Stochastic Resonance in Coupled-Cell Bistable Devices

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Objective

The behavior of dynamical systems in the presence of a noisy perturbative force has been of immense interest over the last 30+ years. One of the most important findings is the phenomenon of Stochastic Resonance (SR). Here we investigate the SR effects on high dimensional ensembles of dynamical systems. These ensembles are called coupled-cell systems and exhibit a wide class of behavioral phenomena on their own regards. Here we wish to extend the theory of stochastic resonance onto coupled-cell systems. Although preliminary findings have been found by a number of others the findings have not shown a commonality in the systems from which a governing result may be drawn. Here we present a first attack in this direction.

Coupled-Cell Dynamics

The dynamics of ensembles of dynamical systems may be described using the theory of coupled-cell systems. Here each cell is a dynamical systems that may or may not be like the remaining cells in the system. To this end all cells may, in fact, be different from one another. If cells are alike in form it does not mean that the parameters in the systems are alike. Using concepts from Modern Algebra and Graph Theory one is capable of predicting what type of behavior(s) may be found in the coupled-cell system based only on the dimension of each cell and the coupling arrangement described as a directed graph.

Stochastic Resonance

Stochastic Resonance (SR) is a noise induced maximization of a measurement of some quantity of the dynamics. This is a specialized behavior not found in all noisy dynamical system but frequent enough to have a prime importance. It is known that when a SR is present that the maximum response does not occur at the same intensity of the noise for all types of measurements. In fact, maximizations generally occur at unique intensities for each different measured quantity.

Coupled-Cells of Bistable Devices

The theory of coupled-cells when applied to cells with bistable dynamics takes on an interesting flavor. Suppose, for instance, that these bistable devices are overdamped and dc-driven. Then in the absence of a coupling the devices will evolve only to a fixed point. However, in the presnece of a coupling of appropriate strength the device dynamics are described by evolution on a heteroclinic cycle. This general behavior even persists in the absence of the dc-drive. This suggests that the region of oscillatory behavior is appropriately described by the strength of the coupling needed to induce the system into oscillations given the strength of the dc-drive. Furthermore, the asymmetry found in the time series for nonzero dc drive allows these system to be used as a sensor for dc (constant magnitude) signals.

SR Measures

The ways in which a SR appears varies nearly as much as the way they are measured. This is jointly due to the fact that a SR does not appear in all quantities, given one even exists, and also each researcher has their own preference in the quantity that is measured. Tantamount to the theory of SR is the measurement of the frequency in the underlying system and the Signal-to-Noise Ratio. Partly due to the importance of these quantities to system performance and their principle use in the quantification of a SR phenomena.

Frequency

Many frequency measurements may generally be of interest. Here we are interested in the fundamental frequency of the system at which the Power Spectral Density (PSD) has a maximum amplitude. This is especially useful for measurement of systems with purely noise induced oscillations.

SNR

Similar to the frequency the SNR measure we employ here is a measure of the SNR at the same frequency where the PSD is maximized. Specifically, let $R(f)$ be the SNR at frequency f, f_0 be the zero frequency, and f_M be the frequency of at maximum PSD. Then $SNR(dB) = 10log_{10}(\frac{R(f_M)-R(f_0)}{R(f_0)})$.

Stochastic Models

We investigate a number of Gaussian noise forcings in two different bistable coupled cell systems in order to investigate and derive a law describing the influence on noise on these particular types of coupled-cell systems.

The two systems are:

1. Coupled Core Fluxgate Magnetometers (CCFM's)

$$dX_i = (-X_i + \tanh(c(X_i + \lambda X_{i+1} + \varepsilon_0)))dt$$

2. Coupled-Cell Electric Field Sensors (CCEFS's)

$$dX_i = (aX_i - bX_i^3 + \varepsilon_0 + \lambda(X_i - X_{i+1}))dt$$

In the above i ranges from 1 to N cyclically, where N is a necessarily odd number. The oddity of N is a requirement of the deterministic behavior but may not be so for N large and even. We will explore a total of 8 different noise sources in each of the systems. That is, for different noise scenarios for each system. The noise sources will be derived from White Gaussian Process characterized by

$$E[\eta(t)] = 0,$$
$$E[\eta(t)\eta(s)] = D\delta(|t-s|),$$

or from a Colored Gaussian process characterized by

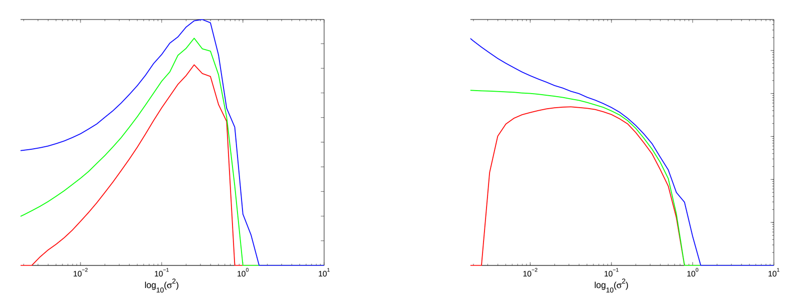
$$E[\eta(t)] = 0,$$
$$E[\eta(t)\eta(s)] = \frac{D}{\tau_c}e^{-\frac{|t-s|}{\tau_c}}.$$

This colored noise is known as an Ornstein Uhlenbeck (OU) Process or OU Noise.

