A Novel Configuration of Coupled Core Fluxgate Magnetometers: Oops = Good

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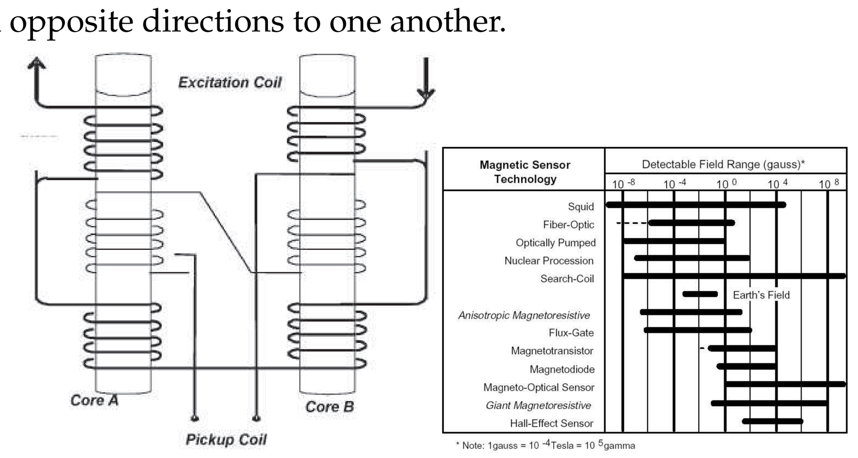
Objective

Recent theoretical and experimental work has shown that unidirectional coupling can induce oscillations in overdamped and undriven bistable dynamic systems that are non-oscillatory when uncoupled; in turn, this has been shown to lead to new mechanisms for weak (compared to the energy barrier height) signal detection and amplification. The potential applications include fluxgate magnetometers, lectric field sensors, and arrays of Superconducting Quantum Interference Device (SQUID) rings. In the particular case of the fluxgate magnetometer, we have developed a "coupled-core fluxgate magnetometer" (CCFM); this device has been realized in the laboratory and its dynamics used to quantify many properties that are generic to this class of systems and coupling. The CCFM operation is underpinned by the emergent oscillatory behavior in a unidirectionally coupled ring of wound ferromagnetic cores, each of which can be treated as an overdamped bistable dynamic system when uncoupled. In particular, one can determine the regimes of existence and stability of the (coupling-induced) oscillations, and the scaling behavior of the oscillation frequency. More recently, we studied the effects of a (Gaussian) magnetic noise floor on a CCFM system realized with $N = 3$ coupled ferromagnetic cores. In this work, we first introduce a variation on the basic CCFM configuration that affords a path to enhanced device sensitivity, particularly for $N > 3$ coupled elements.

Fluxgate Magnetometers

Fluxgate magnetometers are considered to be the most cost-efficient magnetic field sensors for applications that require measuring relatively small magnetic fields in the 0.01mT regime. Originally developed around 1928, today's highly specialized devices can measure magnetic fields in the range of 1-10 pT/$\sqrt{Hz}$ for a variety of magnetic remote sensing applications. In its most basic form, the fluxgate magnetometer consists of two detection coils wound around two ferromagnetic cores (usually a single core configured as an open-ended "racetrack") in opposite directions to one another.



(Left) Basic design of a traditional fluxgate magnetometer. (Right) Sensitivity ranges for different magnetometers.

Residence Time Description

In our ongoing work (on the single core fluxgate as well as the CCFM) we rely on a readout mechanism, based on a threshold crossing strategy, that consists of measuring the "residence times" of the ferromagnetic core(s) in the two stable states of the potential energy function. When the potential energy function is skewed due to the presence of a target dc signal, the residence times are no longer equal. Then either their difference or ratio can be used to quantify the signal. The sensitivity of this residence times distribution (RTD) based readout has been shown to increase with lowered bias frequency and amplitude; these conditions are the opposite of the requirements for enhancing sensitivity in traditional readouts, so that lower onboard power as well as far simpler electronics can be implemented, with benefit, for this (time domain based) readout strategy.

Background of Dynamics: Standard Orientation

A conventional (i.e. single core) fluxgate magnetometer can be treated as a nonlinear dynamic system by assuming the core to be approximately single-domain, and writing down an equation for the evolution of the (suitably normalized) macroscopic magnetization parameter $x(t)$: $\dot{x}(t) = -\nabla_x U(x)$ in terms of the potential energy function $U(x,t) = x^2(t)/2 - c^{-1}\ln\cosh c[(x(t) + A\sin\omega t + \varepsilon(t)]$, where c is a temperature-dependent nonlinearity parameter, which controls the topology of the potential function: the system becomes monostable, or paramagnetic, for $c < 1$ corresponding to an increase in the core temperature past the Curie point. The overdot denotes the time-derivative, $A\sin\omega t$ is the known bias signal that switches the core dynamics between the potential minima, and $\varepsilon(t)$ is an external target signal (taken to be dc throughout this treatment).
The CCFM is, then, constructed by *uni*directionally coupling N (odd) wound ferromagnetic cores with cyclic boundary condition, thereby leading to the dynamics,

