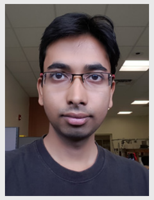
High-Order Stochastic Semi-Lagrangian Method to Solve Transport Equations



A local, explicit high order accurate semi-Lagrangian spectral element method for the solution of stochastic transport equations which model reactive turbulent flows is developed. The semi-Lagrangian method solves for the multi-variate probability density function. The high-dimensional probability distribution function (PDF) that depends on the number of species is obtained using a Monte-Carlo approach, reducing the computational cost as compared to solving the Fokker-Planck type of equations for the PDF. The method is local, highly parallel and explicit and it is therefore consistent with discontinuous Galerkin Navier-Stokes solvers that provide the velocity field for the transport of species. We achieve this by seeding Lagrangian particles at the Gauss quadrature collocation nodes within each element. The particle is advected by integrating the stochastic differential equation in time. A new interpolant is constructed from the advected nodes with a least squares method constrained by boundary conditions and mass conversation. The interpolant maps the function back to the Gauss quadrature nodes. With the stable explicit time restrictions particles cannot leave the element's bounds. The method is hence local and can be easily parallelized. The method is shown to have spectral convergence for a number of one and two dimensional test problems.

Hareshram Natarajan *and Gustaaf Jacobs*

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STOCHASTIC SEMI-LAGRANGIAN SPECTRAL ELEMENT METHOD FOR FILTERED DENSITY FUNCTION MODEL

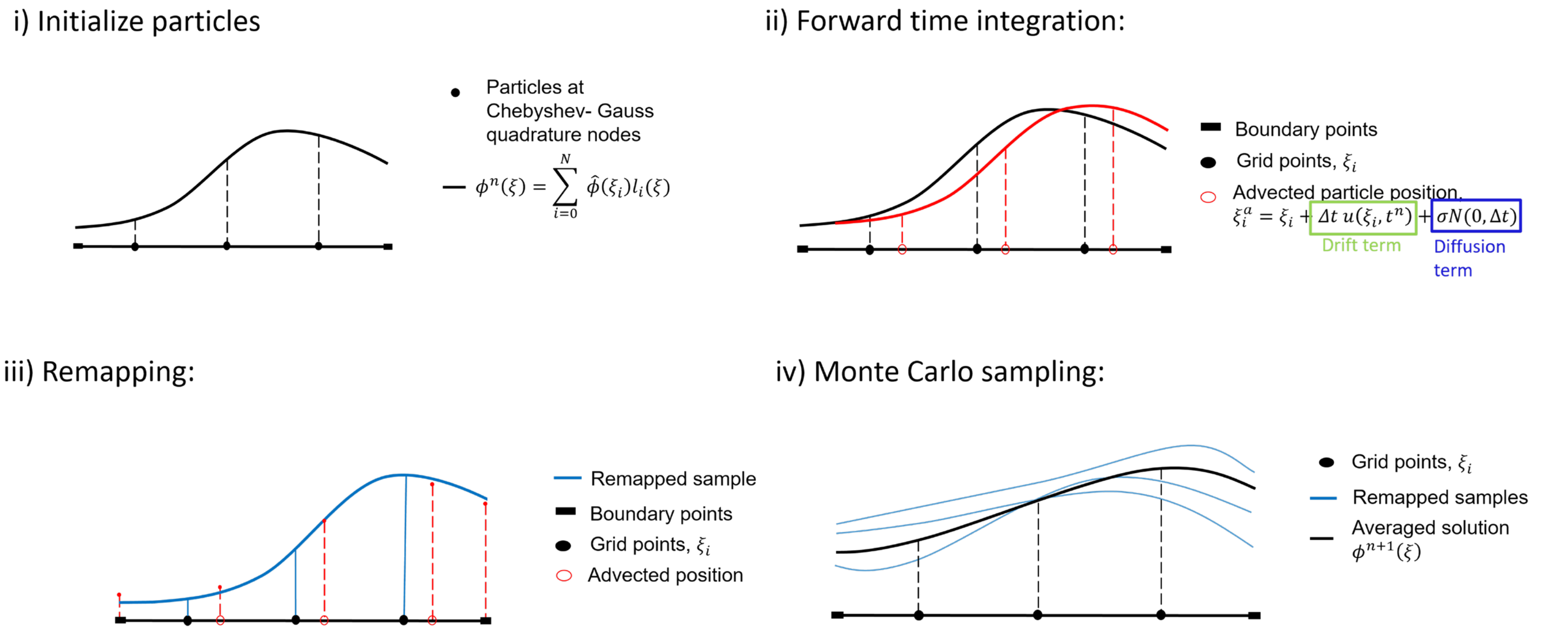

Fig 1: Schematic of the stochastic semi-Lagrangian spectral element methodology which will be used to solve chemically reactive turbulent flows.

SPECTRAL CONVERGENCE

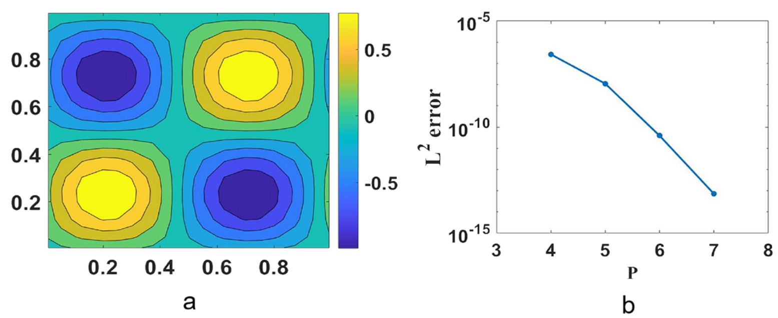

Fig 2: Simulation of a 2D advection equation using the semi-Lagrangian spectral element method (a) plots the instantaneous solution at t=1.98s for number of elements; $H_x = 4, H_y = 4$ and polynomial order $P = 4$. (b) shows spectral convergence in P

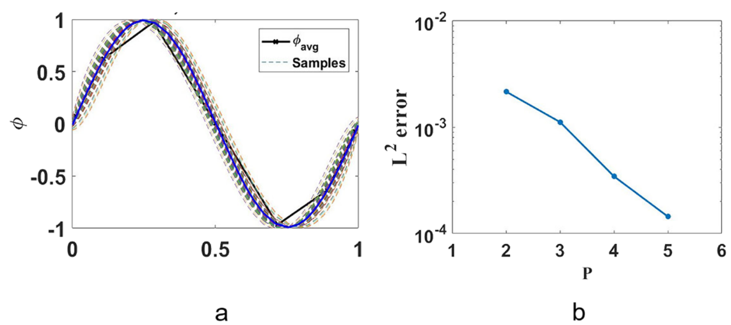

Fig 3: Simulation of a 1D diffusion using the stochastic semi-Lagrangian spectral element method (a) plots the averaged solution including the samples for $H = 1, P = 4$ and #samples=1000 at $T = 0.05$. (b) shows spectral convergence in P