}\|^2 / \lambda^2}, \qquad \lambda = 10^{-2},$$

$$u_{BF} = BF(u), \qquad \sigma_s = 3, \qquad \sigma_r = 10^{-2}.$$

In this model

$$BF(u(\mathbf{x})) = \frac{1}{W(\mathbf{x})} \int_\Omega G_{\sigma_s}(\xi, \mathbf{x}) G_{\sigma_r}(u(\xi), u(\mathbf{x})) * u(\mathbf{x})\, d\xi,$$

$$W(\mathbf{x}) = \int_\Omega G_{\sigma_s}(\xi, \mathbf{x}) G_{\sigma_r}(u(\xi), u(\mathbf{x}))\, d\xi,$$

$$G_{\sigma_s} = \exp\left(-\frac{|\xi - \mathbf{x}|^2}{2\sigma_s^2} \right), \qquad G_{\sigma_r} = \exp\left(-\frac{|u(\xi) - u(\mathbf{x})|^2}{2\sigma_r^2} \right).$$

The basic idea underlying bilateral filtering is to combine domain and range filtering, thereby enforcing both geometric and photometric locality. The parameters σ_s and σ_r are chosen according to the desired amount of low-pass filtering and desired amount of combination of pixel values, respectively [Tomasi and Manduchi 1998]. We loosely followed the recommendations given in [Liu et al. 2006] for choosing σ_s, and the ones in [Paris et al. 2007] for choosing σ_r. They give us a compact kernel that allows a very fast execution of the bilateral filtering.

Experimental Results

For all the cases, we observe that the best image reconstructed by the proposed model is closer to the noise-free image than the best images reconstructed by the other two models tested (see Fig. 1.) We can also observe that the proposed model performs "in between" the other two models in terms of speed of reconstruction. The Catté-Lions-Morel-Coll model accomplishes the fastest reconstruction, *i.e.* it attains its best reconstructed image in fewer iterations than the other two methods. The classic Perona-Malik model achieves a better reconstruction if we were to iterate beyond the optimal stopping points of the three models.

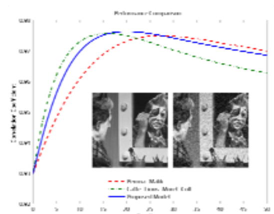

Fig. 1. *Correlation coefficient between the noise-free image of the Clown and the filtered image of the Clown at each iteration, along with the noise-free image of the Clown (left) and the noisy image of the Clown (right) corrupted by additive Gaussian white noise, SNR = 21.2 dB. The maximum value of the correlation coefficient for each model is as follows: Perona-Malik, 0.9749; Catté-Lions-Morel-Coll, 0.9757; Proposed Model, 0.9763.*

Diffusion Stopping Criterion

In order for any of the three models to accomplish its best possible reconstruction, one has to be able to stop the diffusion process at the peak of its performance, in the absence of the noise-free image. In general, this remains an open problem. We propose a new (simple) diffusion-stopping criterion inspired by observation of the behavior of the correlation between the noise-free image and the filtered image, $\operatorname{corr}(f, u)$, and the correlation between the noisy image and the filtered image, $\operatorname{corr}(u_0, u)$ (Fig. 2).

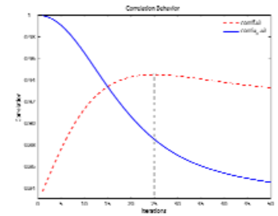

Fig. 2. *The correlation coefficient between the noise-free image and the filtered image increases as the filtered image moves closer to the noise-free image. When the measure reaches a peak it decreases as the filtered image moves slowly towards a constant value. The correlation coefficient between the noisy image and the filtered image decreases gradually from a value of 1.0 (perfect correlation), to a constant value as the filtered image becomes smoother.*

The performance of the proposed stopping criterion can be observed below along with the reconstructed images of the Clown (Fig. 3.) We observe that the stopping criterion is almost optimal, allowing the diffusion process to stop near the point where the filtering methods reach their best possible image reconstructions.

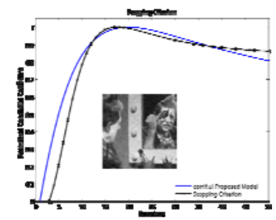

Fig. 3. *Proposed stopping criterion performance along with the reconstructed image of the Clown using the proposed model. The measure $\operatorname{corr}(f, u)$ suggests stopping the diffusion process after 20 iterations, while the proposed stopping criterion suggests to stop the diffusion process after 18 iterations.*

Conclusion

The proposed model is able to obtained the best possible reconstruction of a noisy image as measured by the correlation coefficient between the noise-free image and the reconstructed image. In a real-world situation, the true (unperturbed) image would not be known, hence the correlation coefficient between this and the reconstructed image could not be measured. Therefore, we also propose a new and simple diffusion stopping criterion, based on the second derivative of the correlation between the noisy image and the filtered image.

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