$$\dot{x}_i = -x_i + \tanh(c(x_i + \lambda x_{i+1} + \varepsilon)), \quad i = 1,\ldots,N \text{ mod } N,$$

where $x_i(t)$ represents the (suitably normalized) magnetic flux at the output (i.e. in the secondary coil) of unit i, and $\varepsilon \ll U_0$ is an external dc "target" magnetic flux, U_0 being the energy barrier height (absent the coupling) for each of the elements (assumed identical for theoretical purposes). It is important to note that the oscillatory behavior occurs even for $\varepsilon = 0$, however when $\varepsilon \neq 0$, the oscillation characteristics change; these changes can be exploited for signal quantification purposes, the motivation for this work. A theoretical analysis shows that the system exhibits coupling-induced oscillatory behavior with the following features:

(1) The oscillations commence when the coupling coefficient exceeds a threshold value $\lambda_c = -\varepsilon - x_{inf} + c^{-1}\tanh^{-1}x_{inf}$, with $x_{inf} = \sqrt{(c-1)/c}$; note that in our convention, $\lambda < 0$ so that oscillations occur for $|\lambda| > |\lambda_c|$.

(2) The individual oscillations (in each elemental response) are separated in phase by $2\pi/N$, and have period $T_i = N\pi\left(1/\sqrt{\lambda_c - \lambda} + 1/\sqrt{\lambda_c - \lambda + 2\varepsilon}\right)/\sqrt{cx_{inf}}$.

(3) The *summed* output oscillates at period $T_+ = T_i/N$ and its amplitude (as well as that of each elemental oscillation) is *always supra*threshold.

(4) The RTD can be computed as $\Delta t \approx \pi\left(1/\sqrt{\lambda_c - \lambda} - 1/\sqrt{\lambda_c - \lambda + 2\varepsilon}\right)/\sqrt{cx_{inf}}$, which vanishes (as expected) for $\varepsilon = 0$.

(5) The system responsivity, defined via the derivative $\partial\Delta t/\partial\varepsilon$, is found to increase dramatically as one approaches the critical point in the oscillatory regime.

The New Configuration: Alternating Orientation

Laboratory experiments seem to indicate that the sensitivity of a CCFM-based system of fluxgates increases by simply alternating the orientation of each individual fluxgate. We call this new arrangement a CCFM system with *Alternating Orientation* (AO). We should clarify that the coupling scheme remains the same, i.e., unidirectional coupling via induction. The only feature that changes is the direction at which the individual fluxgates are aimed for signal detection purposes. Thus the sign in front of the target signal ε alternates between $+$ and $-$ so that the governing equations (for the deterministic system) become,

$$\dot{x}_i = -x_i + \tanh(c(x_i + \lambda x_i + (-1)^{i+1}\varepsilon)), \; i = 1,\ldots,N \; mod \; N. \quad (1)$$

The oscillations emerge via global bifurcations of a heteroclinic cycle connecting six saddle-node equilibrium points: $(1,-1,-1)$, $(1,1,-1)$, $(-1,1,-1)$, $(-1,1,1)$, $(-1,-1,1)$, and $(1,-1,1)$. Furthermore, solution trajectories are confined (as is typically the case with heteroclinic cycles) to the intersection of certain invariant planar regions given by (with $\lambda < 0$):

$$\begin{aligned} \delta_i &= \{x_i : \lambda x_i < 1, \quad x_{(i+2 \bmod 3)} = -1\}, \quad & i = 1,2,3, \\ \delta_i &= \{x_i : \lambda x_i > -1, \; x_{(i+2 \bmod 3)} = 1\}, \quad & i = 4,5,6. \end{aligned}$$

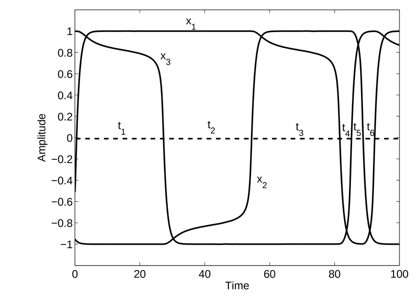
The saddle-node points exist only for $\lambda > \lambda_c^{AO}$ and are annihilated when the periodic solutions appear.

Through analysis, involving existence properties on the saddle-node points for cyclic behavior, it was found that there exists a value of λ,

$$\lambda_c^{AO} = -\epsilon + \frac{1}{c}\ln(\sqrt{c} + \sqrt{c-1}) - \tanh(\ln(\sqrt{c} + \sqrt{c-1})), \quad (2)$$

such that for all $\lambda < \lambda_c^{AO}$, $\lambda_c^{AO} < 0$, that all $x_i(t)$ will exhibit oscillatory behavior.