In all images that follow the blue curve corresponds to a system where the coupling is 1.1 times the strength of coupling where oscillations first appear (super-critical coupling). The green curve corresponds to a coupling where oscillations first appear (The heteroclinic bifurcation, critical coupling). The red curve corresponds to a coupling strenght below the point of onset of bifurcations (sub-critical coupling). The horizontal axis is the intensity of the noise σ^2 ($\sigma^2 = D$ in the white noise case and $\sigma^2 = D/\tau_c$ in the colored noies case). The vertical axis is the frequency where the PSD is maximized for the plots on the left and on the right the vertical axis is the Signal-to-Noise Ratio.

CCFM with Additive White Noise

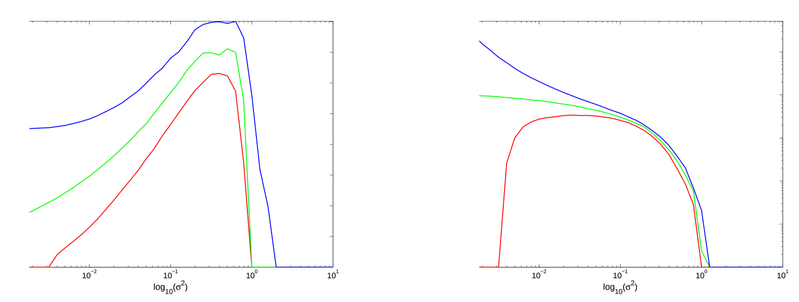
$$dX_i = (-X_i + \tanh(c(X_i + \lambda X_{i+1} + \varepsilon_0)))dt + \sqrt{2D}dB_i$$



CCFM with Additive OU Noise

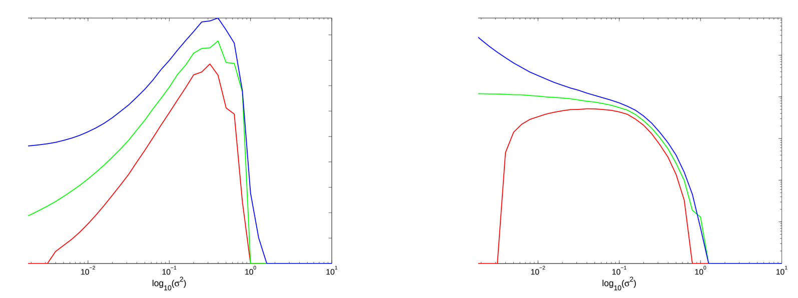
This model was used in [6] where the phenomena of stochastic resonance was investigated and has been a major drive to the current work displayed here.

$$dX_i = (-X_i + \tanh(c(x_i + \lambda X_{i+1} + \varepsilon_0)) + \eta_i)dt$$
$$d\eta_i = -\frac{\eta}{\tau_c}dt + \frac{\sqrt{2D}}{\tau_c}dB_i$$



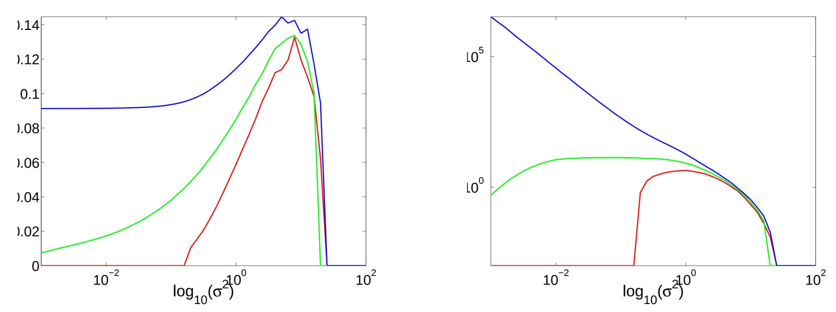
CCFM with Additive OU Noise Inside tanh Nonlinearity

$$dX_i = (-X_i + \tanh(c(X_i + \lambda X_{i+1} + \varepsilon_0 + \eta_i)))dt$$
$$d\eta_i = -\frac{\eta}{\tau_c}dt + \frac{\sqrt{2D}}{\tau_c}dB_i$$



