Translating Quantum Microwave Wave Operators into S-parameter Formalism

Classical S-parameter techniques are advantageous in the construction of multi-port microwave networks. However, classical microwave theory is unable to describe quantum phenomena at the single-photon level. One strategy to describe quantum effects in microwave systems is to perform a quantum input-output network (QION) theory treatment. Another formalism, called SLH theory, incorporates bosonic scattering matrices (S), a coupling vector (L), and a system Hamiltonian (H). We introduce a translation between SLH theory and classical power waves which can retain classical S-parameter intuition. A reformulation of quantum field operators in quantum optics to microwave operators is required. We propose the modification of traditional quantum field operators in the frequency and time domain needed to retain units power waves. This suggests a direct relationship between classical scattering parameters and SLH scattering matrices, and, therefore, a possible extension to signal flow diagrams for quantum networks. To emphasize this result for classical microwave engineers, we translated classical field power waves into QION field operators and modified QION field operators.

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	Microwave Engineering $b(t), b(\omega)$	Quantum Field Operators $\hat{b}_k, \hat{b}(t), \hat{b}(\omega)$	Modified Quantum Field Operators $\hat{\mathbf{b}}_k, \hat{\mathbf{b}}(t), \hat{\mathbf{b}}(\omega)$
Quantized Field Operator $\hat{b}_k, \hat{\mathbf{b}}_k$ (Number Basis)		$\sqrt{\hat{N}_k}\exp\{-i\hat{\phi}_k\}$	$\sqrt{\hat{N}_k}\exp\{-i\hat{\phi}_k\}$
Quantized Field Operator $\hat{b}_k, \hat{\mathbf{b}}_k$ (Phasor Notation)		$\sqrt{\frac{C'\ell}{2\hbar\omega_k}}\hat{V}_0^+$	$\sqrt{\frac{C'\ell}{2\hbar\omega_k}}\hat{V}_0^+$
Frequency Domain $b(\omega), \hat{b}(\omega), \hat{\mathbf{b}}(\omega)$	$\frac{V_0^+(\omega)}{\sqrt{Z_0}}$	$\sqrt{2\pi\frac{v_p}{\ell}}\sum_k \hat{b}_k\delta(\omega-\omega_k)$	$\sqrt{2\pi\hbar\Omega\frac{v_p}{\ell}}\sum_k \hat{b}_k\delta(\omega-\omega_k)$
Time Domain $b(t), \hat{b}(t), \hat{\mathbf{b}}(t)$	$\frac{1}{\sqrt{2\pi}}\int\frac{V_0^+(\omega)\exp\{-i\omega t\}}{\sqrt{Z_0}}d\omega$	$\sqrt{\frac{v_p}{\ell}}\sum_k \hat{b}_k\exp\{-i\omega_k t\}$	$\sqrt{\hbar\Omega\frac{v_p}{\ell}}\sum_k \hat{b}_k\exp\{-i\omega_k t\}$

Table 1: The translation of generalized scattering parameters in microwave engineering language to bosonic and modified bosonic operators in a quantized transmission line. In the frequency domain, classical scattering operators describe $\sqrt{\text{Power}}$ at a given frequency ω. However, in traditional QION theory, bosonic waves are normalized such that the scattering (bosonic) field operator is in units of $\sqrt{\text{Hz}}$. Therefore, we propose a modification of frequency and time quantum bosonic field operators by $\sqrt{\hbar\Omega}$ such that bosonic field operators in the time domain to have the same units of classical power waves, i.e. $\sqrt{\text{Power}}$. The number operator at the kth mode is given as $\hat{N}_k$, the phase operator at the kth wave mode is given as $\hat{\phi}_k$ on the interval $[0, 2\pi]$, the drive frequency is given as Ω, ℓ is the length of a transmission line, the characteristic impedance of the transmission line is Z_0, and the phase velocity given by v_p.