Calculation of the Period and Residence Times for the AO CCFM



Due to behavior of the neighboring fluxgates we may rewrite, for a small time interval, $x_3(t)$ as (when $N = 3$)

$$\dot{x_3}(t) = -x_3 + \tanh c(x_3 + \lambda + \varepsilon). \quad (3)$$

From here we can calculate the time (t_1) it takes $x_3(t)$ to go from $+1$ to 0 as

$$t_1 = \int_1^0 \frac{dx_3}{\tanh c(x_3 + \lambda + \varepsilon) - x_3}, \quad (4)$$

This integral cannot be calculated analytically, however by using consideration of the behavior of $x_3(t)$ we can approximate the integral as

$$t_1 \approx \int_{-\infty}^{\infty} \frac{dx_3}{\lambda_{c3} - \lambda + cx_{inf}(x_3 - x_{m3})^2} = \frac{\pi}{\sqrt{cx_{inf}}\sqrt{\lambda_{c3} - \lambda}}, \quad (5)$$

where $\lambda_{c3} = -\varepsilon - x_{inf} + \frac{1}{c}\tanh^{-1}x_{inf}$, and $x_{m3} = \lambda_{c3} - \lambda + x_{inf}$ ($x_{inf} = \sqrt{(c-1)/c}$)
Curiously, this calculation is valid for all t_i for $i = 1,2,3$. We must now calculate the values t_i for $i = 4,5,6$.
Following a similar prescription we find that

$$t_4 \approx \int_{-\infty}^{\infty} \frac{dx_3}{\lambda_{c3} - \lambda + 2\varepsilon + cx_{inf}(x_3 + x_{m33})^2} = \frac{\pi}{\sqrt{cx_{inf}}\sqrt{\lambda_{c3} - \lambda + 2\varepsilon}} \quad (6)$$

Noting that the period is $T_i = 3(t_1 + t_4)$ for $i = 1,2,3$ and the residence time difference is

$$\Delta_1 t = \frac{3\pi}{\sqrt{cx_{inf}}}\left[\frac{1}{\sqrt{\lambda_{c3} - \lambda}} - \frac{1}{\sqrt{\lambda_{c3} - \lambda + 2\varepsilon}}\right]. \quad (7)$$

and for $i = 2,3$

$$\Delta_i t = \frac{\pi}{\sqrt{cx_{inf}}}\left[\frac{1}{\sqrt{\lambda_{c3} - \lambda}} - \frac{1}{\sqrt{\lambda_{c3} - \lambda + 2\varepsilon}}\right]. \quad (8)$$

We note the absence of the multiplier of three in the residence time difference for $i = 2,3$ which gives a result identical to that obtained for the Standard Orientation. Generalizing this to arbitrarily large N (odd) we find that

$$T_i = \frac{N\pi}{\sqrt{cx_{inf}}}\left[\frac{1}{\sqrt{\lambda_{c3} - \lambda}} + \frac{1}{\sqrt{\lambda_{c3} - \lambda + 2\varepsilon}}\right], \quad (9)$$

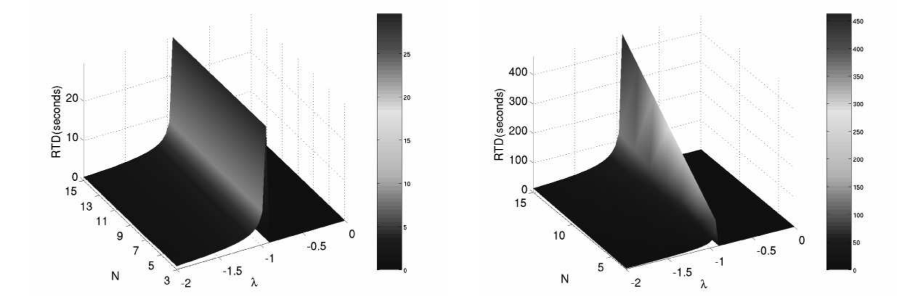
and,

$$\Delta_1 t = \frac{N\pi}{\sqrt{cx_{inf}}}\left[\frac{1}{\sqrt{\lambda_{c3} - \lambda}} - \frac{1}{\sqrt{\lambda_{c3} - \lambda + 2\varepsilon}}\right]. \quad (10)$$

Recall that $\partial\Delta t/\partial\varepsilon$ measures the sensitivity of a CCFM (in the standard orientation). It follows that

$$\frac{\partial\Delta_1 t}{\partial\varepsilon} = N\frac{\partial\Delta t}{\partial\varepsilon},$$

where Δt is the Residence Time Difference for the Standard Orientation.



Future Work

Future work with the Alternating Orientation Coupled Core Fluxgate Magnetometer has many different routes of interest. Among the most prominent/important are the following

1. Investigation of the effects of an ac external signal on the dynamics of the system.
2. Investigation of the noise one the dynamics of the system with both dc and ac external signals and in the absence of both.
3. A full comparison, in all cases, of the AO and SO orientations and the resulting dynamics.
4. Exploration of other known possible couping schemes and their effects on the sensitivity of the system.
5. Calculation of the Residence Times in the presence of Noise.

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