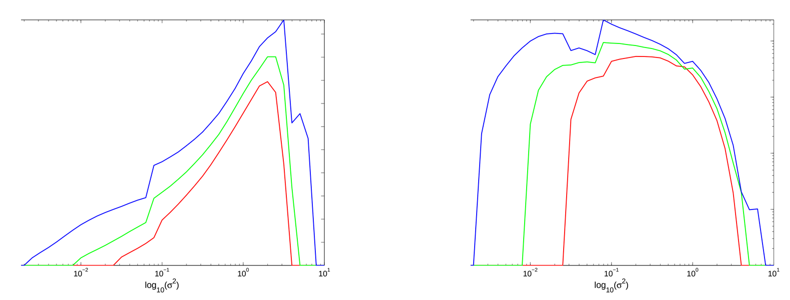
CCFM with Multiplicative OU Noise as a Perturbation to c

$$dX_i = (-X_i + \tanh((c + \eta_i)(X_i + \lambda X_{i+1} + \varepsilon_0)))dt$$
$$d\eta_i = -\frac{\eta}{\tau_c}dt + \frac{\sqrt{2D}}{\tau_c}dB_i$$



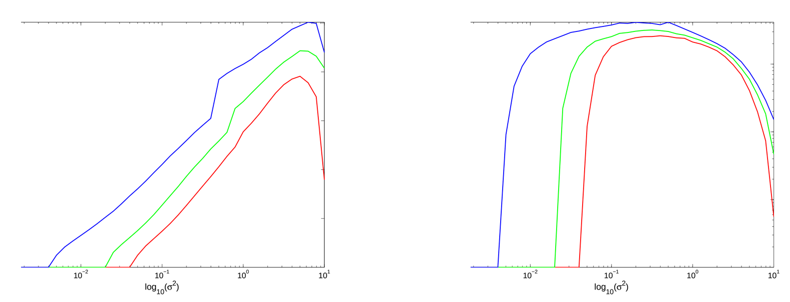
CCEFS with Additive White Noise

$$dX_i = (aX_i - bX_i^3 + \varepsilon_0 + \lambda(X_i - X_{i+1}))dt + \sqrt{2D}dB_i$$



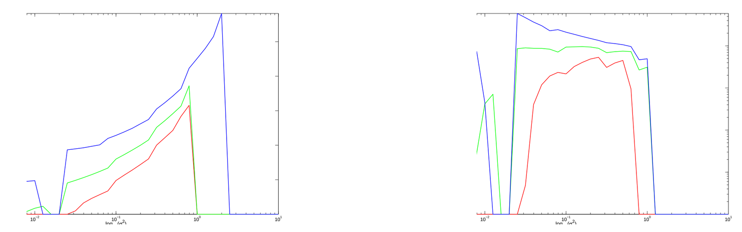
CCEFS with Additive OU Noise

$$dX_i = (aX_i - bX_i^3 + \varepsilon_0 + \lambda(X_i - X_{i+1}) + \eta)dt$$
$$d\eta_i = -\frac{\eta}{\tau_c}dt + \frac{\sqrt{2D}}{\tau_c}dB_i$$



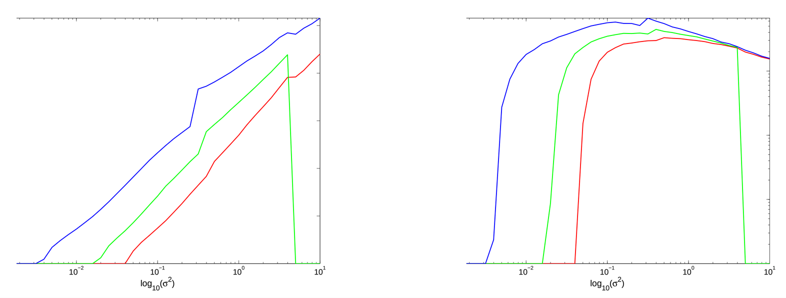
CCEFS with Multiplicative White Noise from Linear Component

$$dX_i = (aX_i - bX_i^3 + \varepsilon_0 + \lambda(X_i - X_{i+1}))dt + \sqrt{2D}X_i dB_i$$



CCEFS with Multiplicative OU Noise from Linear Component

$$dX_i = (aX_i - bX_i^3 + \varepsilon_0 + \lambda(X_i - X_{i+1}) + \eta X_i)dt$$
$$d\eta_i = -\frac{\eta}{\tau_c}dt + \frac{\sqrt{2D}}{\tau_c}dB_i$$



Conclusion

The work still in its infancy we have little to state at this point. However, from preliminary observation due to numerical simulations there are some conclusions that we may draw. For the first three types of noise in the CCFM system the SR behavior is relatively the same. For the Multiplicative Colored Core (4^{th}) case) he response is quite different. In fact, we know little about this as this response does not look anything like what is found in the literature. The E-Field Sensors have a much more standard type of response where SR effects are found here even when the osillations are supercritical (not the case with the CCFM's). One of the more interesting results is that it seems that the E-Sensor systems seem to have a sort of doubly stochastic resonant behavior. This is seemingly caused by a cascade so stochastic P-bifurcations which are currently being investigated by the authors.

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